

# *LECTURE 14*

## *EXTREMAL VALUE THEOREM*

Oliver Knill, Harvard University

October 7 2020

# PLAN

1. Poll

2. Review

3. Extremal Value Theorem

4. Examples

7. Jam

8. Dessert

*POLL*

For how many of the following functions does the extremal value theorem apply on the interval  $[-3,3]$  ?

$$f(x) = \log|x|$$

$$f(x) = \exp(|x|)$$

$$f(x) = |x|^{1/2}$$

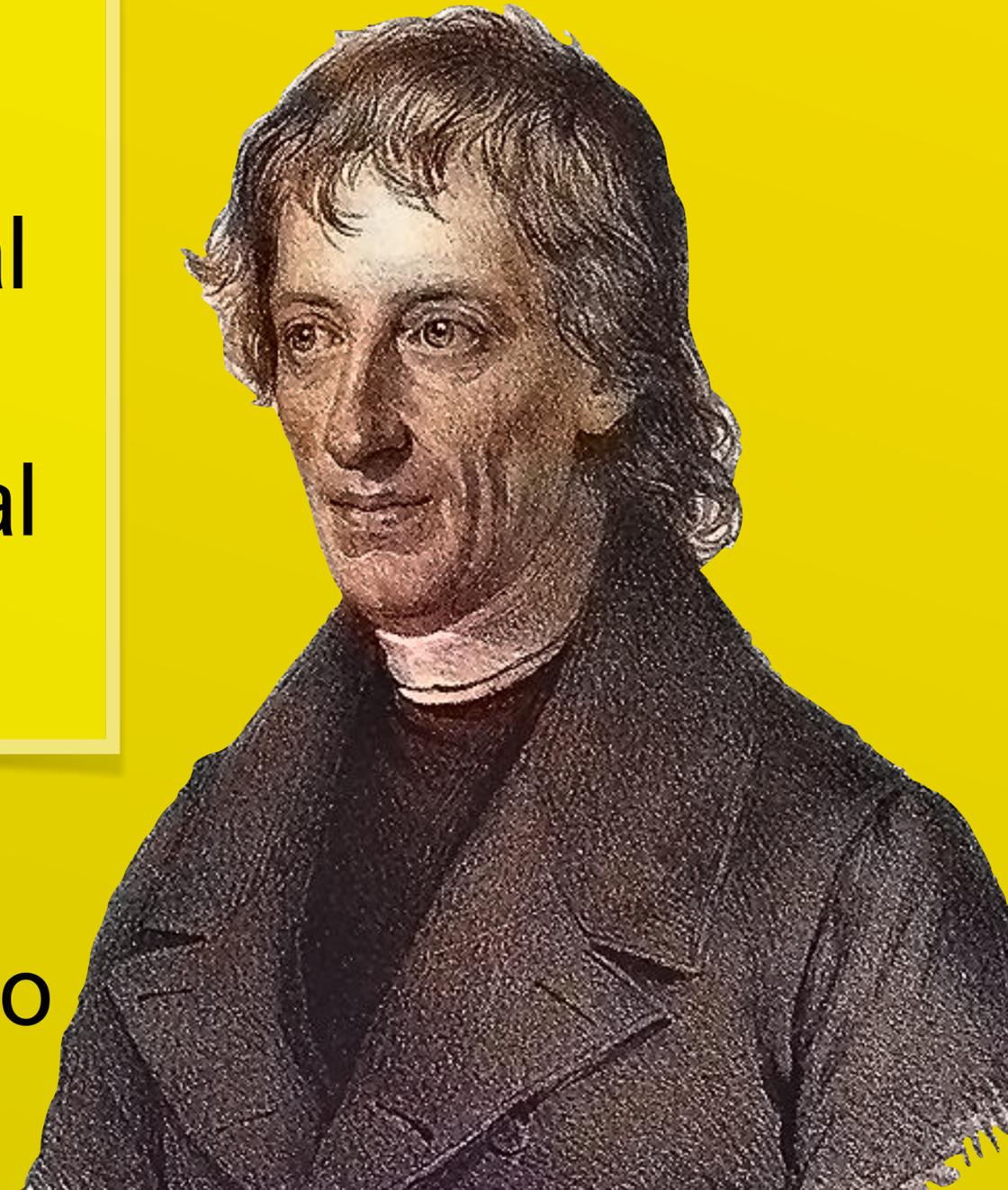
$$f(x) = |x|^{-1/2}$$

$$f(x) = \sin|x|$$

# *BOLZANO THEOREM*

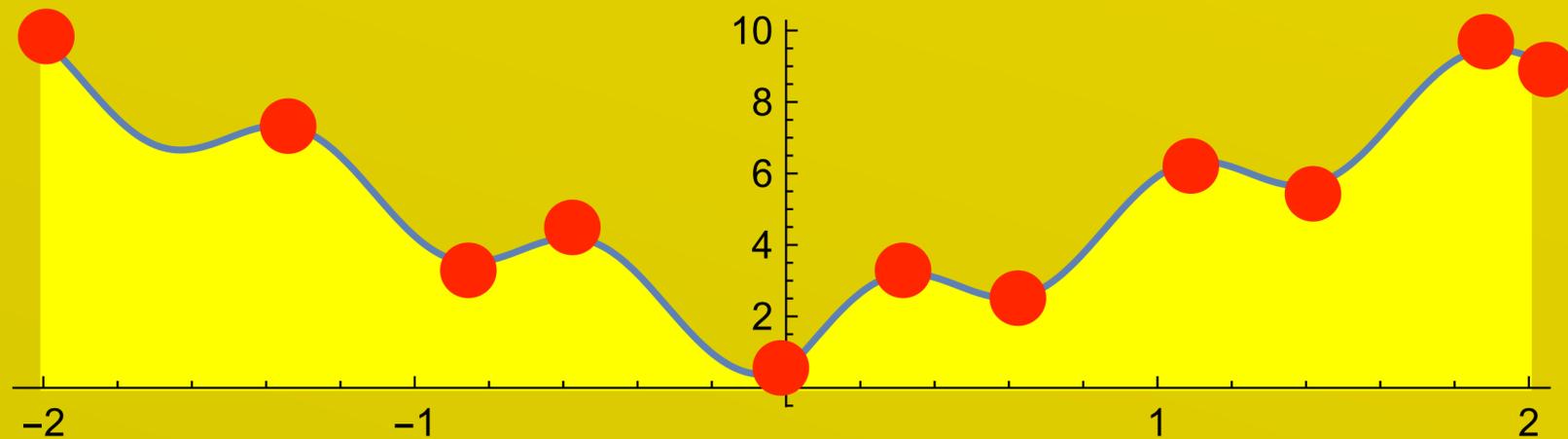
**Theorem:** A continuous function on a compact interval has both a global maximum and a global minimum.

Bernard Bolzano  
1781-1848



# *CLOSED INTERVAL METHOD*

To find a global  
max on  $[a,b]$ ,  
list all critical  
points and  
boundary points



# *MAXIMAL AREA*

$$2-x$$



$x$

$x$

$$2-x$$

What rectangle of total circumference has maximal area?

Which interval of  $x$  values are possible?

# *PROBLEM*

Find the global  
max and min of

$$f(x) = x + 1/x$$

on  $[1/3, 3]$

- a) Does Bolzano apply on  $[-3, 3]$ ?
- b) Does Bolzano apply on  $(1/3, 3)$ ?











# *CHICKEN PROBLEM*

$x$

Area

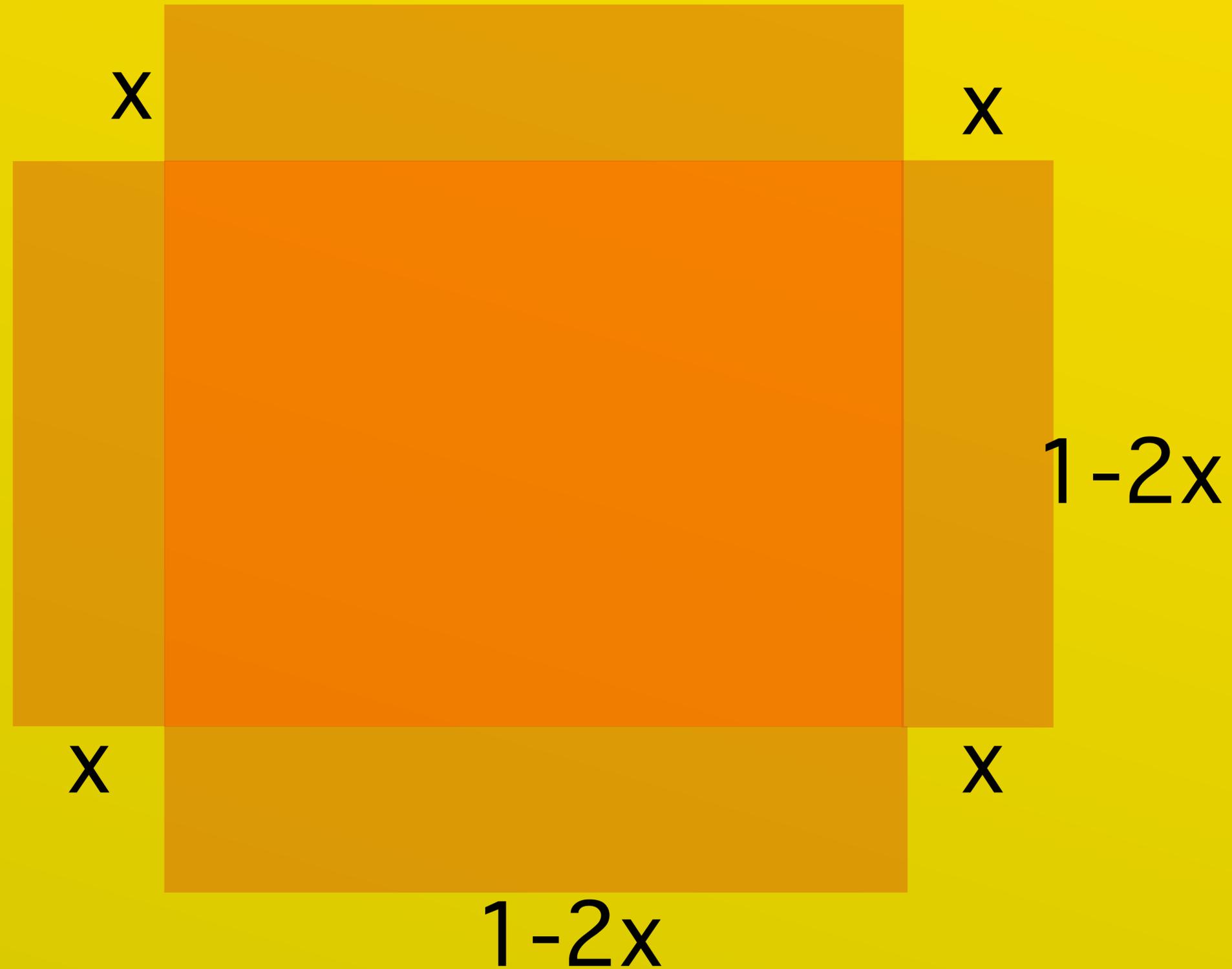
$x$

$10-2x$

We have 10 meters of chicken wire. Maximize the area of the field.



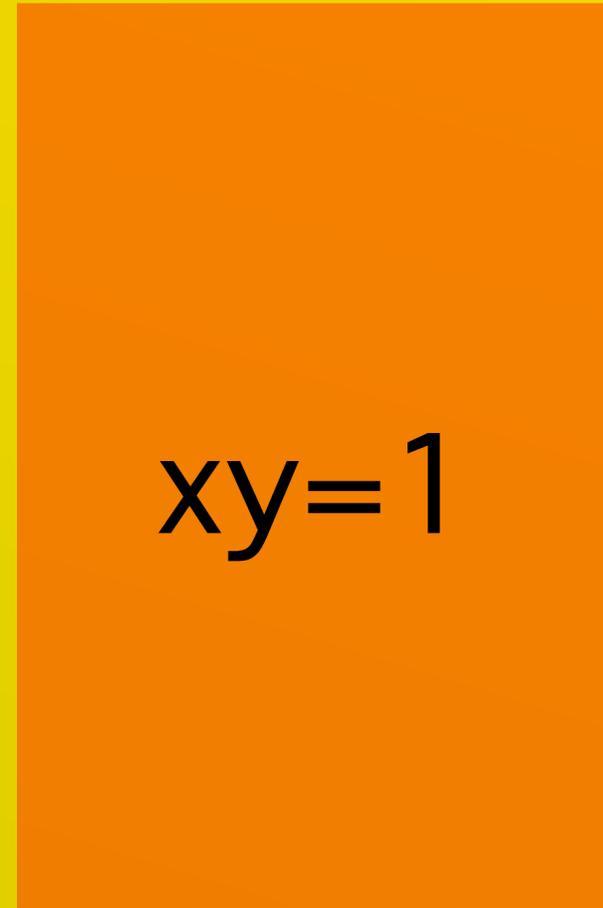
# *ENVELOPE PROBLEM*



Find the global max and min of  $f(x)$  volume of the tray with square base of length  $1-2x$  and height  $x$

*JAM*

x



$$xy=1$$

y

*RECTANGLE*

Which rectangle of fixed area  $xy=1$  has minimal circumference  $2x+2y$ ?

- Build a function  $f(x)$  to minimize
- On which interval is it defined
- Compare the critical values and boundary values?
- Does the extremal value theorem apply?

# CA JAM

if you should finish early. What about a cup, a can in which the top is missing?

## SODA CAN PROBLEM

Which can of volume  
 $V = \pi r^2 h = 2\pi$   
uses minimal material given by  
the surface area

$$A(r) = 2\pi r h + 2\pi r^2 \quad ?$$



Make sure to determine the interval of  $r$  which are possible and also look at boundary points

*THE END*