LECTURE 13

MAXIMA AND MINIMA

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PLAN

1. Poll about reading

2. local and global max and min

3. critical points

4. singular points

5.First derivative test

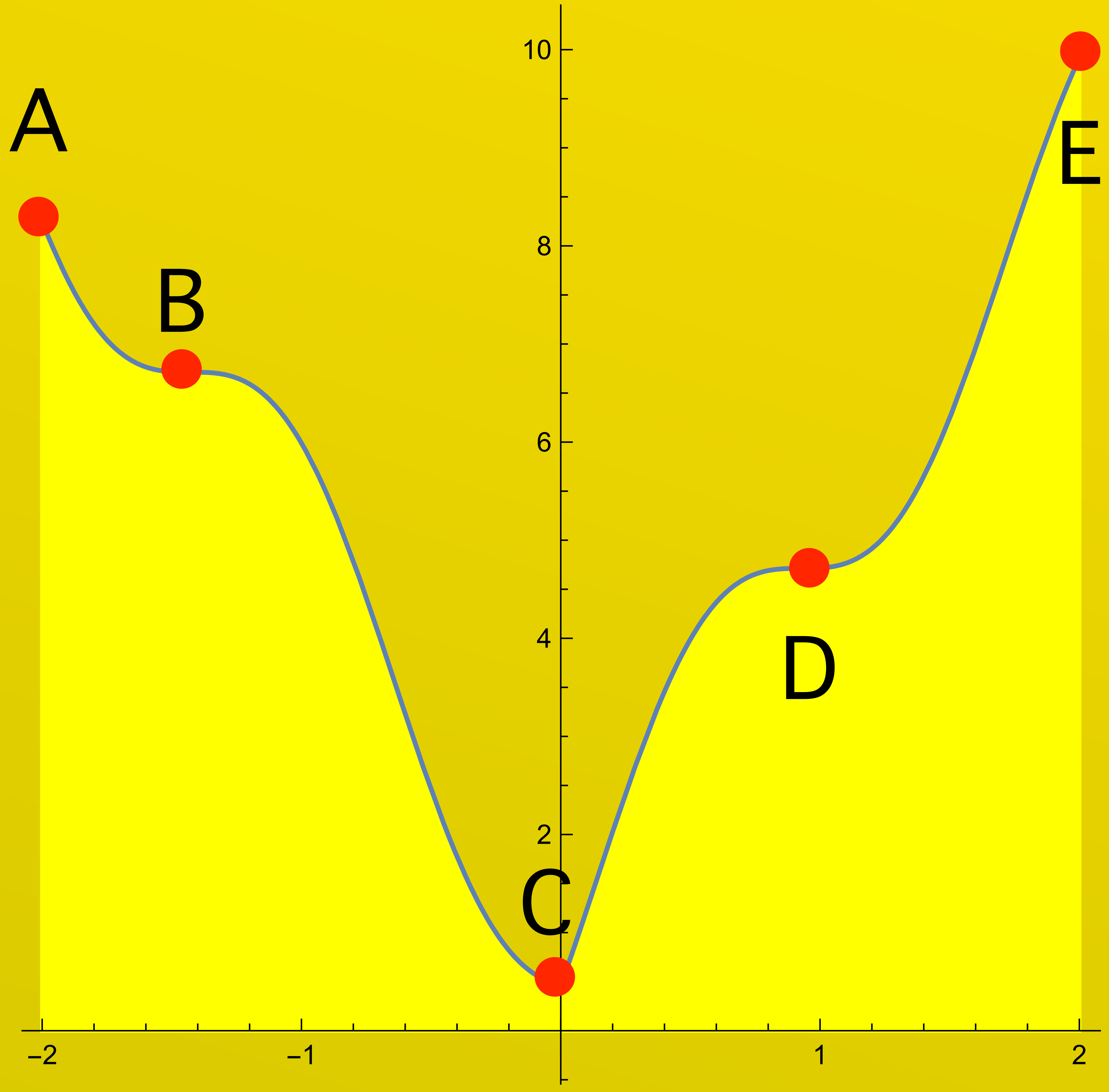
6. Second Derivative test

7. Jam

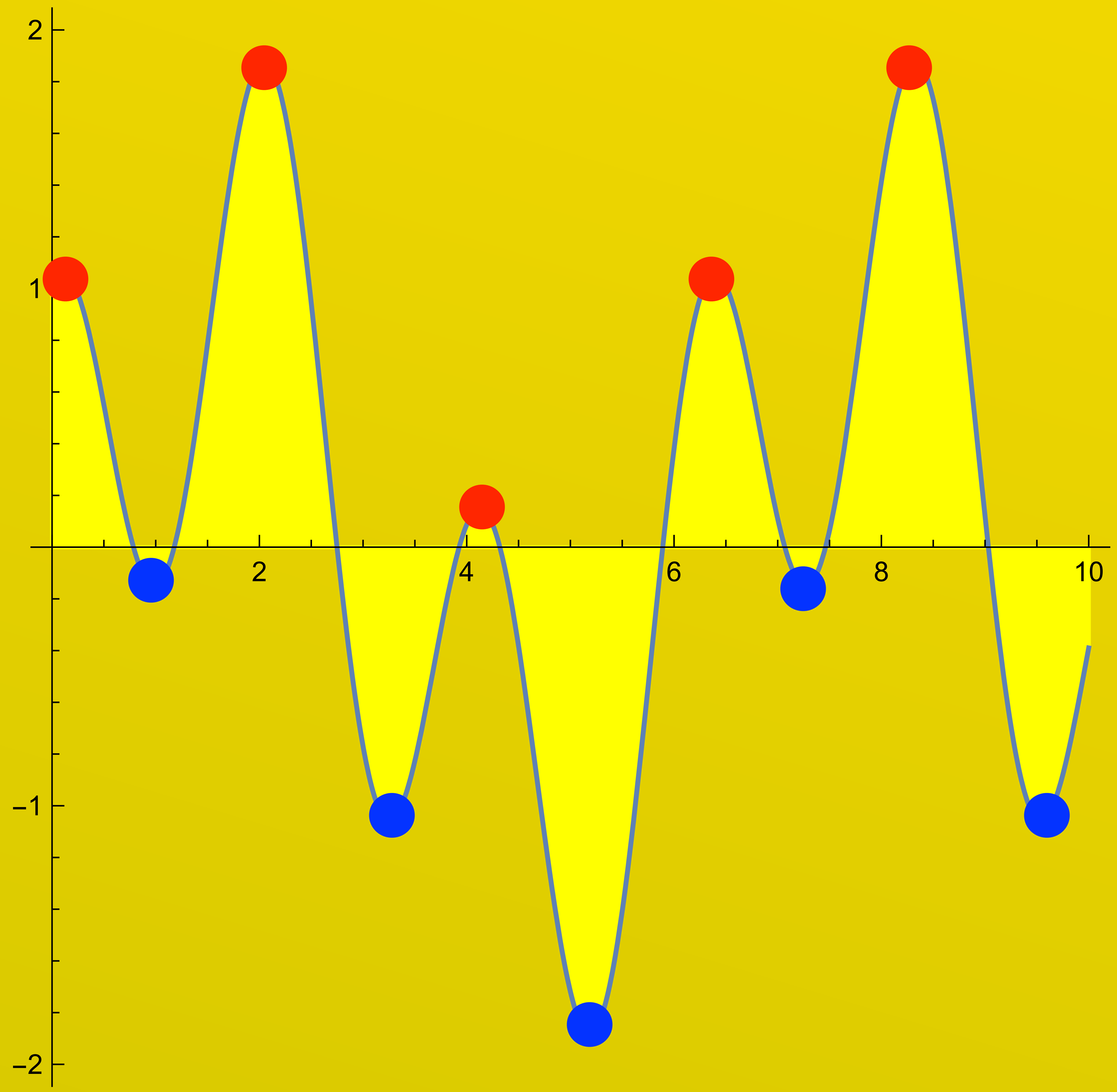
8. Dessert

POLL

It is important whether we specify [-2,2] or (-2,2) as the domain. On (-2,2), there does need to exist a global max or global min.



LOCAL MAX AND MIN



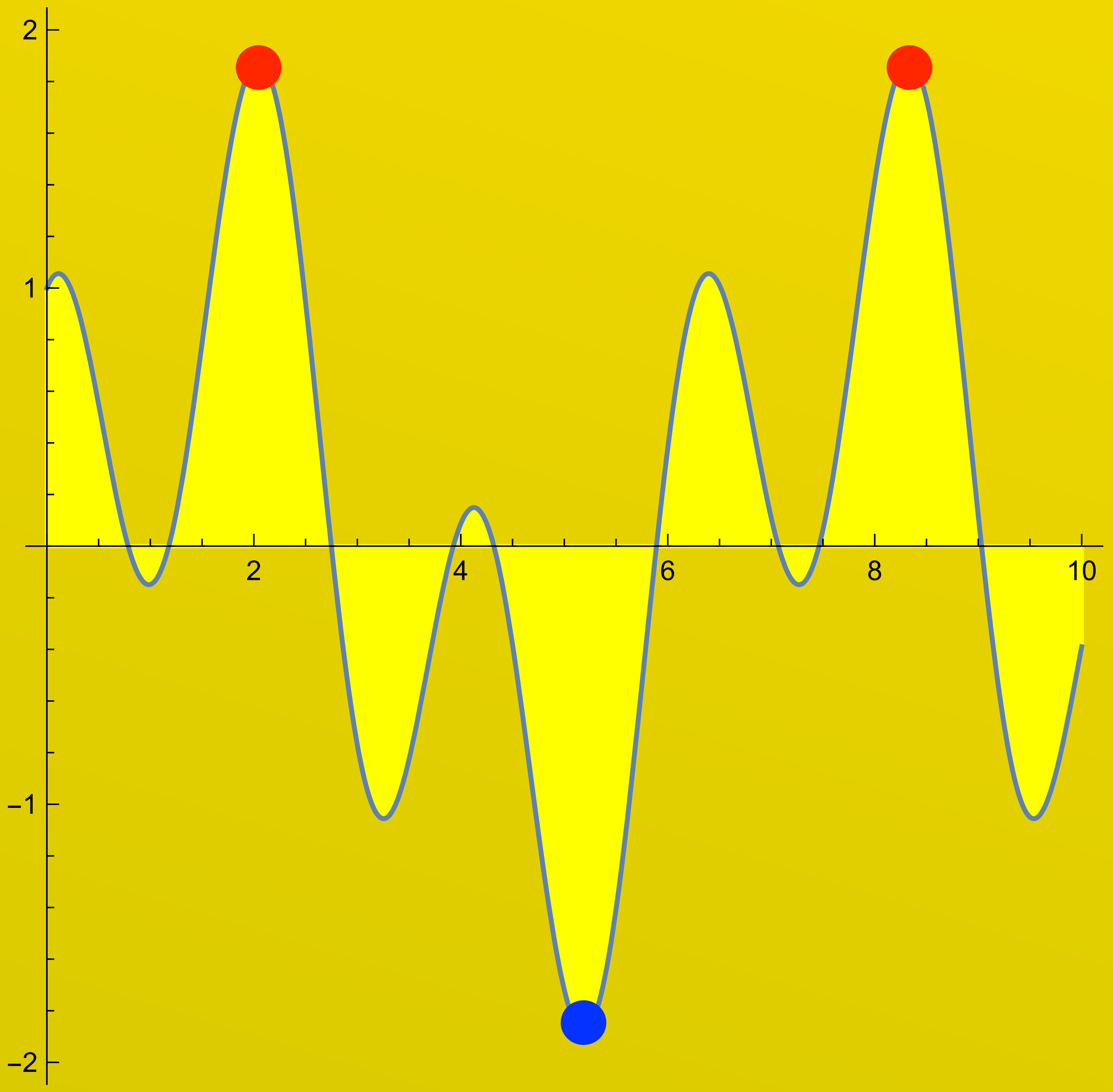
local min

$f(x) \leq f(y)$ for y near x

local max

$f(x) \geq f(y)$ for y near x

GLOBAL MAX AND MIN



- global min

$f(x) \leq f(y)$ for all y in [0,10]

- global max

$f(x) \geq f(y)$ for y in [0,10]

CRITICAL POINTS

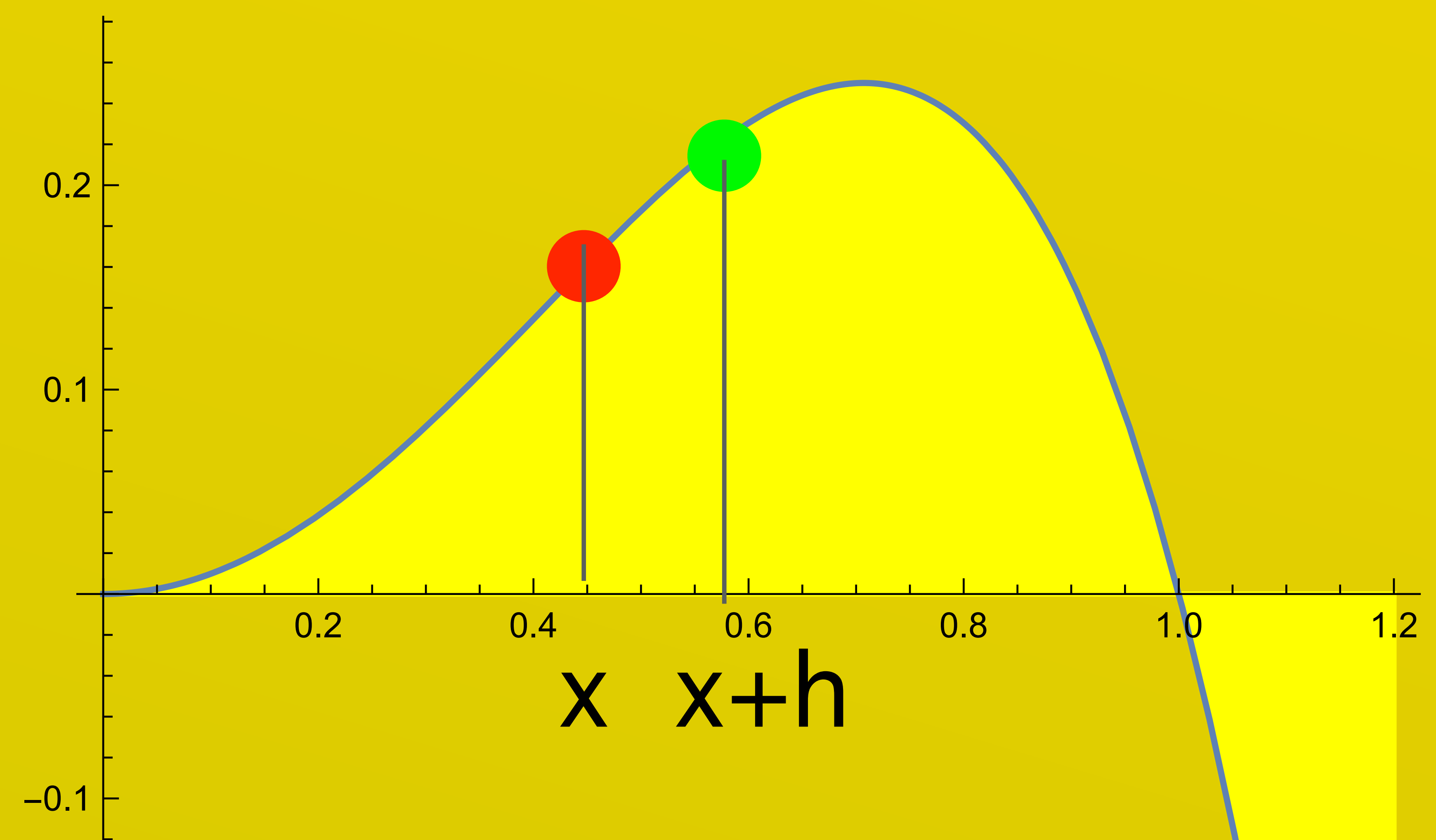
f differentiable near x, and x is local max or local min, then

$$f'(x) = 0$$

One calls this a **critical point.**

Argument of Fermat: assume x is max

and $f'(x) > 0$

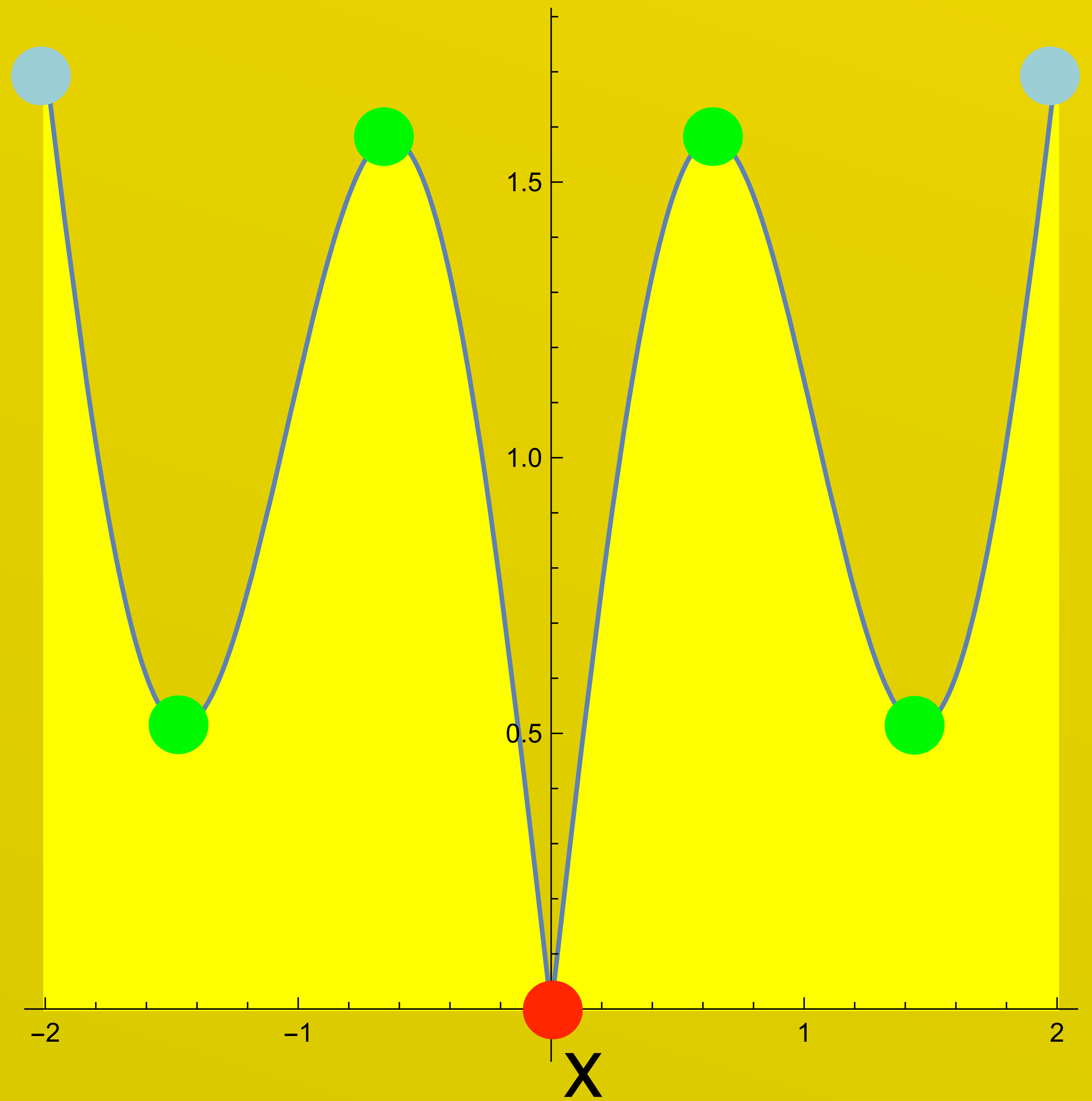


Then [f(x+h)-f(x)]/h >0 for small h and so f(x+h)>f(x) nearby. Similarly for f'(x)<0.

SINGULAR POINTS

If f is not continuous or differentiable at x,
we call it a singular point.

Singular points are candidates for local maxima
or local minima.



singular point

critical point

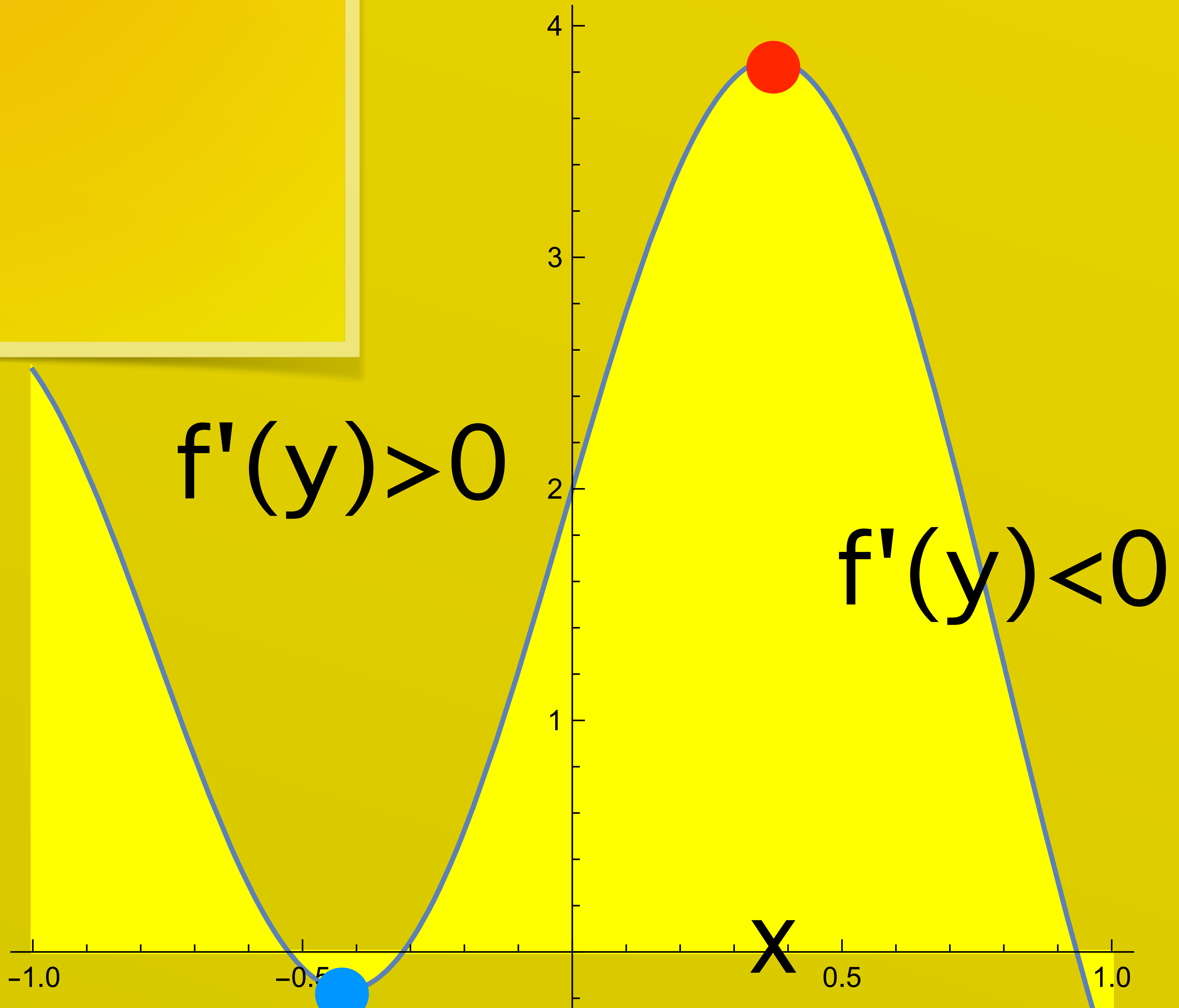
boundary point

We will see in the next class
that boundary points
of the interval on which on
looks for maxima or minima
are candiates too.

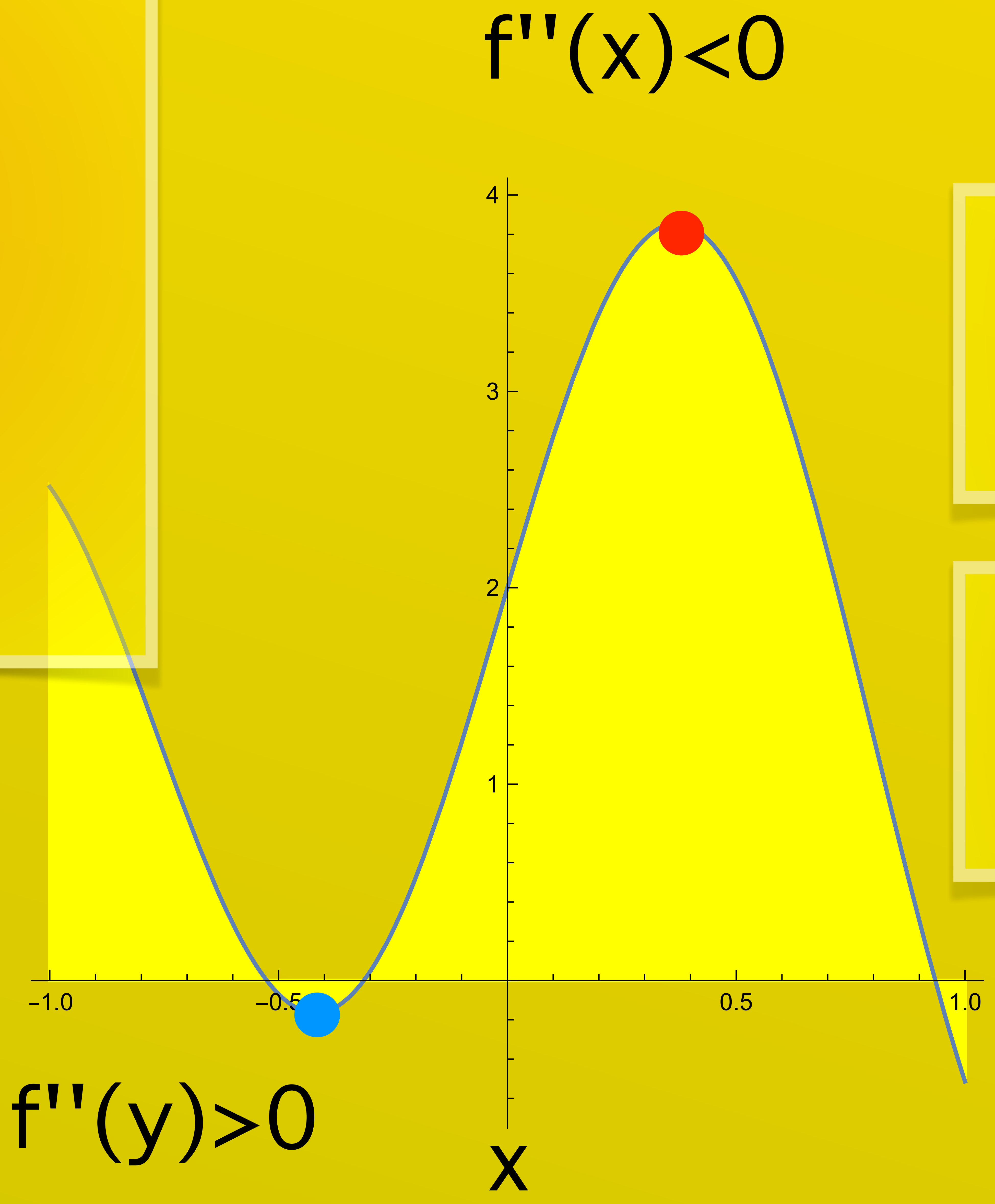
FIRST DERIVATIVE TEST

If f'(y)>0 for y<x and
f'(y)<0 for y>x,
then x is a local maximum

If f'(y)<0 for y<x and
f'(y)>0 for y>x,
then x is a local minimum



SECOND DERIVATIVE TEST



If f'(x)=0 and f''(y)<0
then x is a local maximum

If f'(x)=0 and f''(y)>0
then x is a local minimum

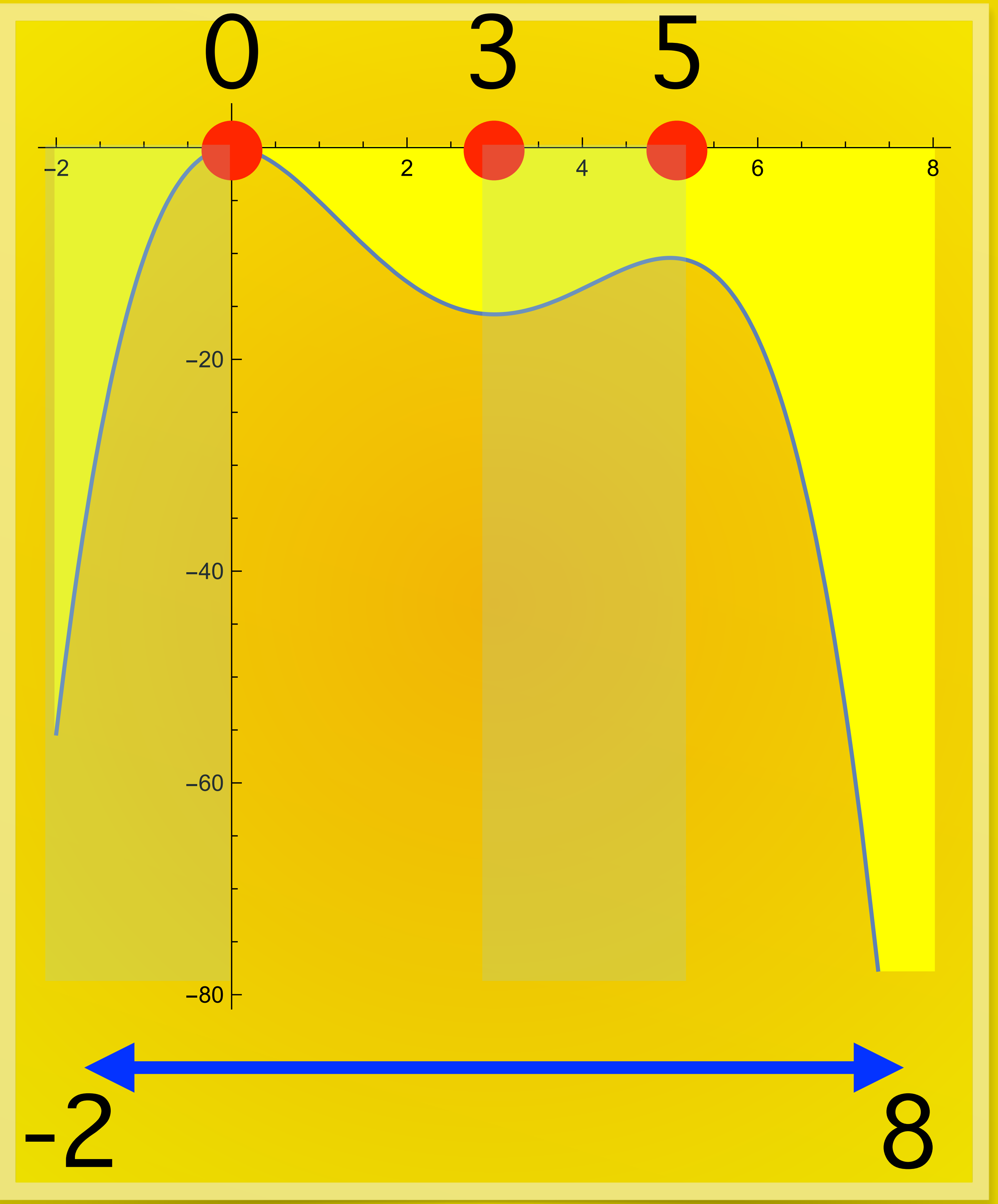
JAM

A

Classify the critical points of $f(x) = 15x - 5e^x$ using the second derivative test

B

Classify the critical points of $f(x) = 3t^4 - 4t^3$ using the first derivative test



Assume f is differentiable and f' is zero on 0,3,5, positive for x<0 and for x in (3,5) and negative else. See an example to the left.

a) does f necessarily have a global minimum?

b) can f have a global minimum?

c) does f necessarily have a global maximum?

d) can f have a global maximum?

Answer the same questions on [-2,8]

CA
JAM

THE END