LECTURE 18

MORE OPTIMIZATION

Oliver Knill, Harvard University

October 19 2020

PLAN

1. Poll

3. Derivative rules

2. Review Optimization

3. 3-4-5 triangle

4. Triathlon Jam

5. Decorative Fan

6. CA Jam

POLL

What is the coolest differentiation rule?

A Chain

B Product

C Reciprocal

D Quotient

RULES

$$(f(g))' = f'(g)g'$$

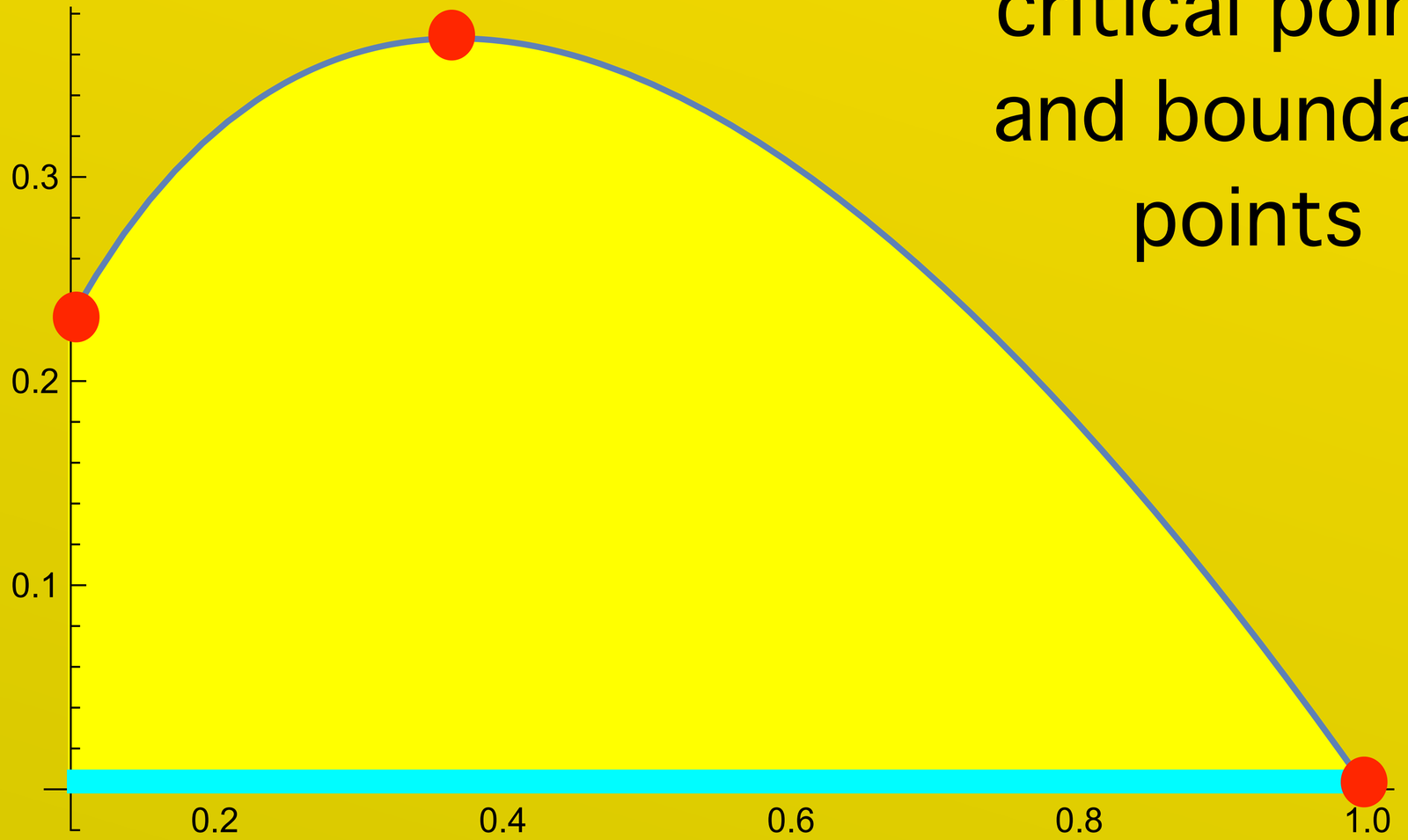
$$(fg)' = f'g + fg'$$

$$(f/g)' = (f'g - fg')/g^2$$

$$(1/g)' = -g'/g^2$$

EXTREMA

check
critical points
and boundary
points



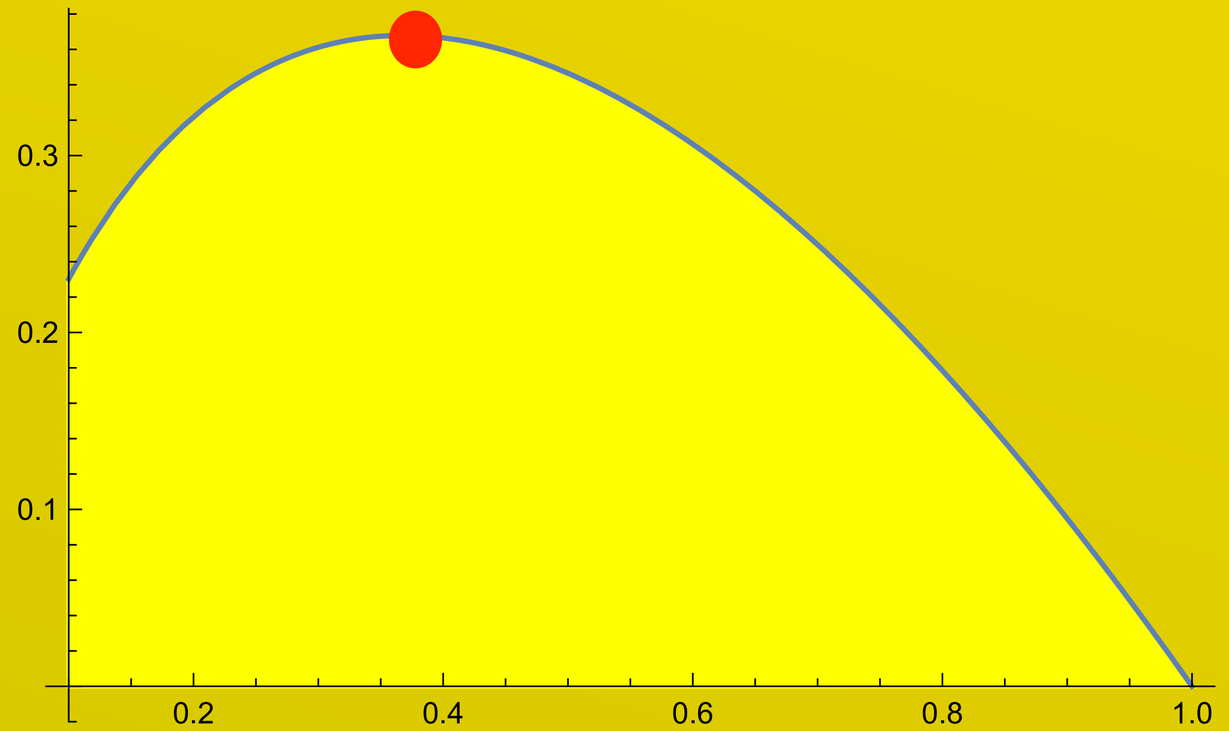
Bolzano
extremal
value
theorem

MOVIE TIME

Rauland
LOVE SIMON 2018

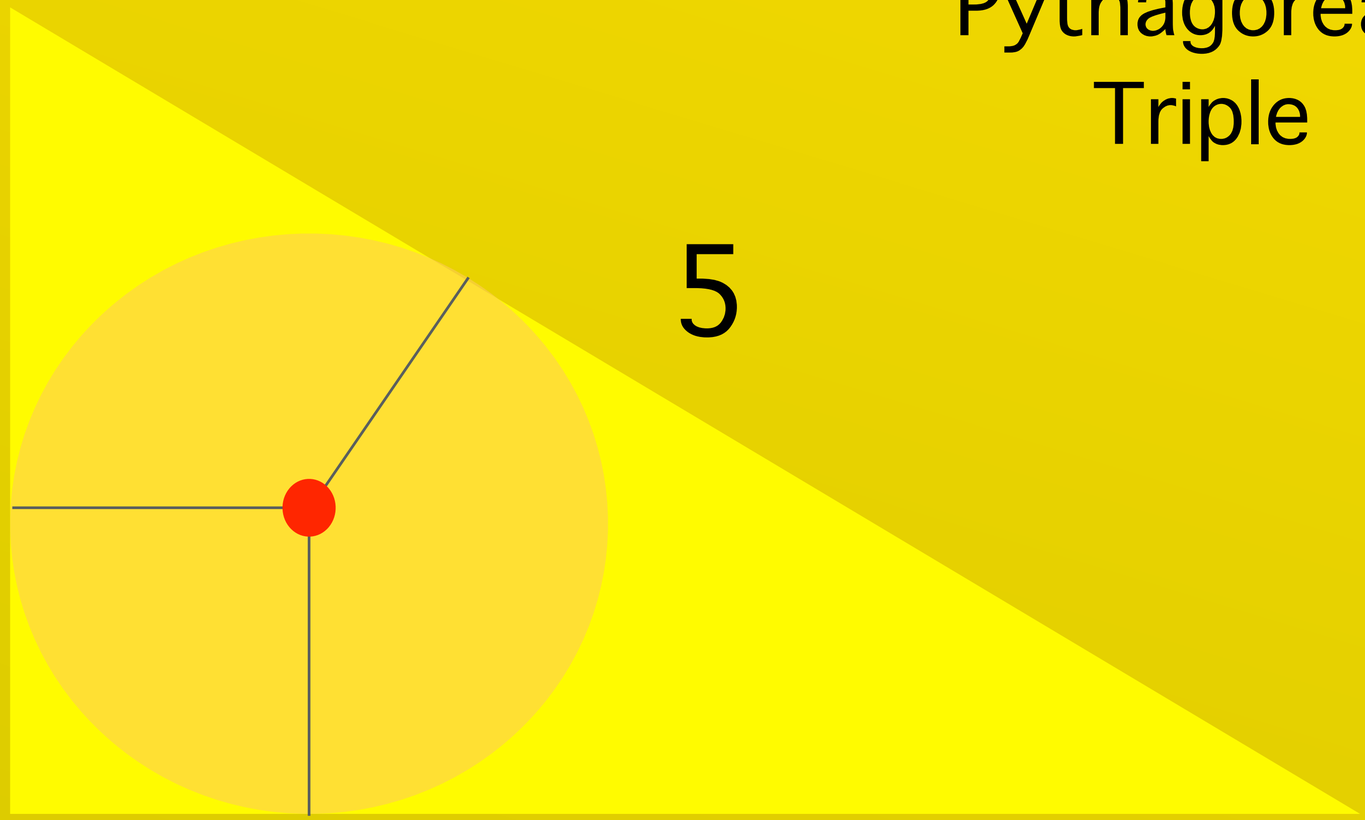
ENTROPY

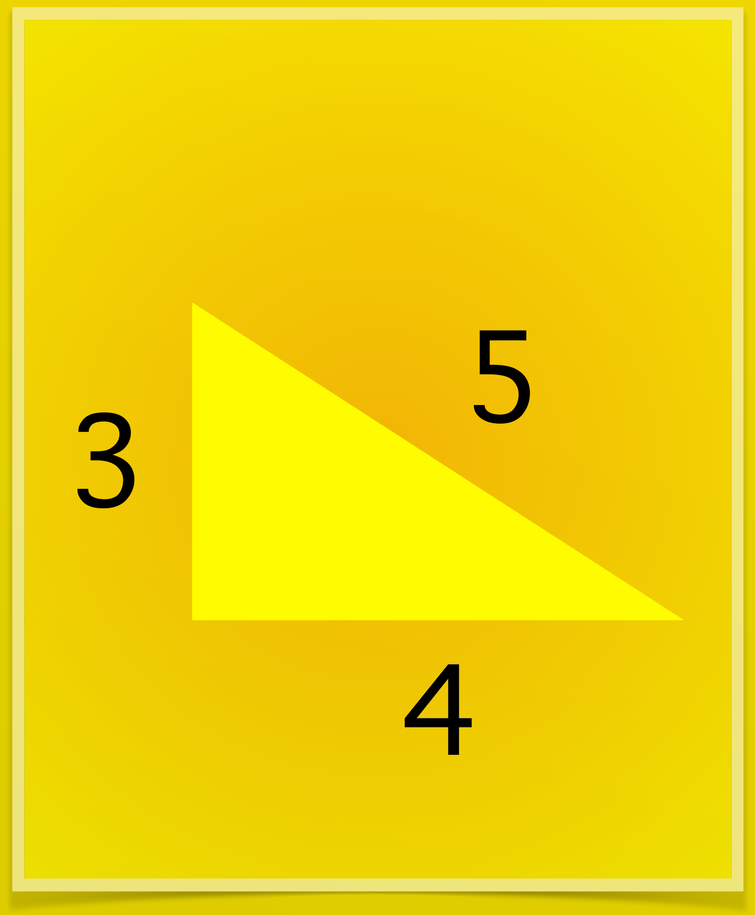
What is the
number of maximal entropy
$S(x) = -x \log(x)$



3-4-5 TRIANGLE

Pythagorean Triple





HISTORY

569-475 BC Pythagoras

1900-1600 BC Plympton 322

HISTORY

Table 2

n	$\overline{n}$	n	$\overline{n}$	n	$\overline{n}$
2	30	16	3.45	45	1.20
3	20	18	3.20	48	1.15
4	15	20	3	50	1.12
5	12	24	2.30	54	1.06.40
6	10	25	2.24	1	1
8	7.30	27	2.13.20	1.04	56.15
9	6.40	30	2	1.12	50
10	6	32	1.52.30	1.15	48
12	5	36	1.40	1.20	45
15	4	40	1.30	1.21	44.26.40

In the case of a right triangle, the notions of *indanum* and *ukullû* overlap, and was also used to measure the steepness of walls (Robson, 1999, 90) and a grain pile (Robson, 1999, 222). Hence both Egyptian and OB cultures had a practical ratio-based measurement which is the reciprocal of our notion of slope—at least 1000 years before angles were introduced.

OB scribes also knew that the sides of *similar triangles* are in the same ratio. For example in YBC 8633 #1 the $(3, 4, 5)$ triangle is enlarged to a triangle with measurements $(1.0, 1.20, 1.40)$ through multiplication by a scaling factor called the *makşarum*, which translates as "to bind, tie", determined by the ratio of the lengths $\frac{1.20}{4} = 20$ (Neugebauer and Sachs, 1945, 53).

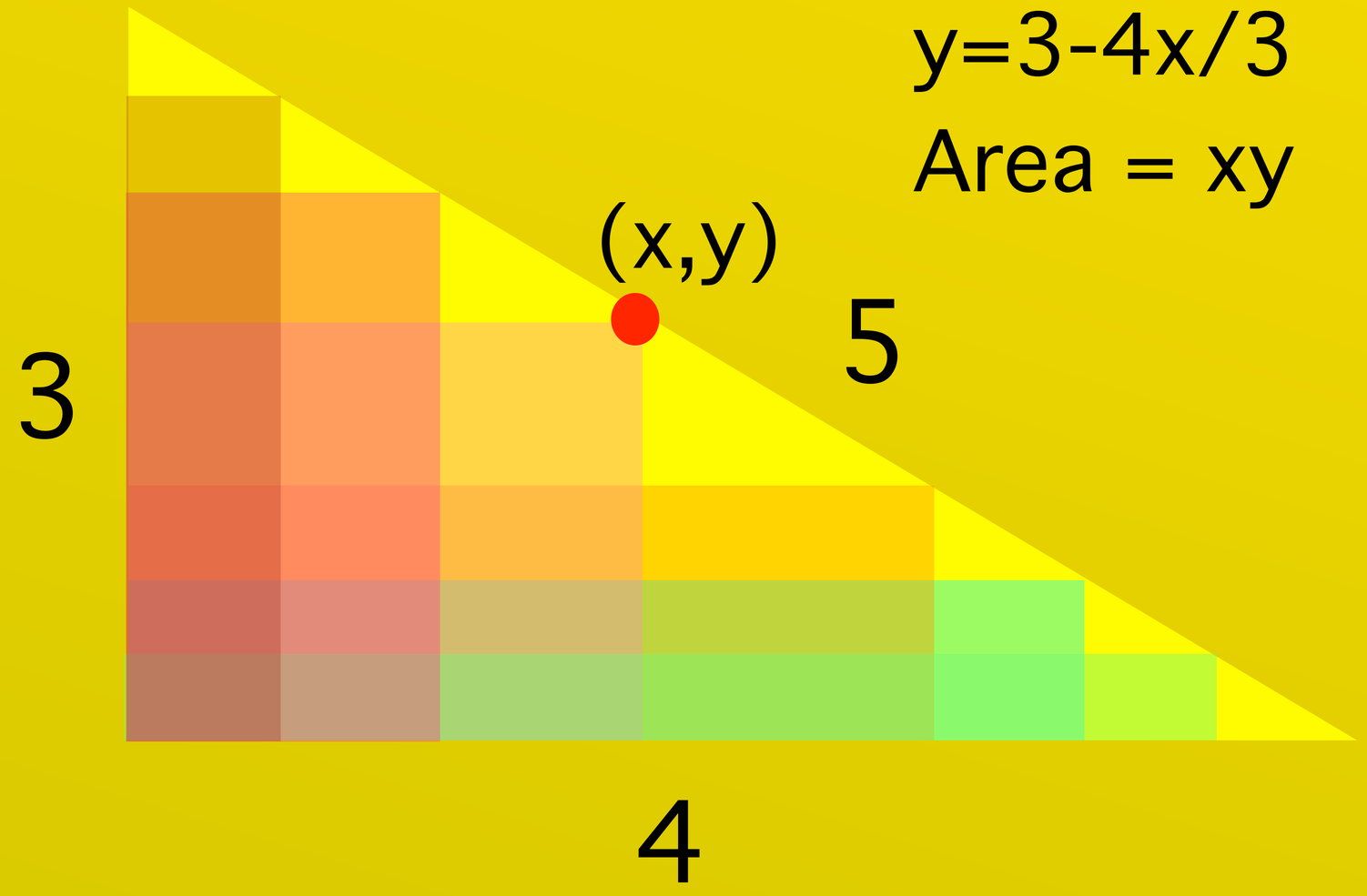
3.1. The Diagonal rule

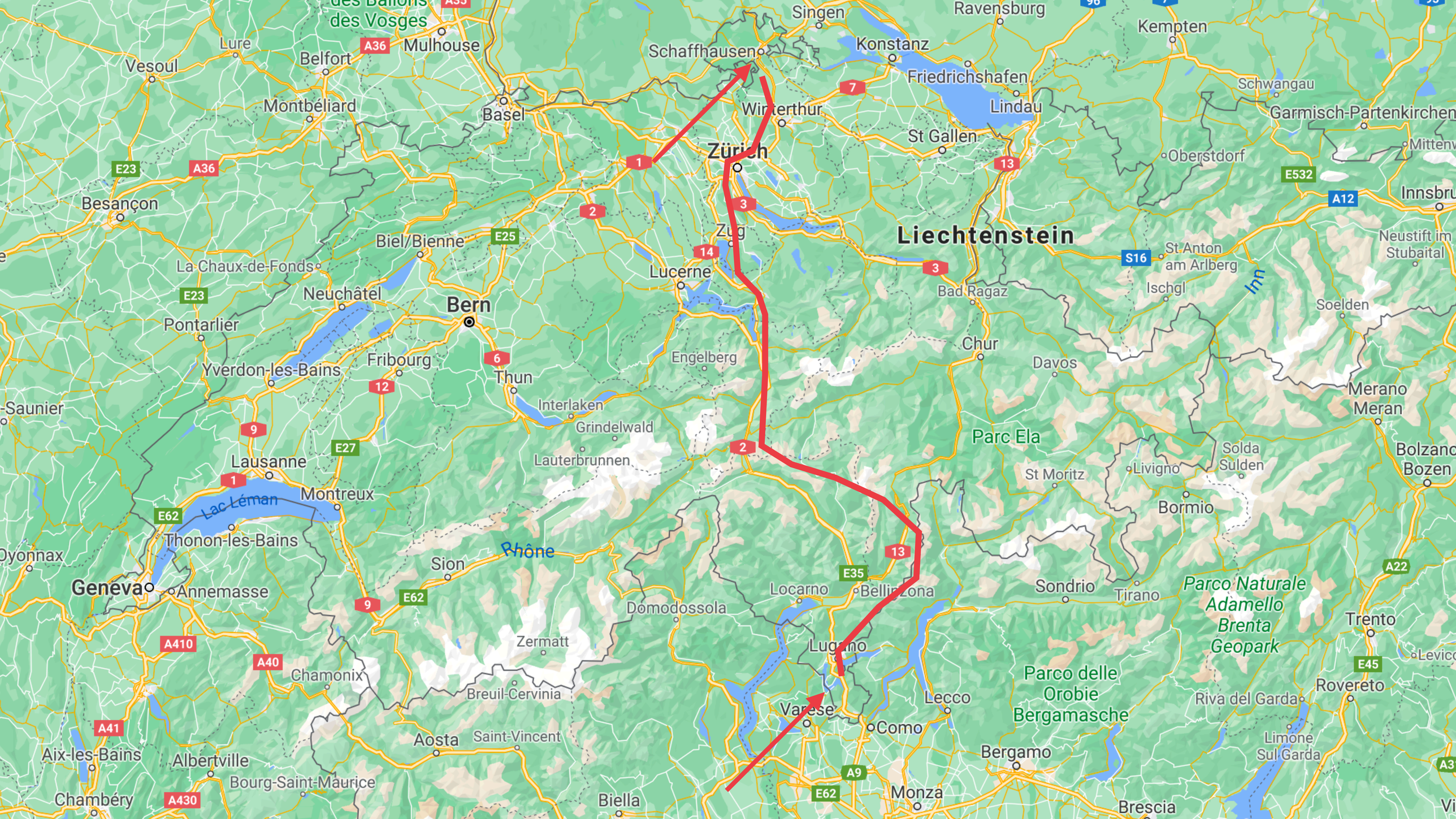
Evidence of the *Diagonal rule*, which now we call Pythagoras' theorem, can be found in quite a few tablets (Høyrup, 1999, 396–401; Melville, 2004, 150–152; Friberg, 2007, 450). This demonstrates the crucial metrical understanding that the three sides of a right triangle, or the sides and diagonal of a rectangle, (b, l, d) are related by the equation

$$b^2 + l^2 = d^2. \tag{1}$$

Right triangles and rectangles of unit height $l = 1$ appear to have been particularly significant, and we call a right triangle (or rectangle) with sides $(\beta, 1, \delta)$ **normalized** when β and δ are sexagesimal numbers. These

3-4-5 TRIANGLE





des Vosges
Mulhouse
Lure
Belfort
Vesoul
Montbéliard
Basel
Schaffhausen
Singen
Ravensburg
Kempten
Konstanz
Friedrichshafen
Lindau
Schwangau
Garmisch-Partenkirchen
Winterthur
St Gallen
Zürich
Oberstdorf
Besançon
Biel/Bienne
Zug
Liechtenstein
Neustift im Stubaital
La Chaux-de-Fonds
Lucerne
St Anton am Arlberg
Ischgl
Bad Ragaz
Inn
Neuchâtel
Bern
Soelden
Pontarlier
Chur
Engelberg
Davos
Fribourg
Yverdon-les-Bains
Thun
Merano
Meran
Interlaken
Grindelwald
Parc Ela
Lauterbrunnen
Solda
Sulden
Bolzano
Bozen
Lausanne
St Moritz
Livigno
Montreux
Lac Léman
Bormio
Thonon-les-Bains
Rhône
Sion
Geneva
Annemasse
Locarno
Bellinzona
Sondrio
Tirano
Parco Naturale Adamello Brenta Geopark
Domodossola
Trento
Zermatt
Lugano
Chamonix
Parco delle Orobie Bergamasche
Breuil-Cervinia
Lecco
Rovereto
Riva del Garda
Varese
Como
Aosta
Saint-Vincent
Limone Sul Garda
Aix-les-Bains
Albertville
Bergamo
Bourg-Saint-Maurice
Chambéry
Monza
Biella
Brescia

TRIATHLON

swim

bike

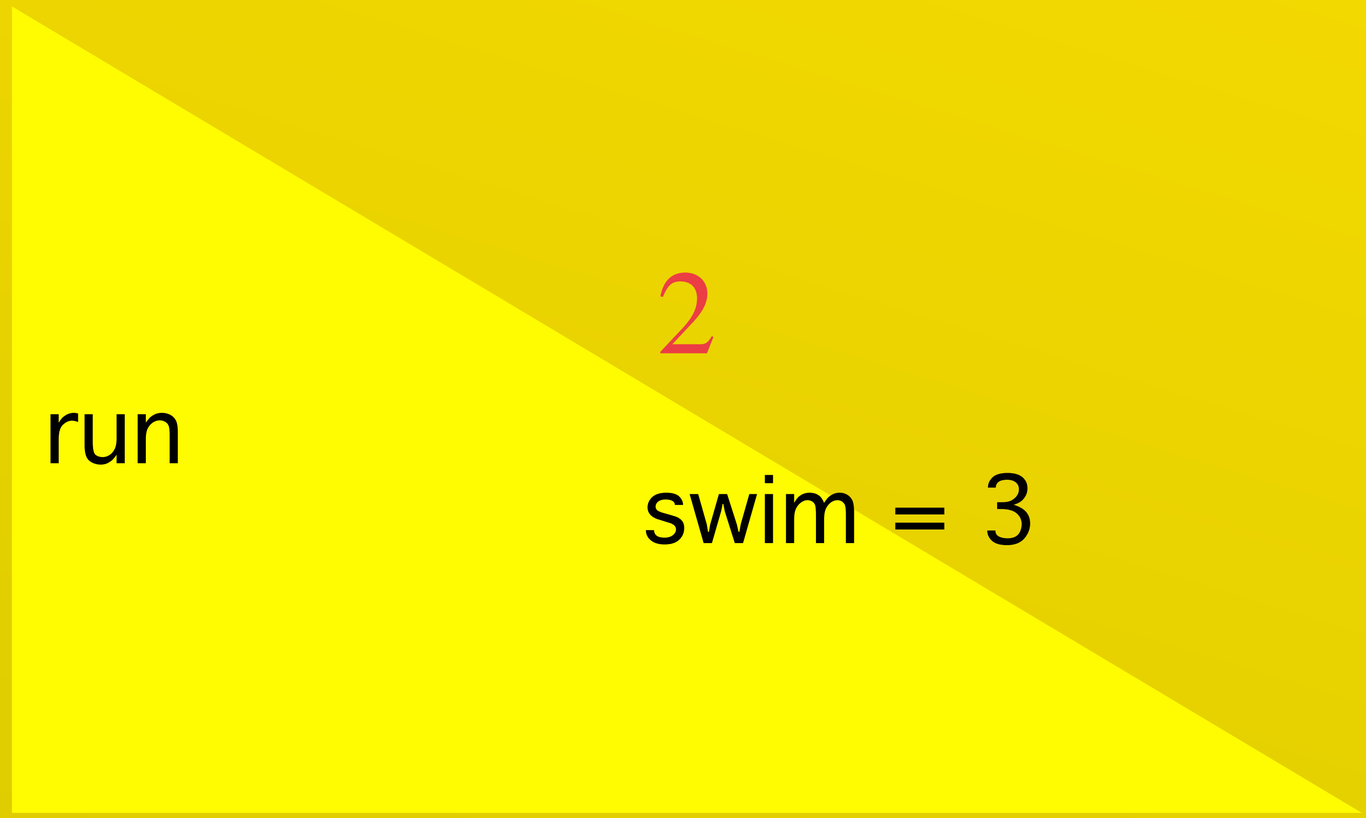
run

1998 and 2008

FAT
EIDG. FORSCHUNGSANSTALT
FÜR BETRIEBSWIRTSCHAFT UND
LANDTECHNIK

TRIATHLON

speed

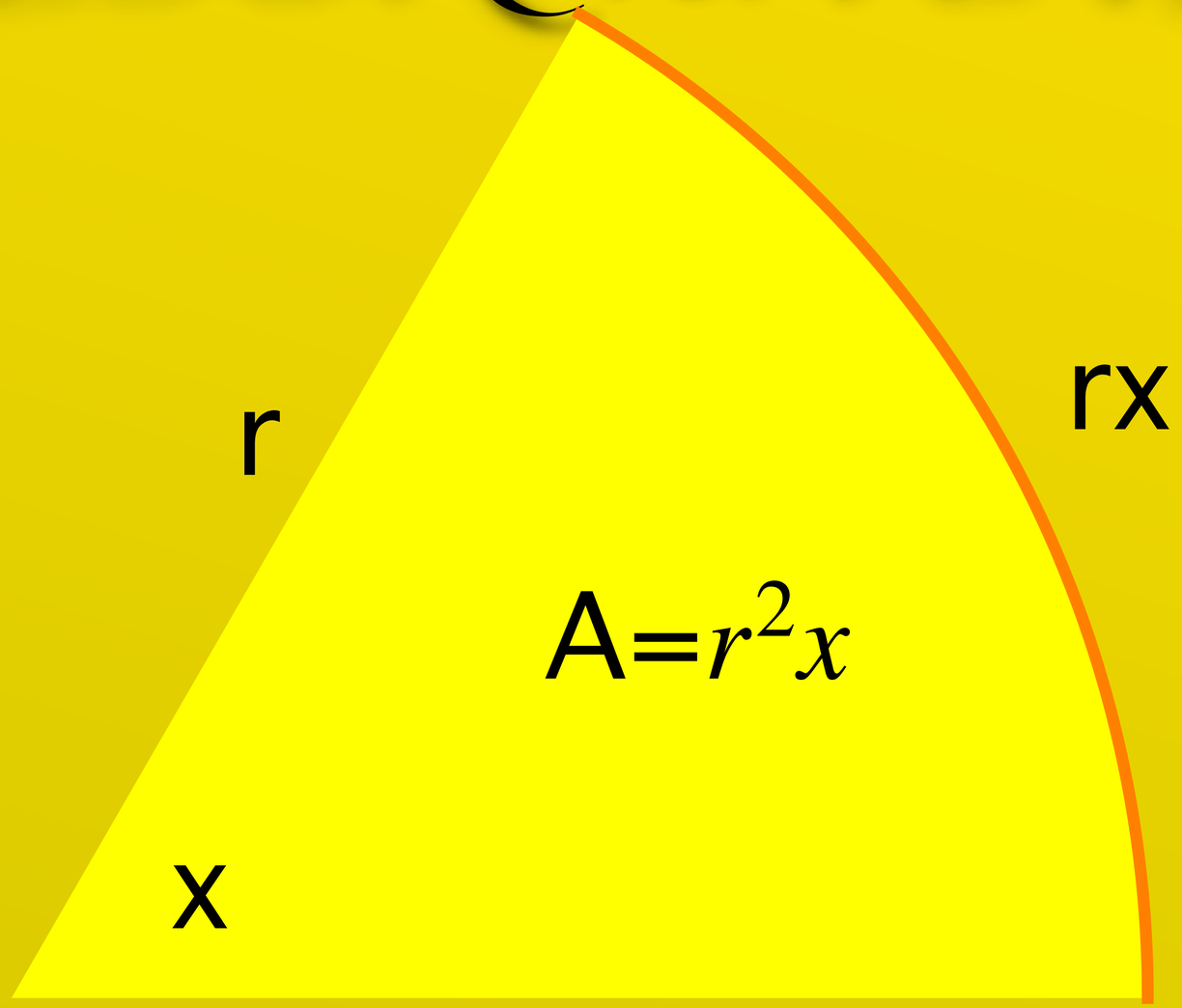


maximize time

$$f = x/30 + 3/2 + \sqrt{9 - x^2}/12$$

DECORATIVE FAN

JAM



$$f = 10r^2x + 15rx$$

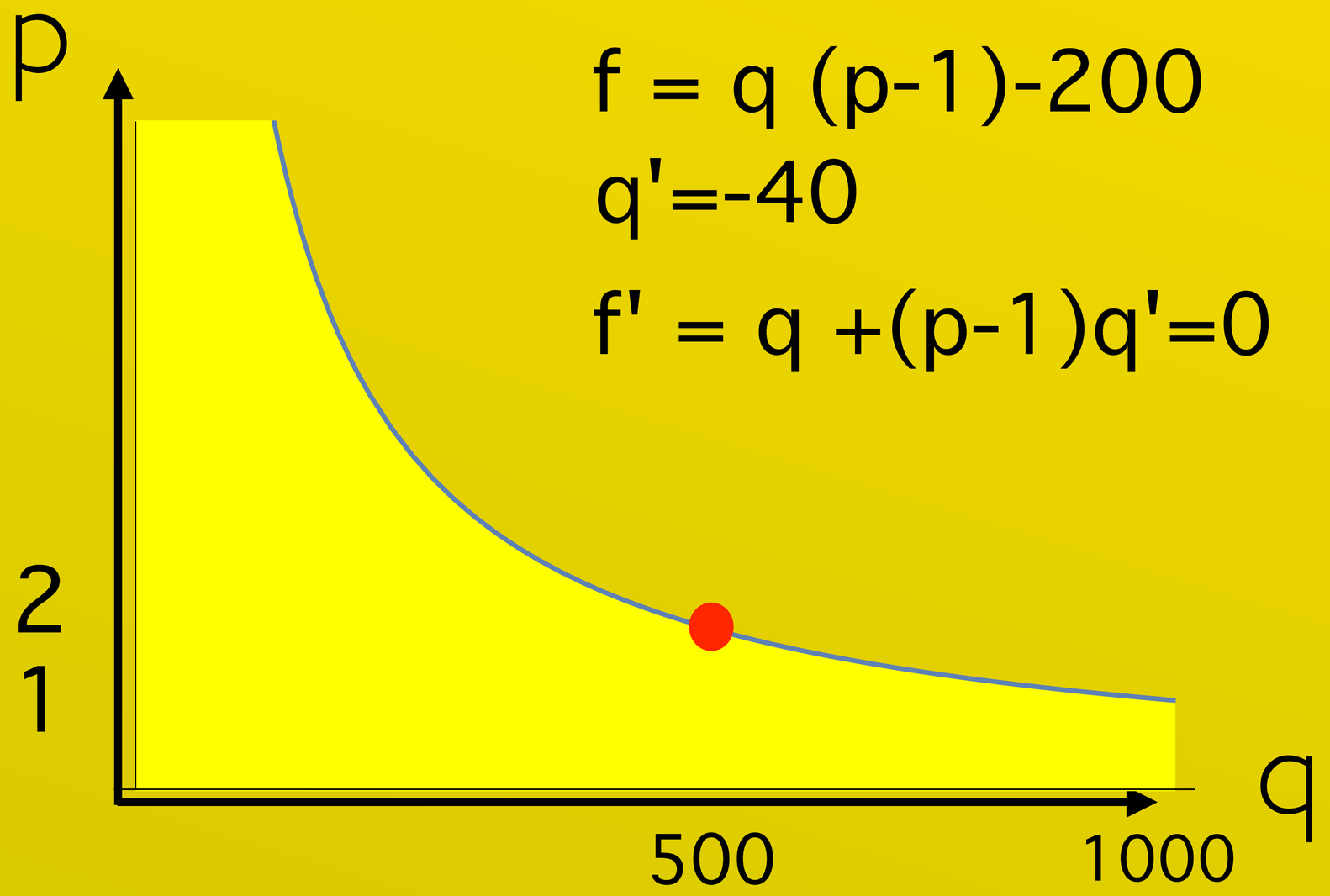
$$2r + rx = 60$$

. Traditional Chinese fans are often shaped like circular sectors.[1] Marvin wants to buy a large decorative fan of perimeter 60 cm to hang on his wall, so he commissions a fan designer to design such a fan for him. Marvin will pay 10 cents per square cm for the fan's surface, and he would like the top curved edge of the fan is to be lined with gold thread costing 15 cents per cm. If the fan designer would like to maximize her revenue, what dimensions should she make the fan?

AVOCADO JAM

After a lot of experimentation, you have perfected your recipe for a creamy avocado dip. You join a stall at the Farmer's Market on the Science Center Plaza to sell your product by the jar. Somehow, a rumor gets started (not traceable to you) that your dip is an excellent snack to help keep skin healthy. Sales take off. At a price of $2.00 per jar, you can sell 500 jars a day. For every quarter that you increase the price, you sell ten fewer jars.

Assuming that your fixed costs are $200 per day (for your patent lawyer) and the cost per jar is $1.00, determine the price for which you should sell your dip in order to maximize profit.



$$f = q\,(p-1)-200$$

$$q' = -40$$

$$f' = q + (p-1)q' = 0$$

THE END