

# *LECTURE 12*

## *LINEARIZATION AND EXPONENTIAL FUNCTION*

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# PLAN

1. Poll about exponentials

2. how to define  $\exp(x)$

3. experiments

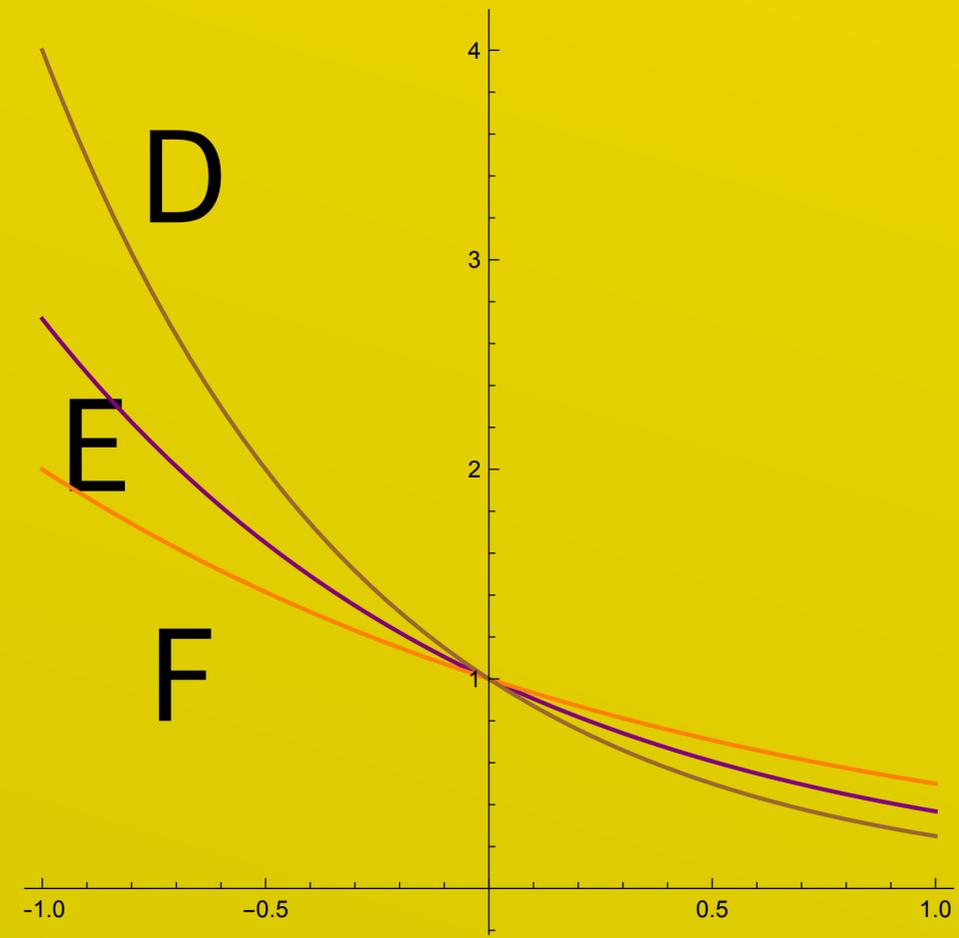
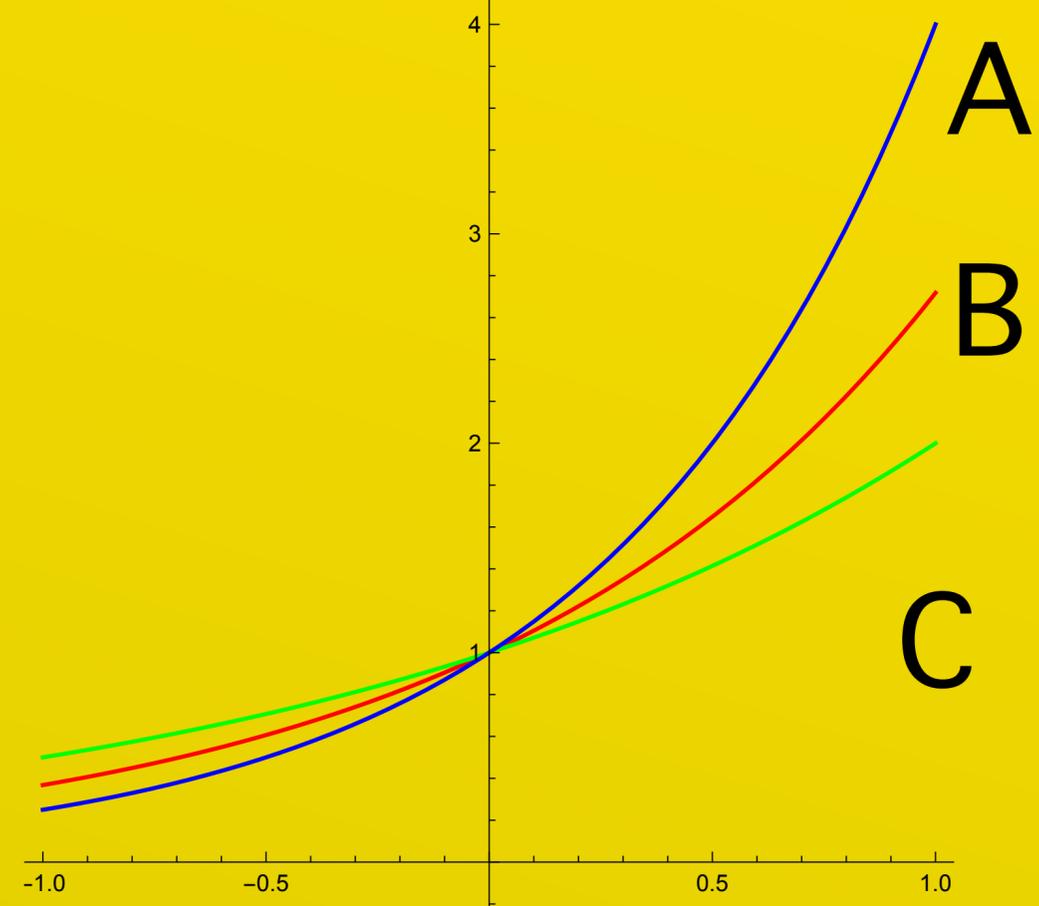
4.  $f'(x) = f(x)$

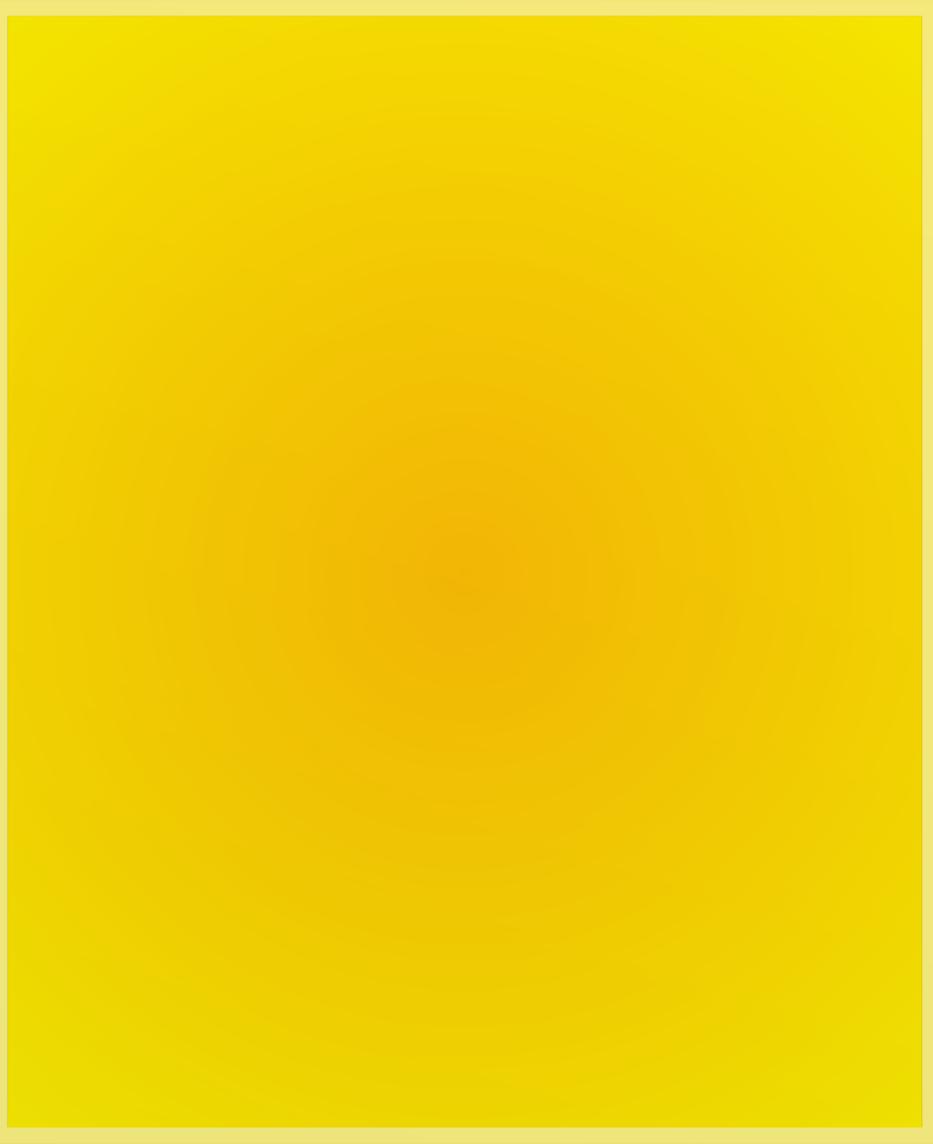
5. linear approximation

6. Jam

7. Jam

*POLL*

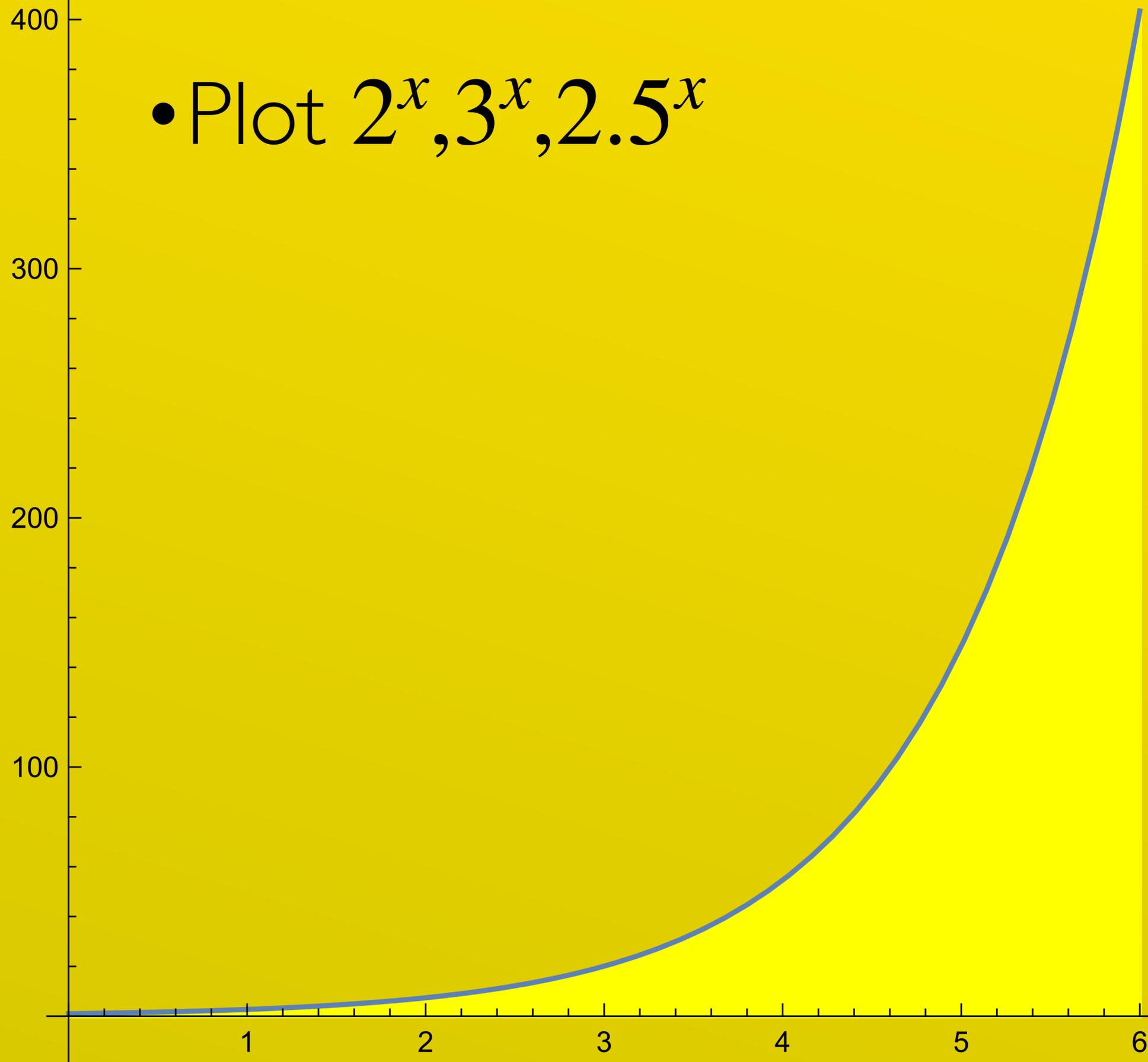




*PART 1*

*REVIEW*

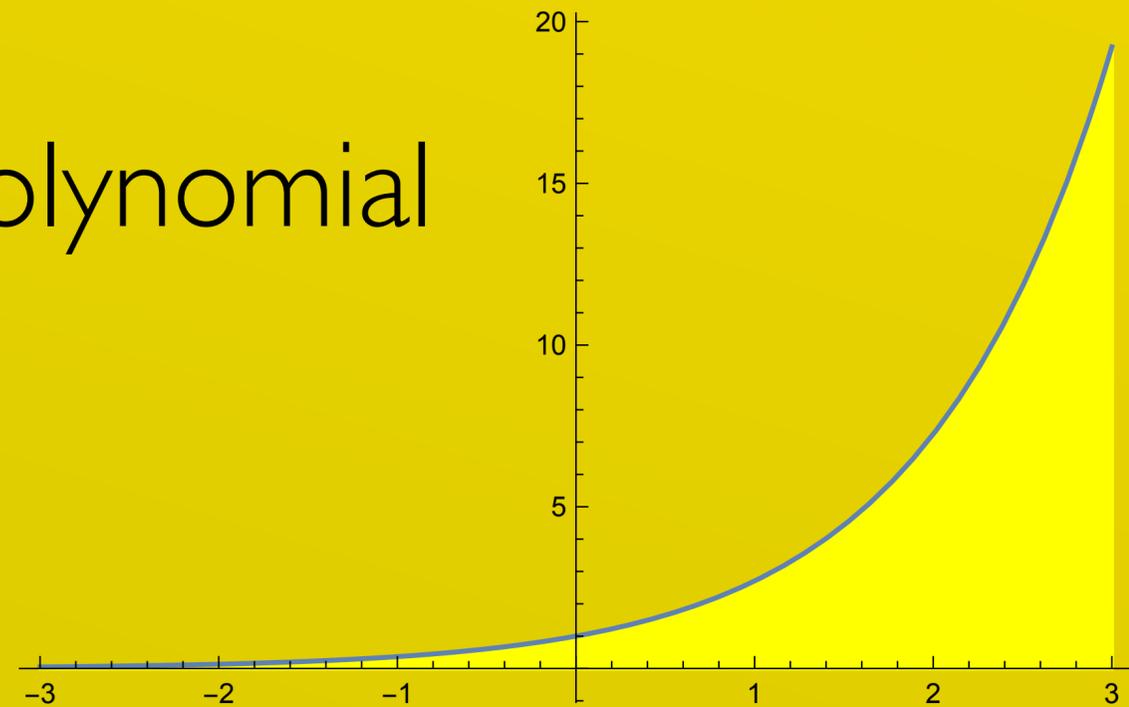
• Plot  $2^x, 3^x, 2.5^x$



# *USUAL DEFINITION*

$$f(x) = \left(1 + \frac{x}{n}\right)^n$$

This is a polynomial



$$\left(1 + \frac{1}{1000000}\right)^{1000000} = 2.7182804693193768838$$

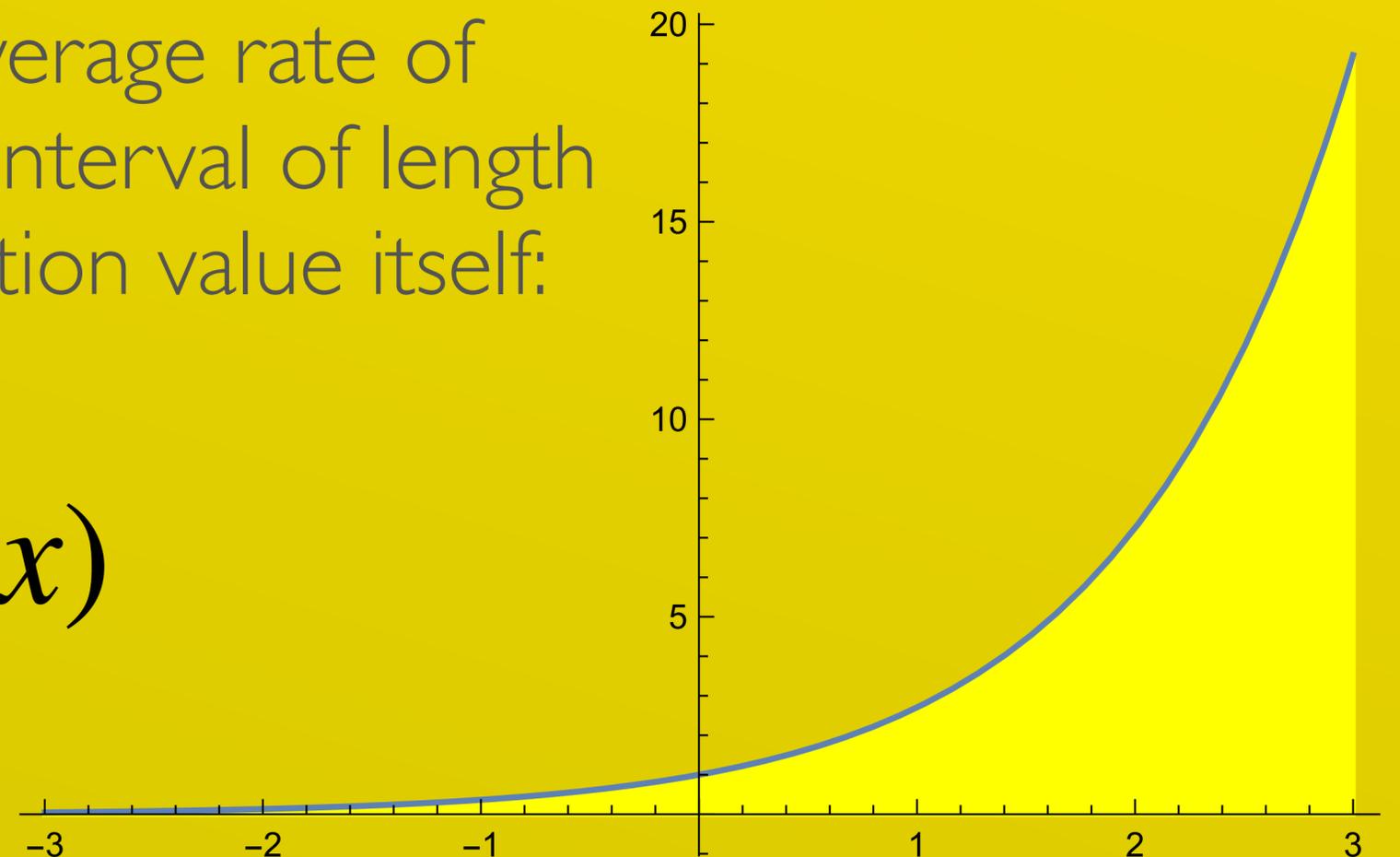
# COMPOUND INTEREST

$$f(x) = (1 + h)^{x/h}$$

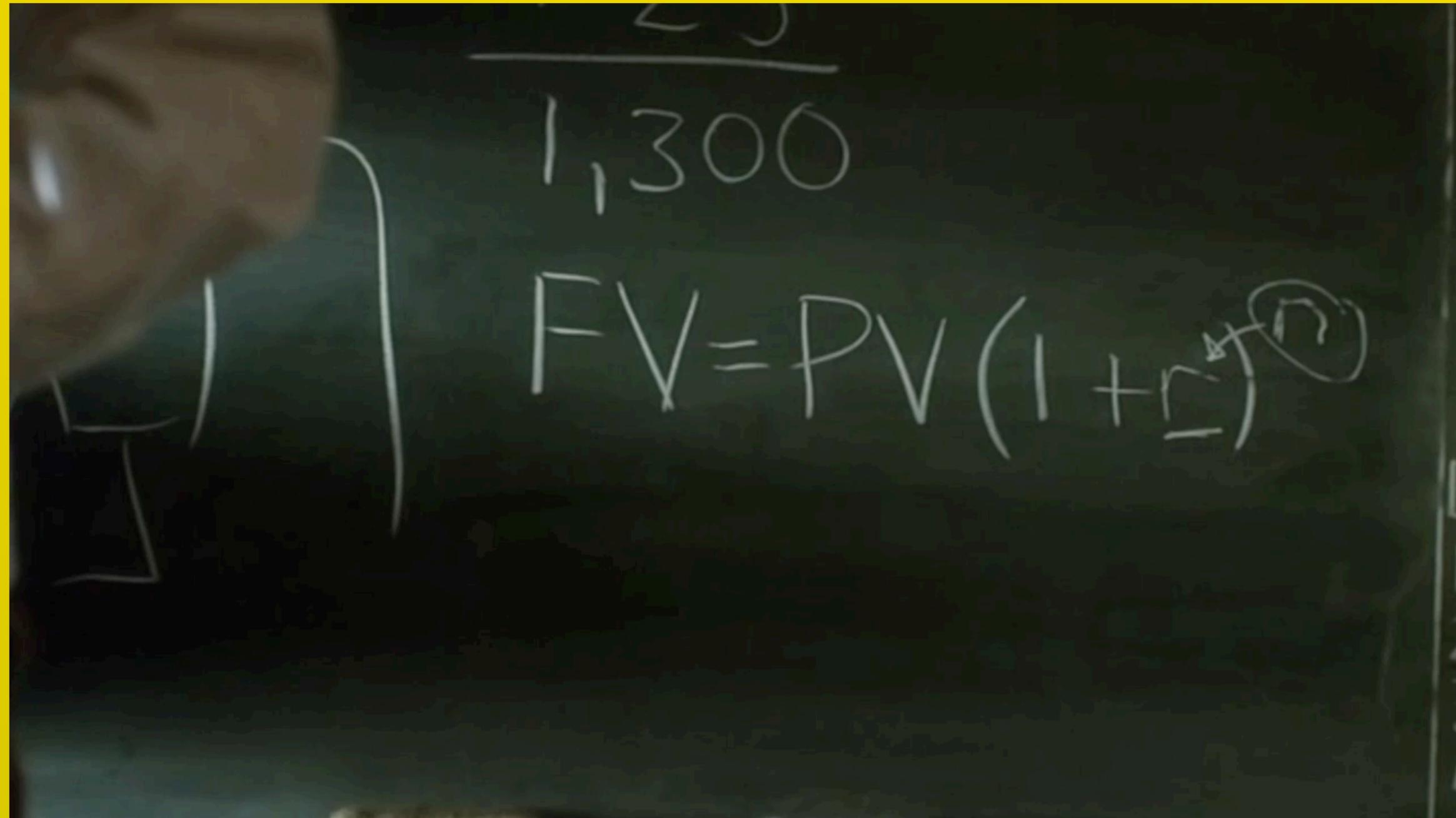
$n$

This function has the property that the average rate of change on an interval of length  $h$  is the function value itself:

$$f(x + h) - f(x) = hf(x)$$



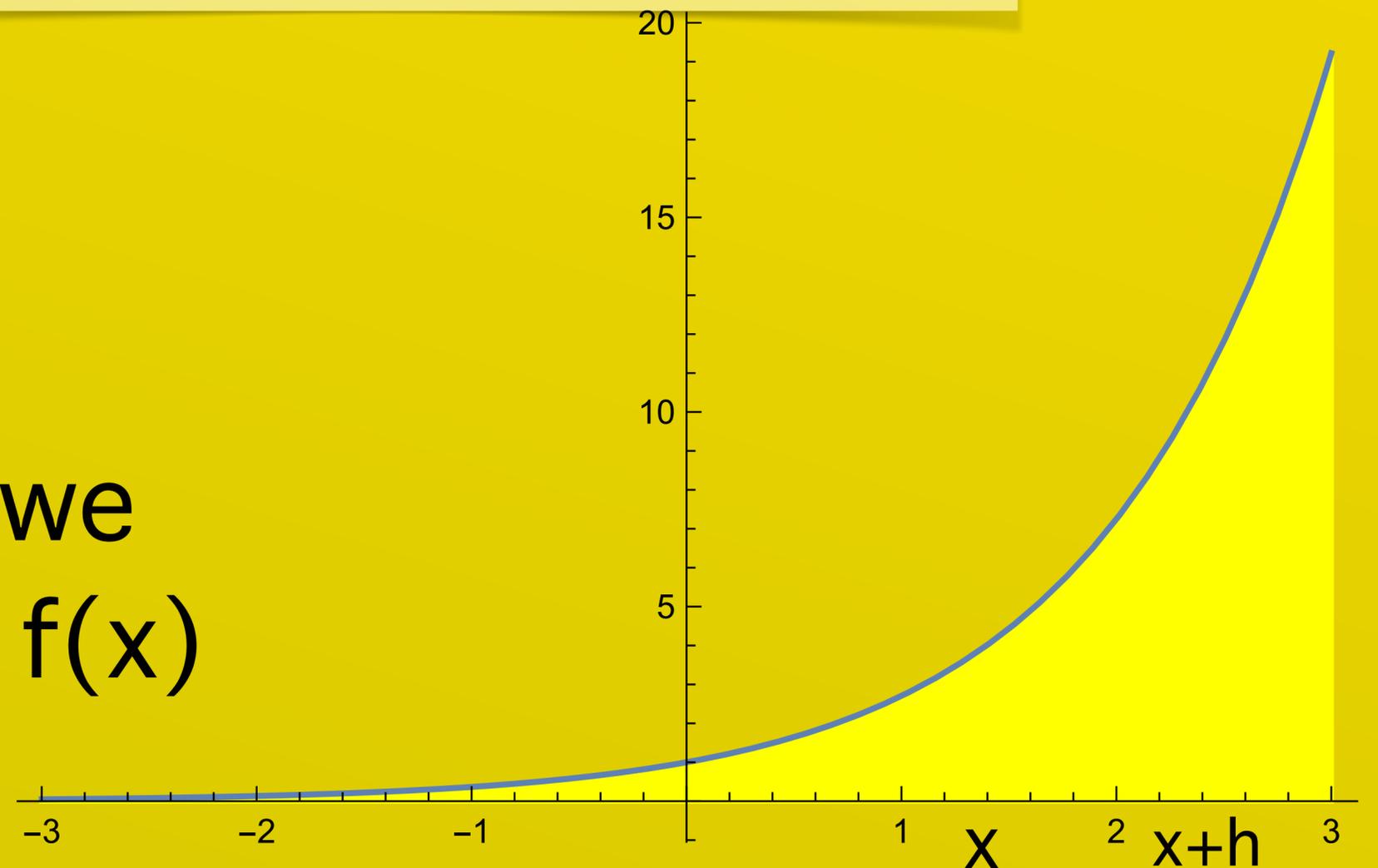
# COMPOUND INTEREST

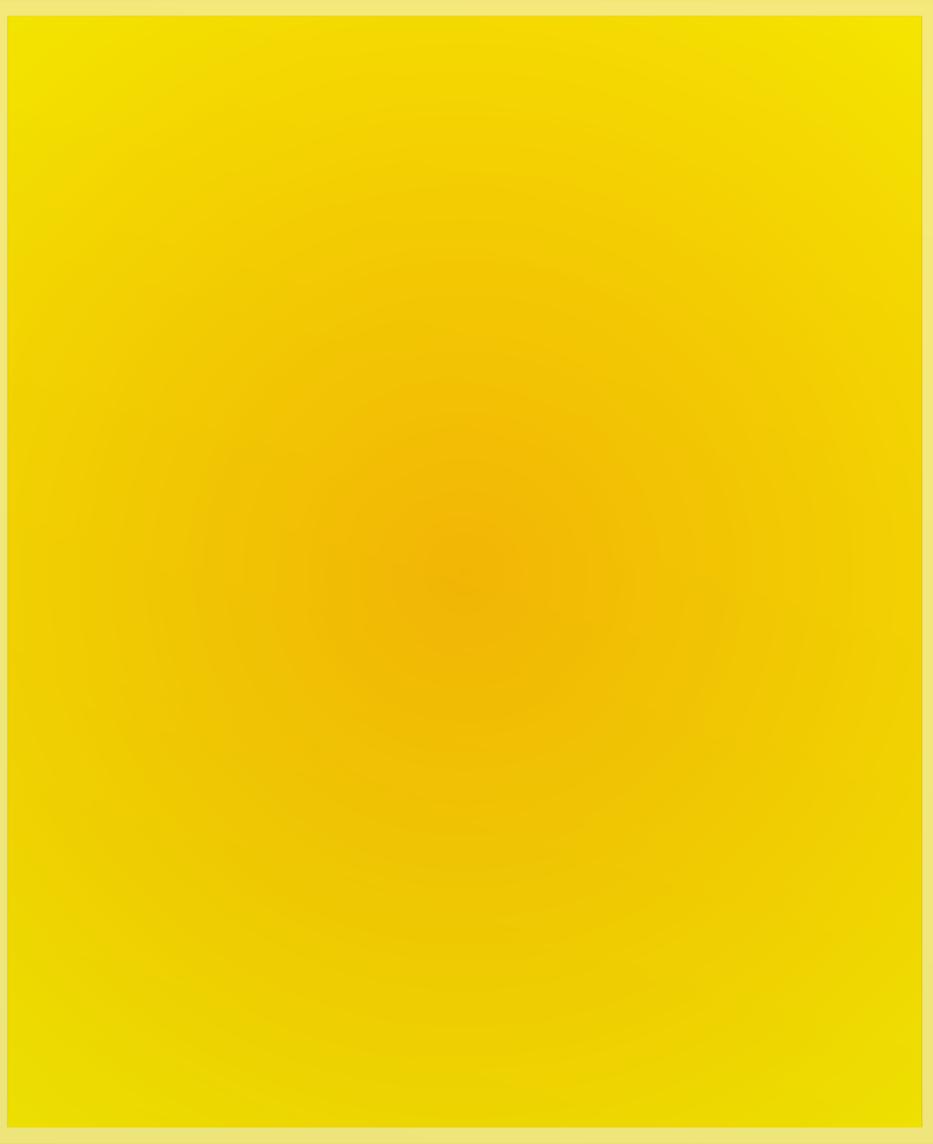


# FOR EXPONENTIAL

$$f(x+h) \sim f(x) + h f'(x)$$

for  $f(x) = (1+h)^{x/h}$  we  
have  $f(x+h) = f(x) + h f'(x)$





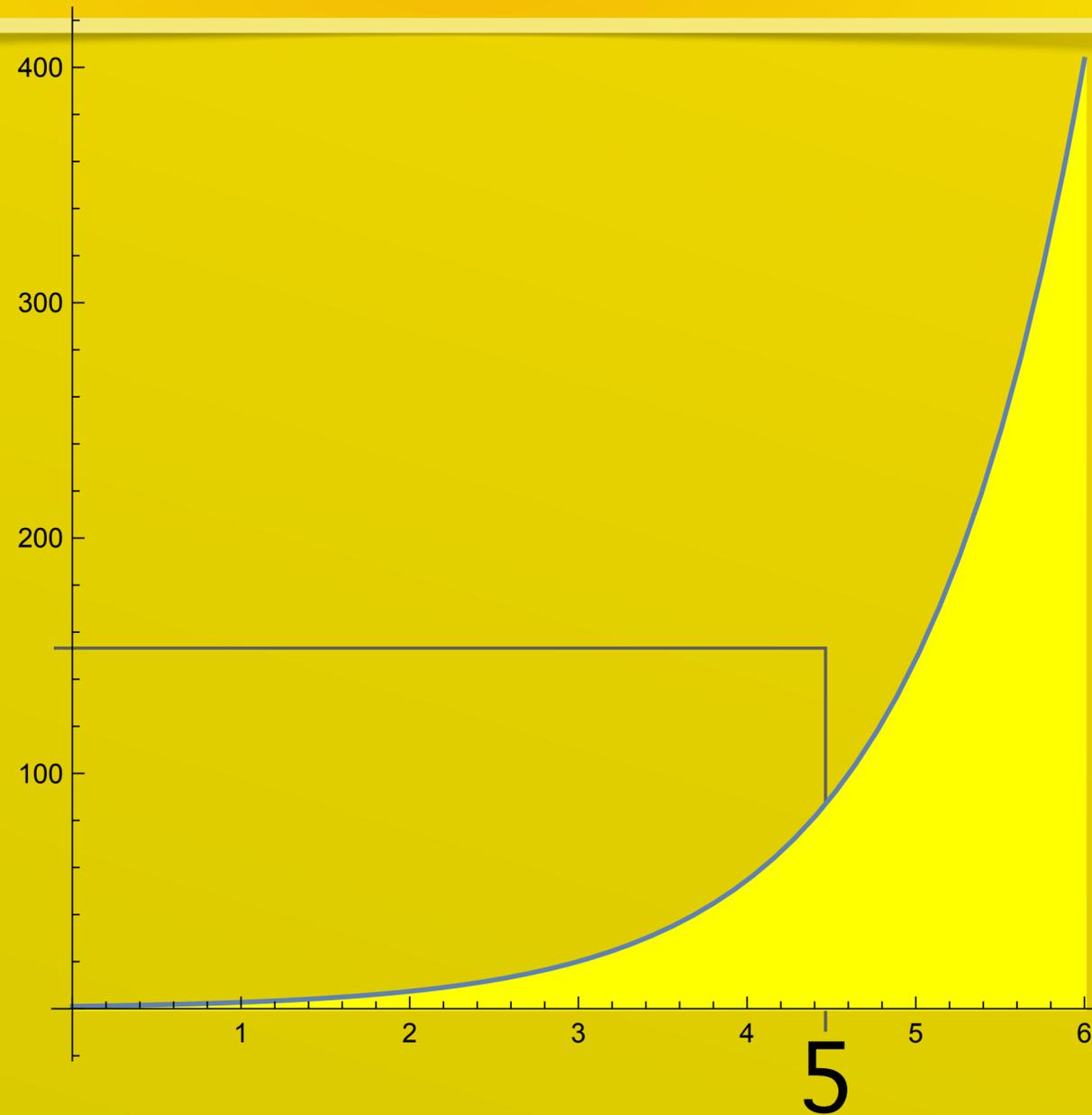
*PART II*

# *ESTIMATE*

estimate

`exp(5.1)`

150

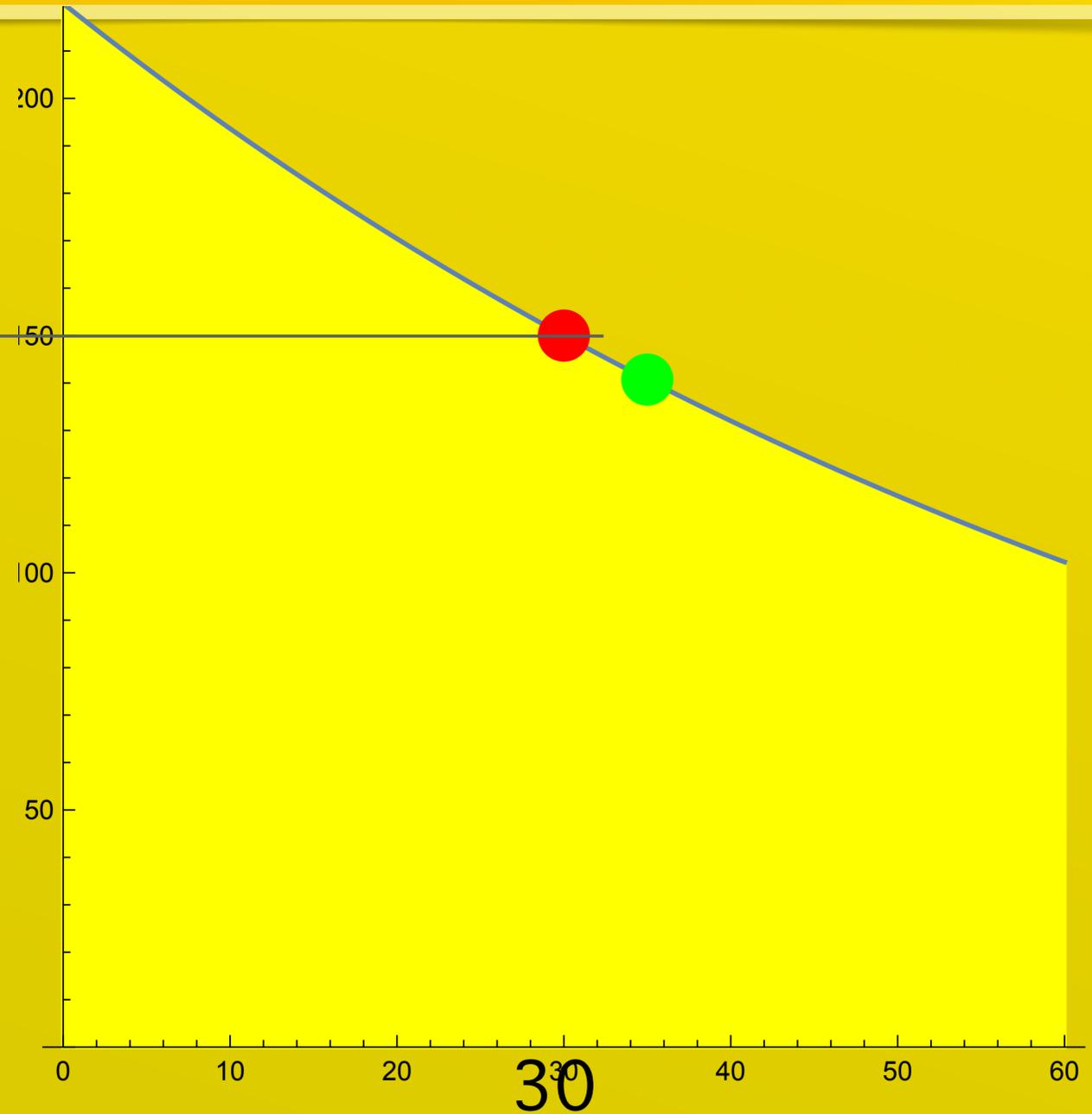


# COFFEE PROBLEM

estimate the coffee temperature  
at 8:35

150

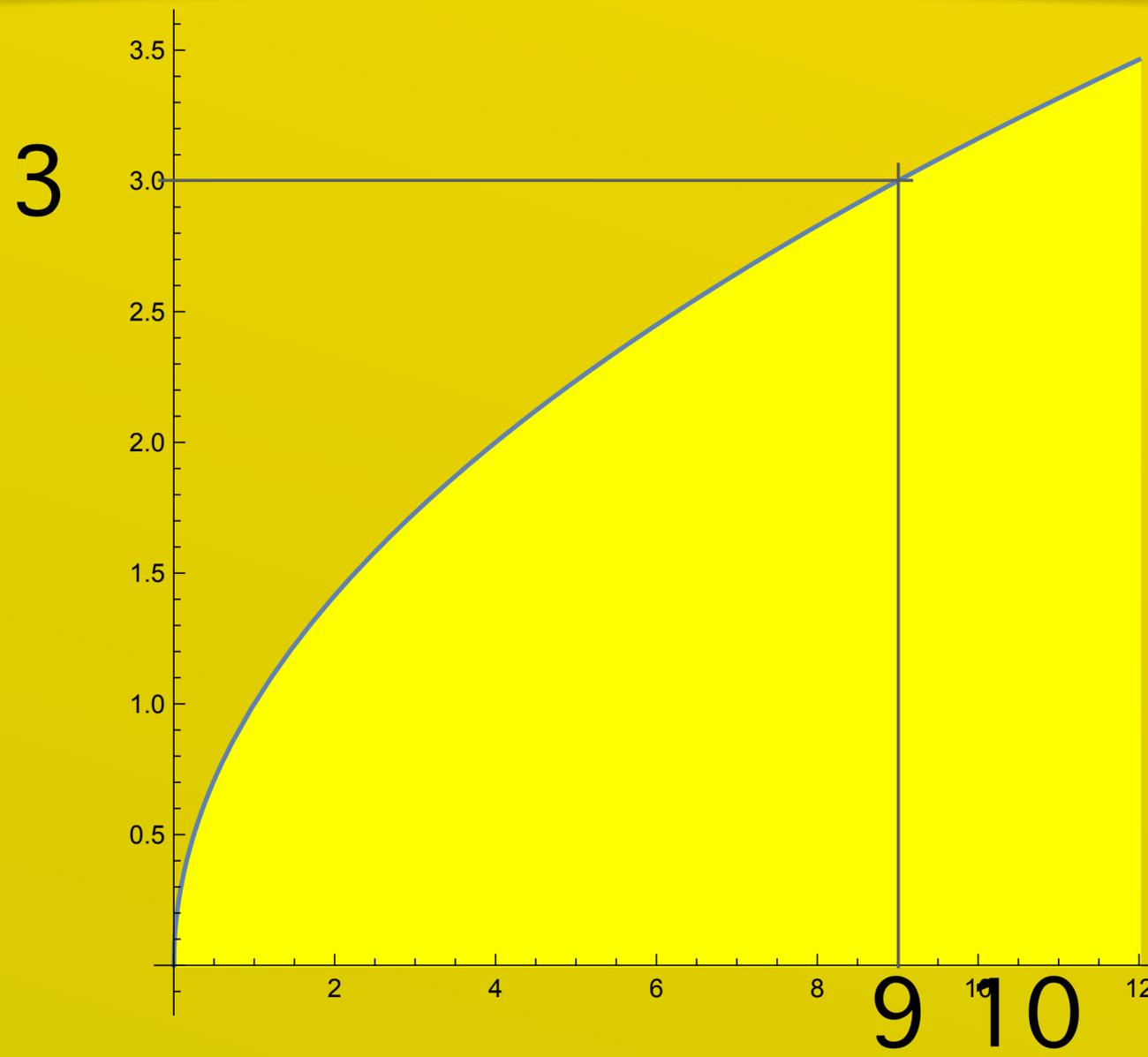
$$f'(30) = -2$$



# LINEARIZATION

$$f(x+h) \sim f(x) + h f'(x)$$

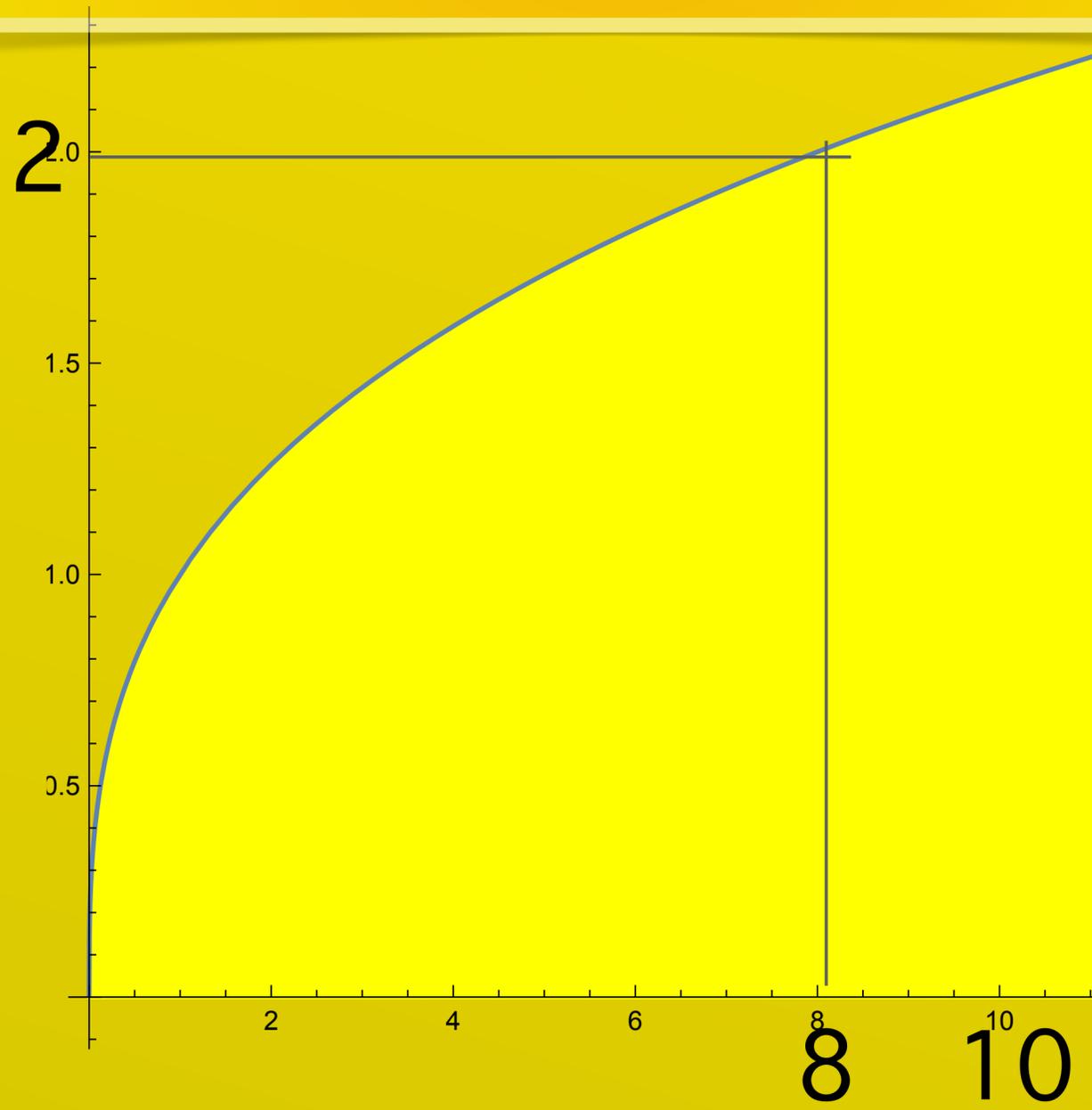
How to estimate  
the square root of 10?



# LINEARIZATION

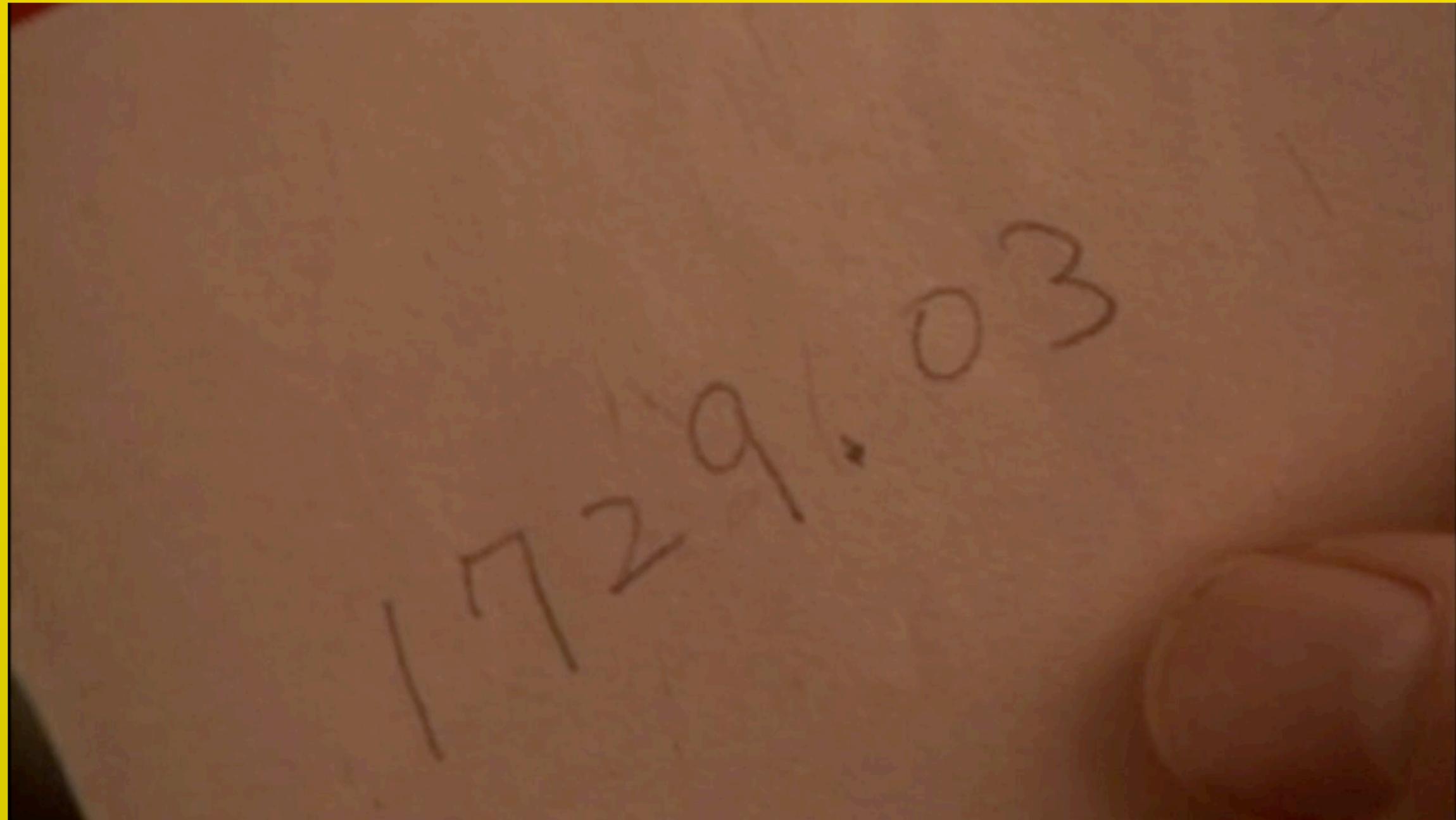
$$f(x+h) \sim f(x) + h f'(x)$$

How to estimate  
the cube root of 10?



*WORK ON PAPER*

# FEYNMAN



Movie Infinity 1996

# *JAM*

Compute the cube root of 1003  
using linearization  $f(x) = f(a) + f'(a)(x-a)$

$$f(x) = x^{1/3}$$

# CAJAM

$$f(x) = 2e^x - 6x - e^\pi$$

Draw the graph of  $f$ .

Does  $f$  have a maximum or minimum?

*THE END*