LECTURE 17

CHAIN RULE

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PLAN

1. Poll
2. Chain rule
3. Examples
4. One ring
5. Flask problem
6. Jam
7. CA Jam

POLL

What is the derivative of

$$e^{(e^x)}$$

A $e^{(e^x)}$

B $e^x e^x$

C $e^{(e^x)} e^e$

D $e^{e^x} e^x$

CHAIN RULE

$$(f(g))' = f'(g)g'$$

Proof:

$$\frac{f(g(x+h)) - f(g(x))}{h} = \frac{[f(g(x) + (g(x+h) - g(x))) - f(g(x))]}{[g(x+h) - g(x)]} \cdot \frac{[g(x+h) - g(x)]}{h}$$

VARIABLE-FUNCTION

$$\frac{d}{dx}\sin(x^2)$$

$$= \cos(x^2)2x$$

For a moment, the function x^2 becomes a variable.

ONE RING

$$f(x) = 1/x$$

Chain rule

$$f(g(x)) = 1/g(x)$$

$$f(g(x))' = -g'(x)/g(x)^2$$

The chain rule implies the reciprocal rule!


One ring to bring them all and
in the Darkness bind them.

ONE RING

Chain rule

$$\frac{d}{dx}(f+g)^2 = 2(f+g)(f'+g')$$

$$= 2(ff' + gg' + gf' + fg')$$

Chain rule

$$\frac{d}{dx}(f^2 + g^2 + 2fg) = 2ff' + 2gg' + 2(fg)'$$

The chain rule implies the product rule!

ONE RING

One Ring to rule them all
One Ring to find them
One Ring to bring them all,
and in the darkness bind them

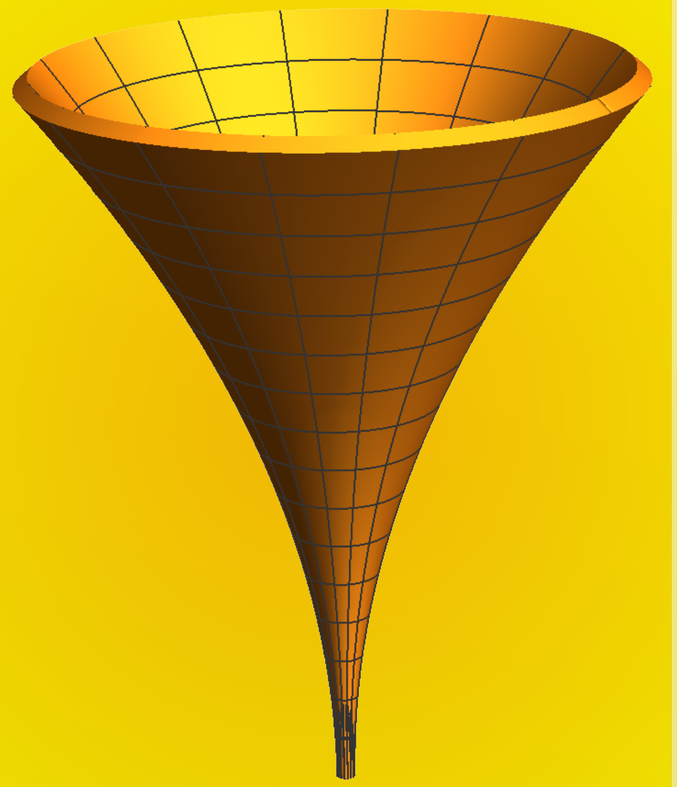
EXAMPLES

A $\frac{d}{dx}\sin(\log(x))$

B $\frac{d}{dx}e^{-x^2}$

C $\frac{d}{dx}\exp(\log(x))$

D $\frac{d}{dx}\sqrt{x^2+4}$



FLASK PROBLEM

$V(t) =$ Volume $\qquad h(t) = \sqrt{V(t)}$ height

V'(t) = 2

A $\qquad h'(t) = ?$

B $\qquad V(t) = t^2 + 2t + \cos(t) \qquad h'(0) = ?$

JAM

A $\frac{d}{dx}\sin(\sin(\sin(\sin(x))))$

B $\frac{d}{dx}e^{e^{e^{e^x}}}$

C $\frac{d}{dx}\sin(\cos(\tan(x)))$

CA JAM

A

$g(x) = \cos^2(x) + \sin(x)$

Find the absolute maximum and minimum values of g on [0,2π].

B

Compute both

$\frac{d}{dx}[f(x)^3]$ $\frac{d}{dx}f(x^3)$

C

$\frac{d}{dx}y^3$

where y=y(x) is a function of x

D

Compute $\frac{d}{dx}\frac{(2e^x)^3 \cdot \sqrt{25e^x}}{e^7 e^{x^2}}$

Can you simplify first?

THE END