LECTURE 16

DERIVATIVE RULES

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PLAN

1. Poll

2. Product rule

3. Quotient rule

4. Examples

4. Trig functions

5. Examples

6. CA Project Check

POLL

$$\frac{d}{dx} e^x \log(x)$$

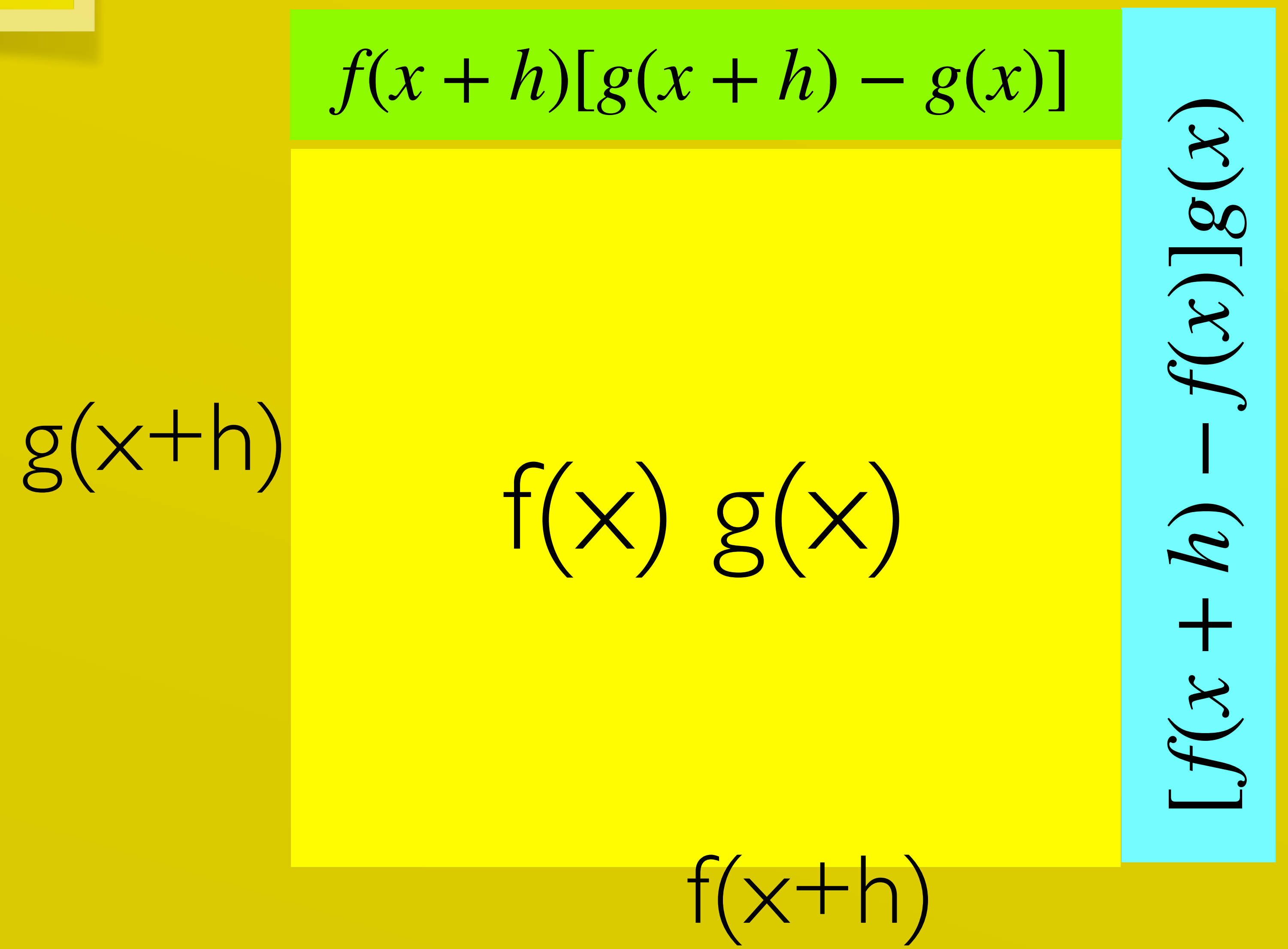
$$\frac{d}{dx} \log(x) \log(x)$$

PRODUCT

$$(fg)' = f'g + fg'$$

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h} = \frac{[f(x+h) - f(x)]g(x)}{h} + \frac{f(x+h)[g(x+h) - g(x)]}{h}$$

1646-1716



Now divide by h

Taking limits $h \to 0$ gives the product rule

EXAMPLE

$$\frac{d}{dx} x \log(x)$$

EXAMPLE

$$\frac{d}{dx}e^{x}e^{3x}$$

EXAMPLE

$$\frac{d}{dx}(x^4 + 1)^2$$

SOURCE

NOVA METHODUS PRO MAXIMIS ET MInimis, itemque tangentibus, quæ nec fractas, nec irrationales quantitates moratur, & singulare pro illis calculi genus, per G. G. L.

Sit axis AX, & curvæ plures, ut VV, WW, YY, ZZ, quarum ordinatæ, ad axem normales, VX, WX, YX, ZX, quæ vocentur respective, v, w, y, z; & ipsa AX abscissa ab axe, vocetur x. Tangentes sint VB, WC, YD, ZE axi occurrentes respective in punctis B, C, D, E. Jam recta aliqua pro arbitrio assumta vocetur dx, & recta quæ sit ad dx, ut v (vel w, vel y, vel z) est ad VB (vel WC, vel YD, vel ZE) vocetur dv (vel dw, vel dy vel dz) sive differentia ipsarum v (vel ipsarum w, aut y, aut z) His positis calculi regulæ erunt tales: TAB. XII.

Sit a quantitas data constans, erit da æqualis 0, & d $\overline{ax}$ erit æqu. a dx: si sit y æqu v (seu ordinata quævis curvæ YY, æqualis cuivis ordinatæ respondenti curvæ VV) erit dy æqu. dv. Jam *Additio & Subtractio*: si sit z − y + w + x æqu. v, erit d $\overline{z - y + w + x}$ seu dv, æqu dz − dy + dw + dx. *Multiplicatio*, d $\overline{xv}$ æqu. x dv + v dx, seu posito y æqu. xv, fiet dy æqu x dv + v dx. In arbitrio enim est vel formulam, ut xv, vel compendio pro ea literam, ut y, adhibere. Notandum & x & dx eodem modo in hoc calculo tractari, ut y & dy, vel aliam literam indeterminatam cum sua differentiali. Notandum etiam non dari semper regressum a differentiali Æquatione, nisi cum quadam cautione, de quo alibi. Porro *Divisio*, d $\frac{v}{y}$ vel (posito z æqu. $\frac{v}{y}$) dz æqu. $\frac{\pm v\,dy \mp y\,dv}{yy}$

Quoad *Signa* hoc probe notandum, cum in calculo pro litera substituitur simpliciter ejus differentialis, servari quidem eadem signa, & pro + z scribi + dz, pro − z scribi − dz, ut ex additione & subtractione paulo ante posita apparet; sed quando ad exegesin valorum venitur, seu cum consideratur ipsius z relatio ad x, tunc apparere, an valor ipsius dz sit quantitas affirmativa, an nihilo minor seu negativa: quod posterius cum fit, tunc tangens ZE ducitur a puncto Z non versus A, sed in partes contrarias seu infra X, id est tunc cum ipsæ ordinatæ

QUOTIENT

$$(f/g)' = (f'g - fg')/g^2$$

the product rule gives

$$f' = (g\frac{f}{g})' = g'\frac{f}{g} + g(\frac{f}{g})'$$

$$f'g = g'f + g^2(\frac{f}{g})'$$

Now solve for (f/g)'

1646-1716

The Song

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$$

Low Di High
Take High Di Low
Cross the line
and square the low

EXAMPLES

$$\frac{d}{dx}(x^2 + x)e^x$$

$$\frac{d}{dx}\frac{x^2 + x}{x^4 + 1}$$

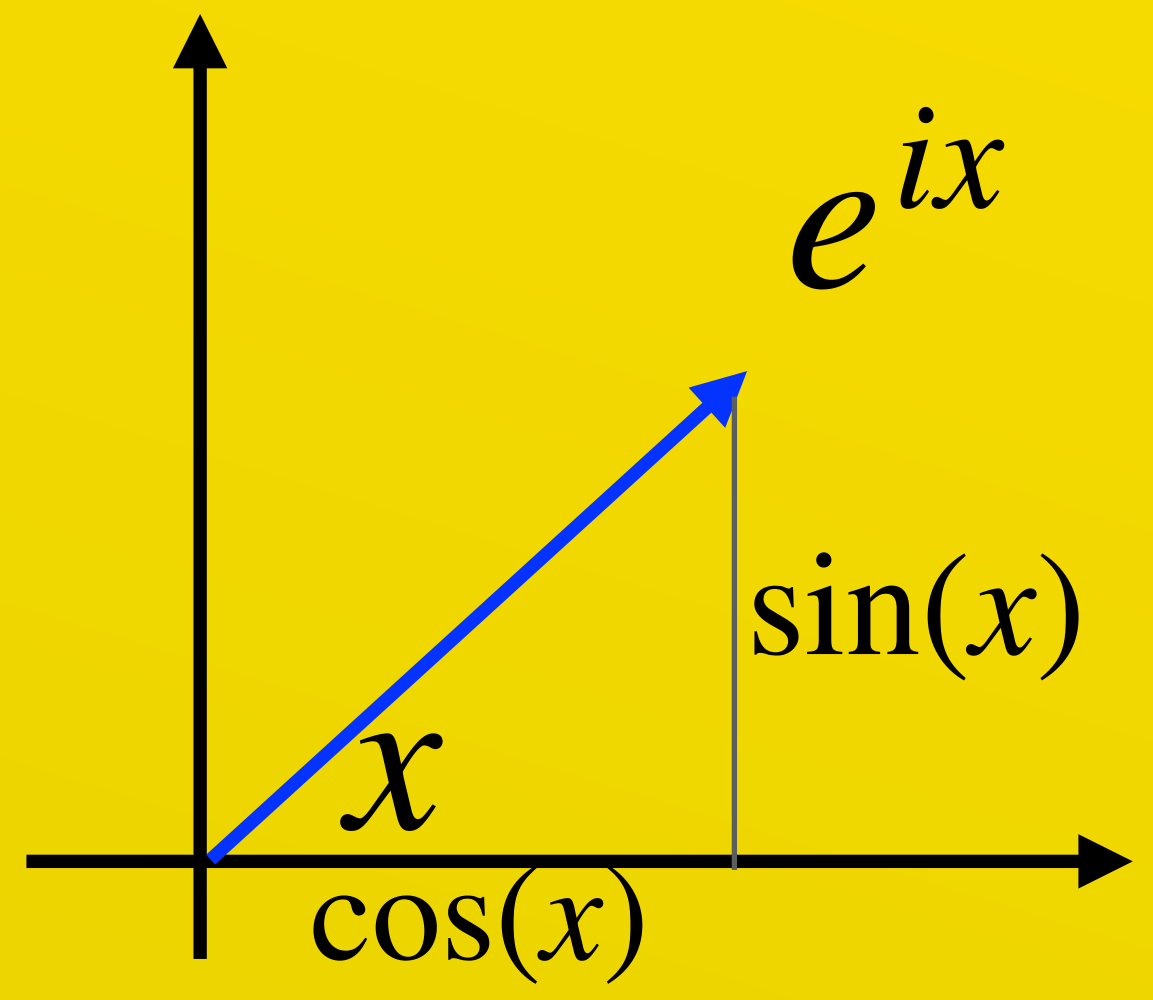
$$\frac{d}{dx}\frac{1}{\log(x)}$$

$$\frac{d}{dx}\frac{x^4}{x^6}$$

TRIG

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$



Most beautiful formula in math

Take this as the definition of cos and sin

$$\cos(x) + i\sin(x) = e^{ix}$$

$$\cos'(x) + i\sin'(x) = ie^{ix} = i\cos(x) - \sin(x)$$

EXAMPLES

$$\frac{d}{dx}(x^2 + x)e^x$$

$$\frac{d}{dx}\frac{x^2 + x}{x^4 + 1}$$

$$\frac{d}{dx}\frac{1}{\log(x)}$$

$$\frac{d}{dx}\frac{x^4}{x^6}$$

$$\frac{d}{dx}\sqrt{x^2 + 5}$$

JAM

$$\frac{d}{dx}\tan(x)$$

$$\frac{d}{dx}\sec x$$

$$\frac{d}{dx}\sin^2(x)$$

$$\frac{d}{dx}x\sin(x)$$

$$\frac{d}{dx}x\sin(x)\cos(x)$$

$$\frac{d}{dx}\sqrt{\sin(x)}$$

THE END