

LECTURE 19

IMPLICIT DIFFERENTIATION

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PLAN

1. Poll

2. Implicit differentiation

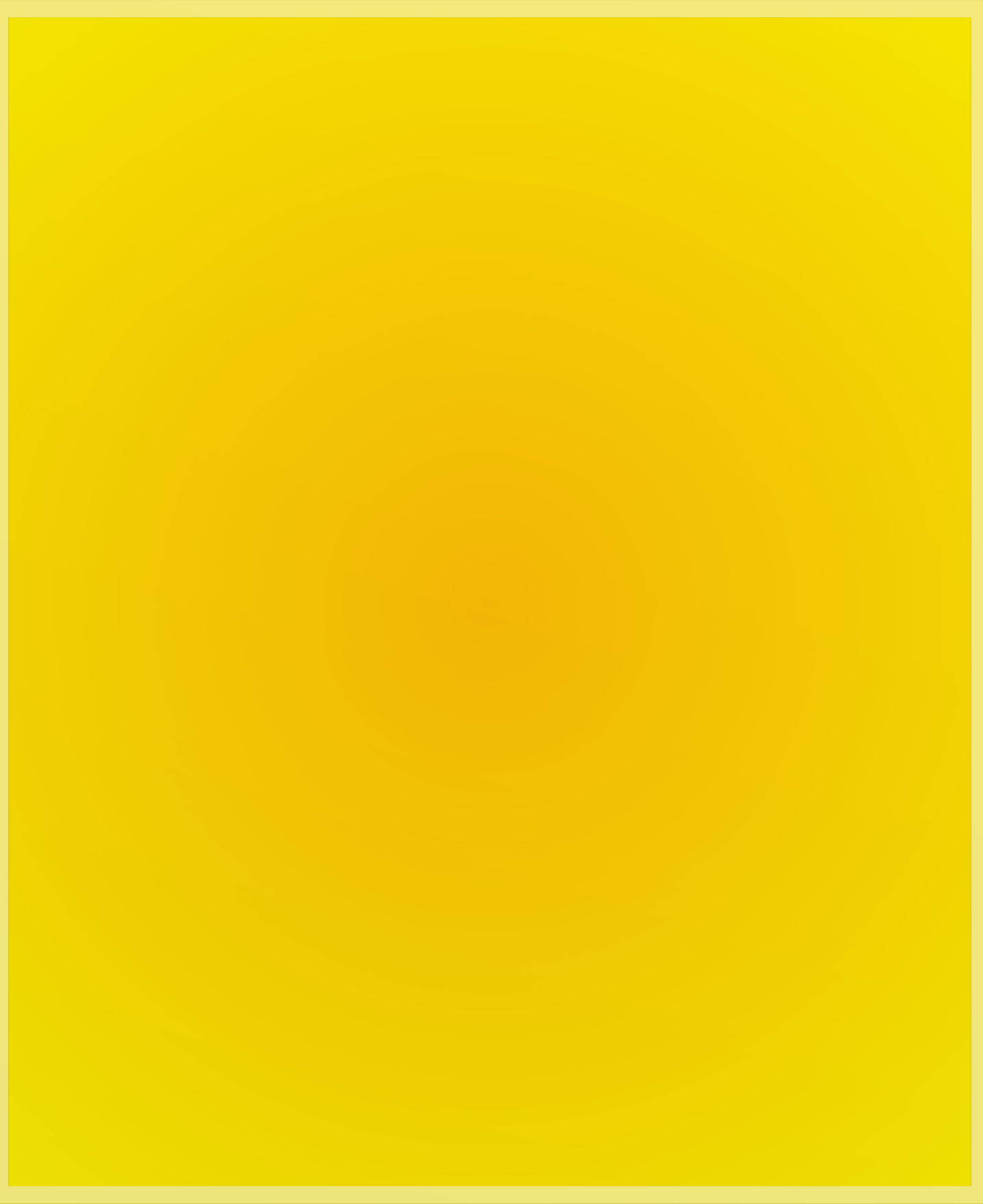
3. Examples Foil of Descartes

4. log derivative

5. Drill Jam

6. Bug Jam

POLL


$$xy^2 = 1$$

What is
 $y'(1)$?

A

-1/2

B

2

C

1/2

D

0

IMPLICIT DIFFERENTIATION



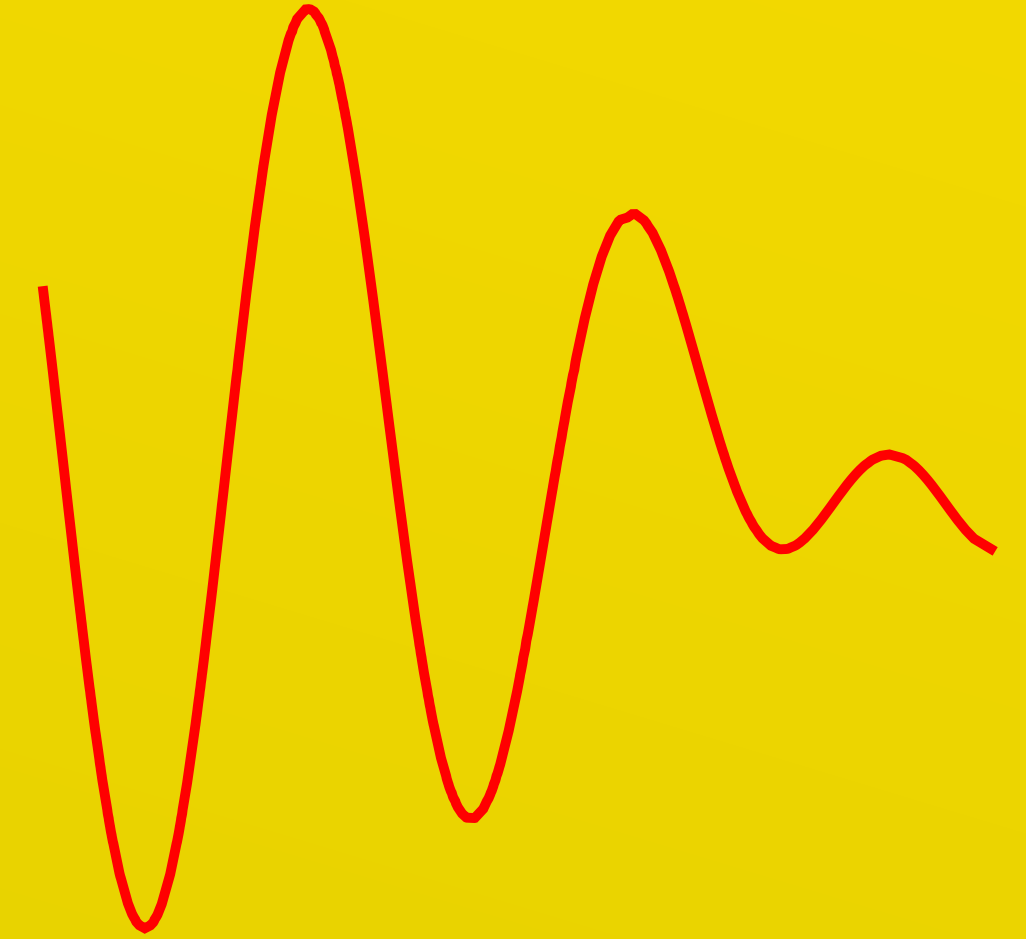
$$x^2 + y^2 = 5$$

Find $y'(1)$ for $x=1$

$$2x + 2y y' = 0$$

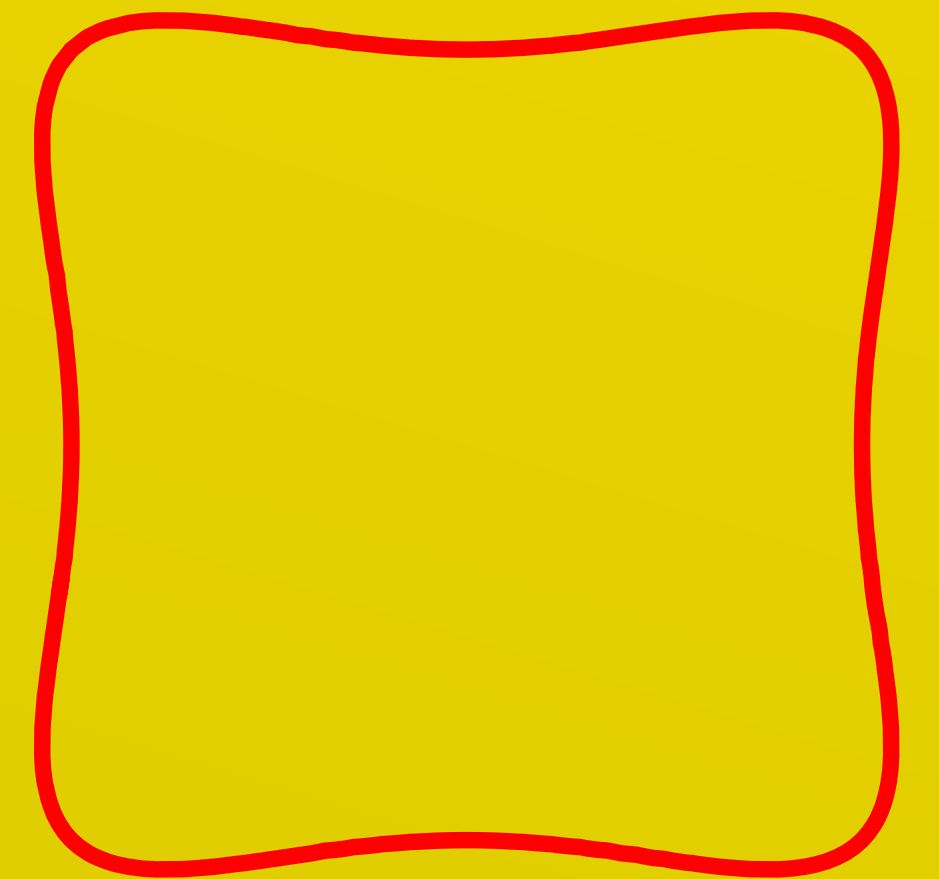
IMPLICIT-EXPLICIT

$$y = \sin(5x) - \cos(4x)$$

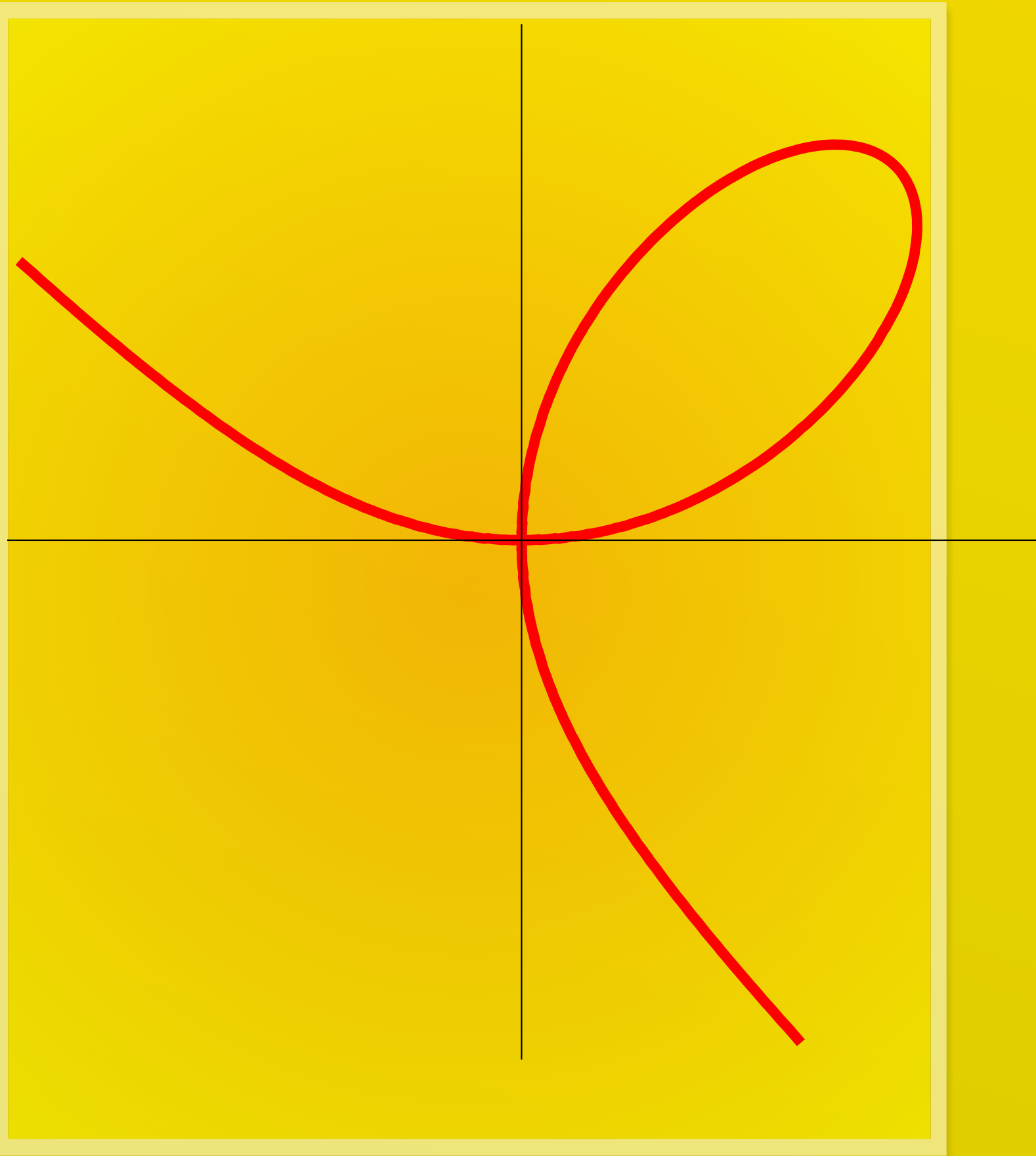


Explicit:
solve for y

$$x^4 + y^4 - x^2y^2 = 1$$



THE FOLIUM



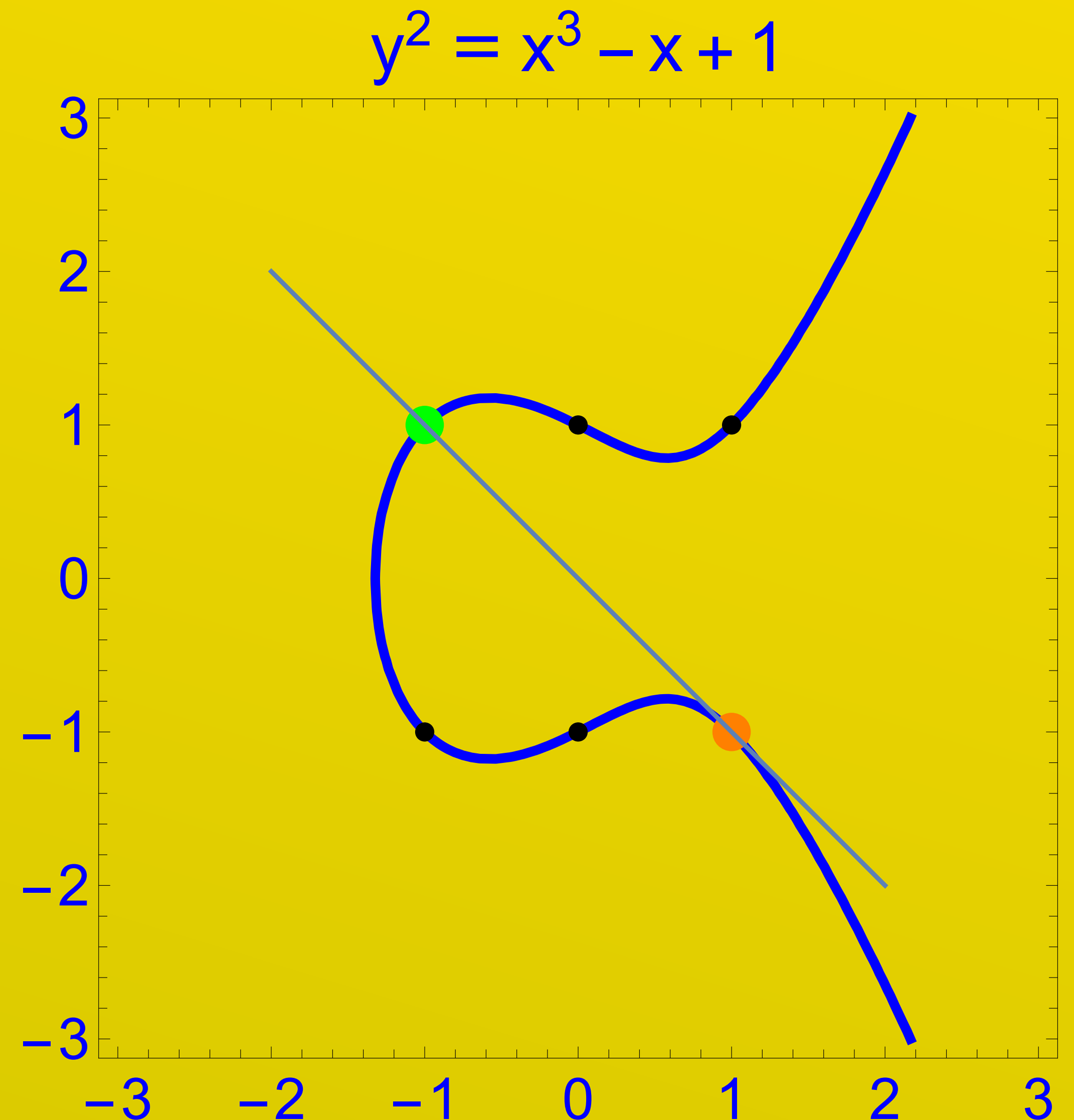
$$x^3 + y^3 = 9xy/2$$

Find $y'(1)$

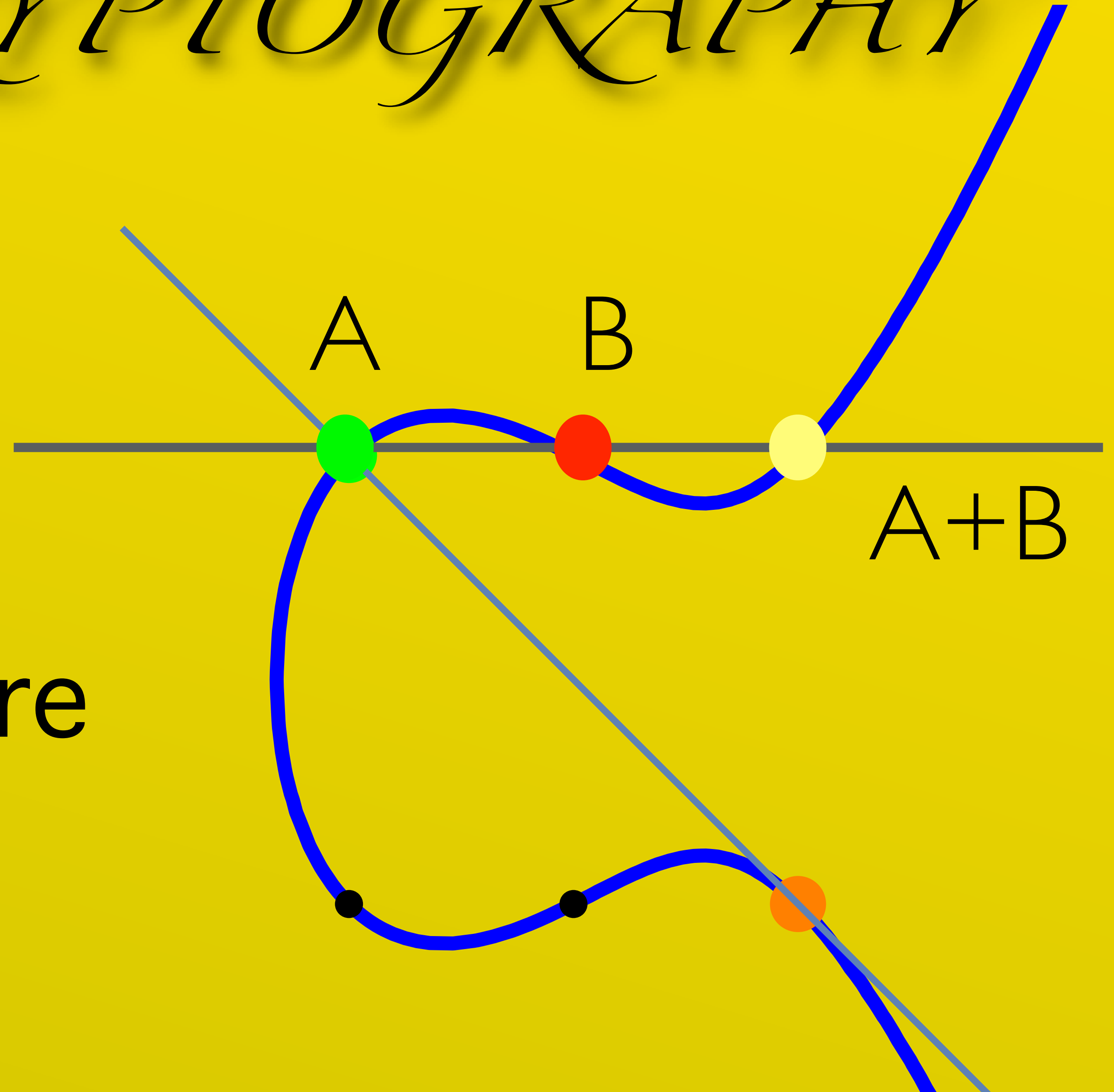
ELLIPTIC CURVE

Find the
tangent line
at
 $(1, -1)$

$$y^2 = x^3 - x + 1$$



CRYPTOGRAPHY



Elliptic curves are
used for
computations



Sneakers, 1992

DERIVATIVE OF LOG

$$\log'(x) = 1/x$$

$$y = \log(x)$$

Find $y'(x)$

$$\exp(y) = x$$

$$\exp'(y) y' = 1$$

CHANGE OF BASE

$$b^x = e^{x \log(b)}$$

$$\log_b(x) = \frac{\log(x)}{\log(b)}$$

$$y = \log_b(x)$$

Find $y'(x)$

$$\frac{d}{dx}(b^x) = \log(b)b^x$$

$$\frac{d}{dx} \log_b(x) = \frac{1}{x \log(b)}$$



Jost Bürgi
1552
-1632



	228000	228500	229000	229500	230000	230000
0	977556601	982456378	987380714	992329732	0000	997303557
10	...654356	...554623	...479452	...428965	10000	...403287
20	...752122	...652879	...578200	...528208	20000	...503027
30	...849397	...751144	...676958	...627461	30000	...602778
40	...947682	...849419	...775726	...726724	40000	...702538
50	978045477	...947704	...874503	...825996	50000	...802308
60	...143281	983045999	...973291	...925279	60000	...902088
70	...241096	...144304	988072088	993024572	70000	998001879
80	...338920	...242618	...170895	...123874	80000	...101679
90	...436754	...340942	...269712	...223187	90000	...201489
100	...534597	...439276	...368539	...322509	230100000	...301309
110	...632451	...537620	...467376	...421841	10000	...401139
120	...730314	...635974	...566223	...521183	20000	...500979
130	...828187	...734338	...665080	...620535	30000	...600829
140	...926070	...832711	...763946	...719898	40000	...700690
150	979023962	...931094	...862822	...819269	50000	...800560
160	...121865	984019488	...961709	...918651	60000	...900440
170	...219777	...127890	989060605	994018043	70000	999000330
180	...317699	...226303	...159511	...117445	80000	...100239
190	...415631	...324726	...258427	...216857	90000	...200140
200	...513572	...423158	...357353	...316278	230200000	...300060
210	...611524	...521601	...456288	...415710	10000	...399990
220	...709485	...620053	...555234	...515152	20000	...499930
230	...807456	...718515	...654190	...614603	30000	...599880
240	...905437	...816987	...753155	...714065	40000	...699840
250	980003427	...915468	...852130	...813536	50000	...799810
260	...101427	985013960	...951115	...913017	60000	...899790
270	...99438	...112461	990050111	995012509	230270000	999999779
280	...297457	...210973	...149116	...117010	70	879
290	...395487	...309494	...248130	...211521	80	979
300	...493527	...408025	...347155	...311042	90	89
310	...591576	...506565	...446190	...410573	230270022	999999999
320	...689635	...605116	...545235	...510115		
330	...787704	...703677	...644289	...609666		
340	...885783	...802247	...743353	...709227		
350	...983872	...900827	...842428	...808797		
360	981081970	...99417	...941512	...908378		
370	...180078	986098017	991040606	996007969		
380	...278196	...196627	...139711	...107570		
390	...376324	...295247	...238825	...207181		
400	...474462	...393876	...337948	...306801		
410	...572609	...492516	...437082	...406432		
420	...670766	...591165	...536226	...506073		
430	...768934	...689824	...635380	...605723		
440	...867110	...788493	...734543	...705384		
450	...965297	...887172	...833717	...805054		
460	982063494	...985861	...932900	...904735		
470	...161700	987084559	992032093	997004425		
480	...259916	...183268	...131296	...104126		
490	...358142	...281986	...230510	...203836		
500	...456378	...380714	...329732	...303557		

Also enden sich die
zwo Summen das
len in 9. Zypfern/ vn
ist die Rote

230270022
230270023

Die Schwarze
aber ist ganz mit 9.
nollen als 1000000000
vnd so dieselben ganz
zen Zalen/nicht genug
geben mögen / so mag
man dieselben 2.3.4.
5.6.7.8.9.zusammen
addieren.

JAM

A

$$\frac{d}{dx} \log(5x^3 + 1)$$

B

$$\frac{d}{dx} x^2 \log_2(x^4)$$

C

$$\frac{d}{dx} \pi^x$$

D

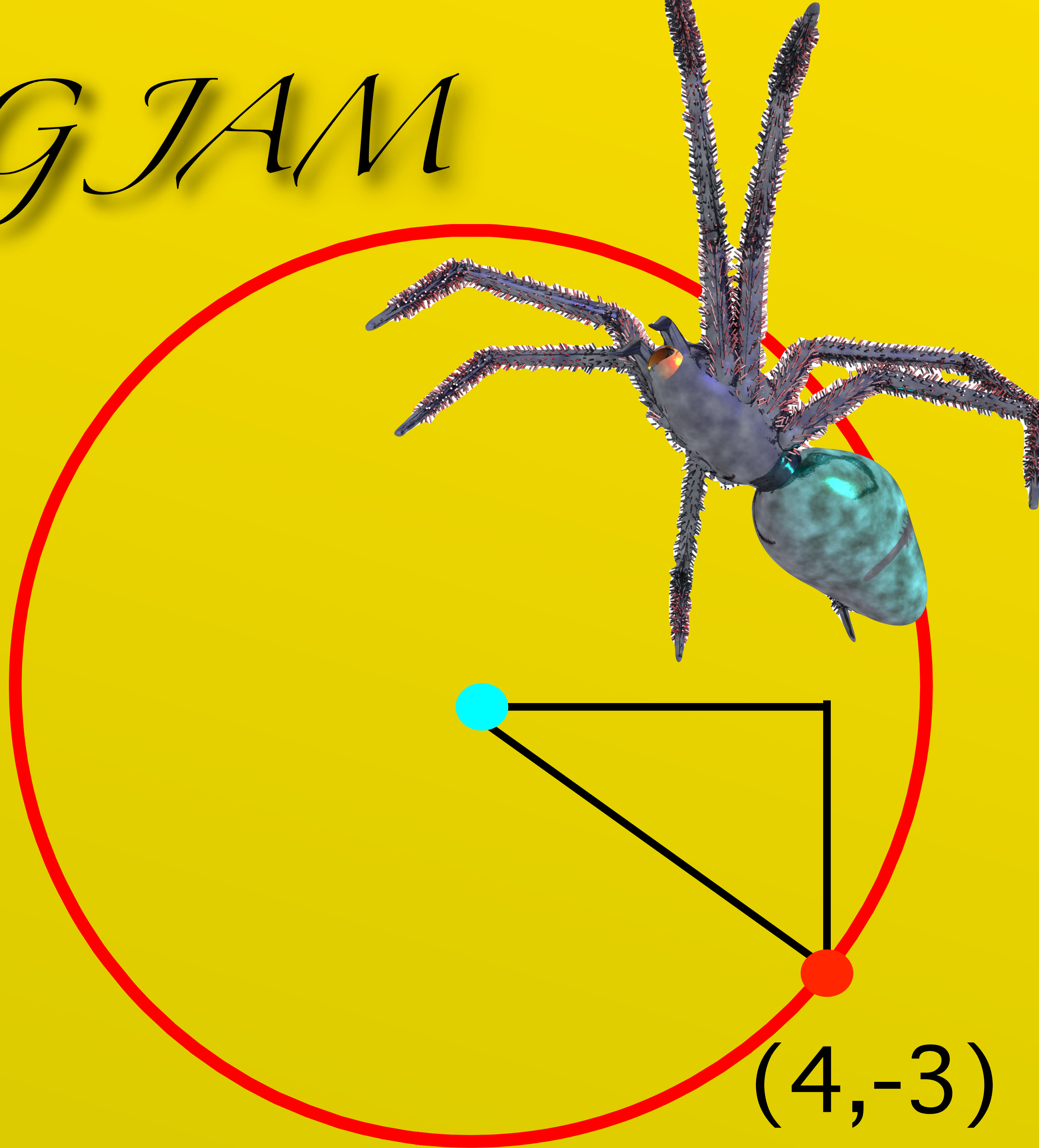
$$\frac{d}{dx} x^\pi$$

BUG JAM

Find
 x'

$$x^2 + y^2 = 25$$

$$y'(4) = -2$$





THE END