LECTURE 20

LOGARITHMIC DIFFERENTIATION

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PLAN

1. Poll

2. Logarithmic differentation

3. Example type exp

4. Example type products

5. Challenge Jam

6. Large Numbers

POLL

$$\frac{d}{dx}2^{\sin(x)}$$

A $e^{\sin(x)\log(2)}$

B $e^{\sin(x)\log(2)}\cos(x)$

C $e^{\sin(x)\log(2)}\log(2)\cos(x)$

D $e^{\cos(x)\log(2)}\log(2)$

LOGARITHMIC DIFFERENTIATION

$$\frac{d}{dx}\log(x)^x$$

Howdo we differentiate this?

DIRECT

$$e^{x \log(\log(x))}$$

DIRECT

$$e^{x \log(\log(x))}$$

$$e^{x \log(\log(x))} \quad [\log \log(x) + 1/(\log(x)x)]$$

LOG DIFFERENTATION

$$y = \log(x)^x$$

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$$y' = y[\log \log(x)) + x/(\log(x)x]$$

OTHER EXAMPLE

$$\frac{d}{dx}x\sqrt{1+x^2}$$

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Direct

$$\sqrt{1+x^2} - x^2/\sqrt{1+x^2}$$

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$$\frac{d}{dx} x\sqrt{1+x^2}$$

Direct

$$\sqrt{1+x^2} - x^2/\sqrt{1+x^2}$$

Log derivative

$$\frac{d}{dx}\log(y) = \frac{y'}{y} = \frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{1+x^2}$$

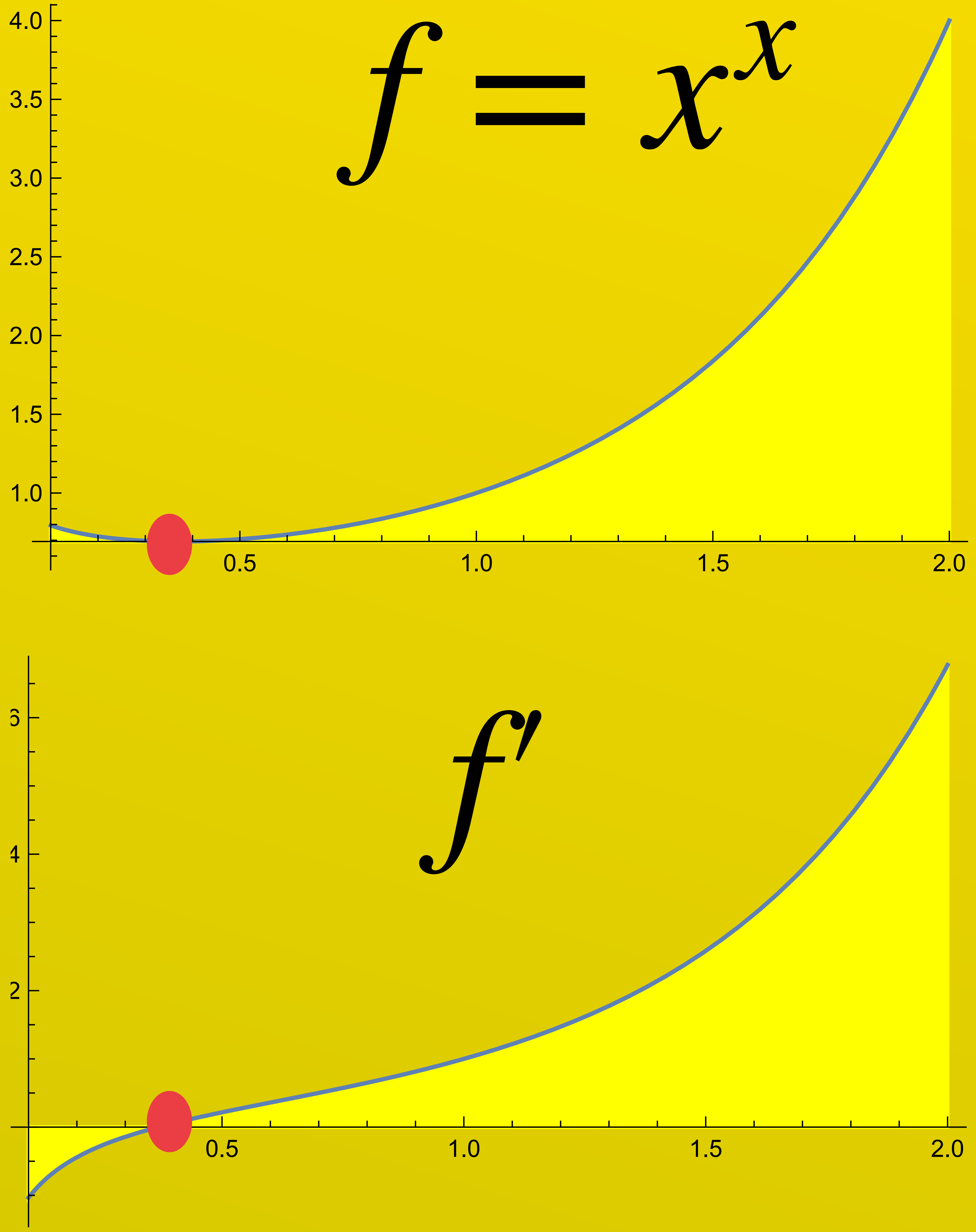
$$y' = (x\sqrt{1+x^2})(\frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{1+x^2})$$

x^x

Find the derivative of

$$x^x$$

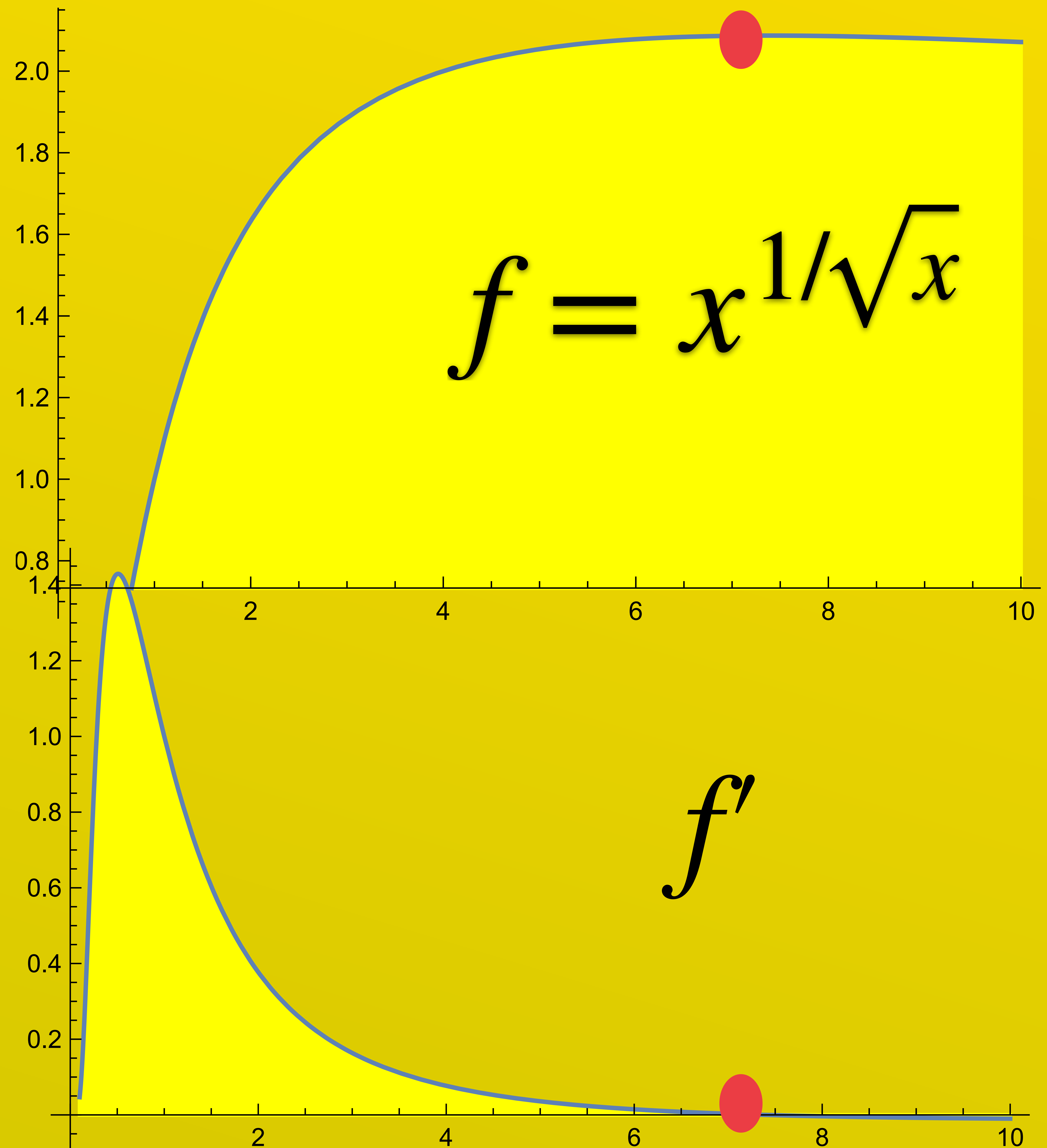
Where is f minimal?


$f = x^x$
f'
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
1.0
1.5
2.0
6
4
2
0.5
1.0
1.5
2.0

$$x^{1/\sqrt{x}}$$

Find the derivative of

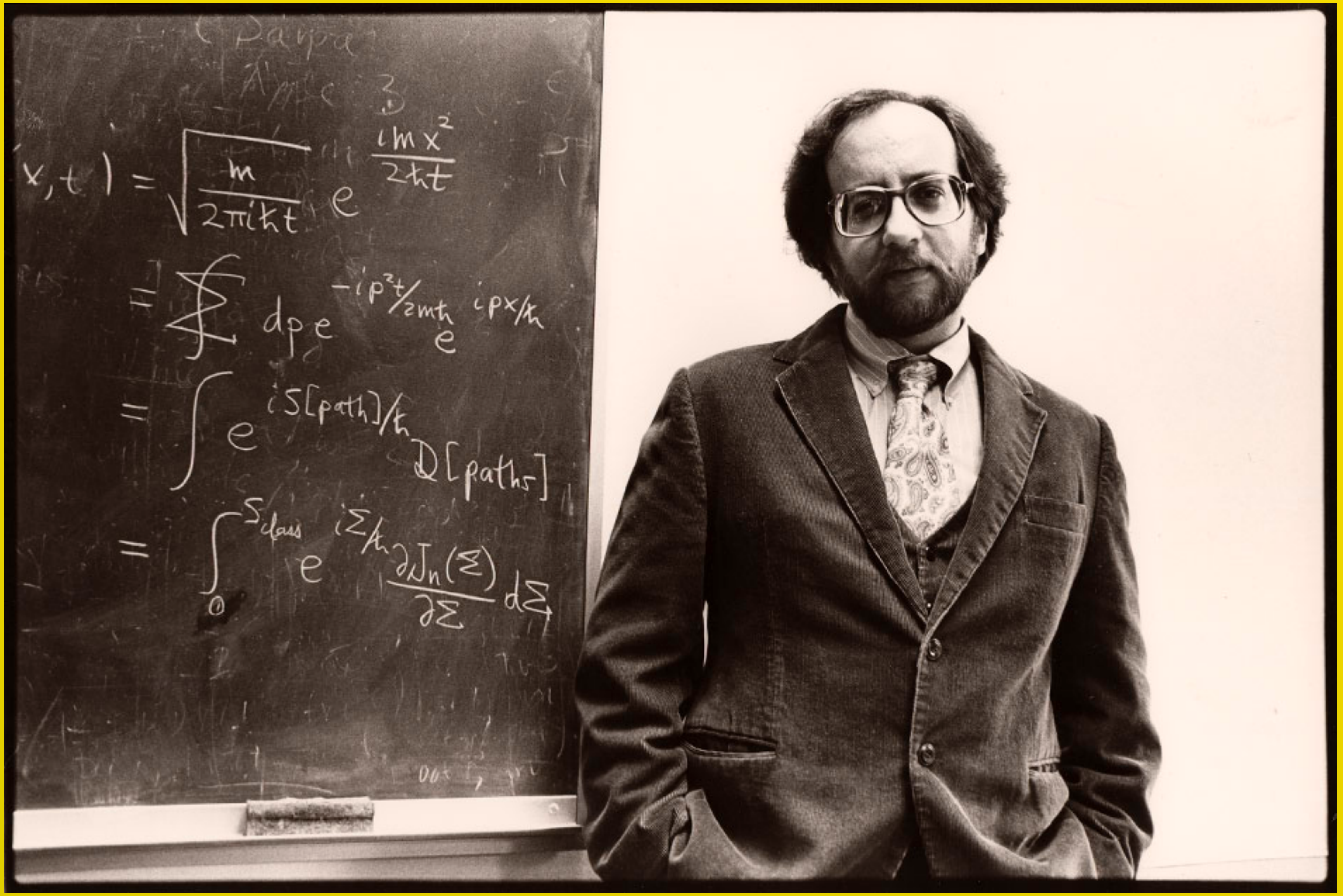
$$x^{1/\sqrt{x}}$$


$f = x^{1/\sqrt{x}}$
f'
2.0
1.8
1.6
1.4
1.2
1.0
0.8
2
4
6
8
10
1.4
1.2
1.0
0.8
0.6
0.4
0.2
2
4
6
8
10

LARGE NUMBERS

10^{100} googol

$10^{(10^{100})}$ googolplex



1947-2012) physisist

apple chief cryptographer

Next Chief scientist

RICHARD CRANDALL

1987 article in Scientific American

prime: $2^{1,398,269}-1$. This number, which has over 400,000 decimal digits, is the largest known prime number as of this writing. It is, like most other record holders, a so-called Mersenne prime. These numbers take the form 2^q-1, where q is an integer, and are named after the 17th-century French mathematician Marin Mersenne.

For this latest discovery, Woltman optimized an algorithm called an irrational-base discrete weighted transform, the theory of which I developed in 1991 with Barry Fagin of Dartmouth College and Joshua Doenias of NeXT Software in Redwood City, Calif. This method was actually a by-product of cryptography research at NeXT.

Blaine Garst, Doug Mitchell, Avadis Tevanian, Jr., and I implemented at NeXT what is one of the strongest—if not the strong-

MARY BONO

COLOSSI become somewhat easier to contemplate—and compare—if one adopts a statistical view. For instance, it would take approximately $10^{3,000,000}$ years before a parrot, pecking randomly at a keyboard, could reproduce by chance *The Hound of the Baskervilles*. This time span, though enormous, pales in comparison to the $10^{10^{33}}$ years that would elapse before fundamental quantum fluctuations might topple a beer can on a level surface.

John Littlewood: probability of mouse surviving on sun for 1 week

$$p = 10^{-(10^{42})}$$

John Littlewood: probability of mouse surviving on sun for 1 week

$$p = 10^{-(10^{42})}$$

probability to find yourself on Mars by quantum teleportation

$$p = 10^{-(10^{51})}$$

MORE EXAMPLES

R.E. Crandall, Scient. Amer., Feb. 1997

10^4	One "myriad". The largest numbers, the Greeks were considering.
10^5	The largest number considered by the Romans.
10^10	The age of our universe in years.
10^22	Distance to our neighbor galaxy Andromeda in meters.
10^23	Number of atoms in two gram Carbon (Avogadro).
10^26	Size of universe in meters.
10^41	Mass of our home galaxy "milky way" in kg.
10^51	Archimedes's estimate of number of sand grains in universe.
10^52	Mass of our universe in kg.
10^80	The number of atoms in our universe.
10^153	Number mentioned in a myth about Buddha.
10^(10^7)	Years, ape needs to write "hound of Baskerville"
10^(10^(33))	Inverse is chance that a can of beer tips
10^(10^(42))	Inverse is probability that mouse survives on sun for a week.
10^(10^{51})	Inverse is chance to find yourself on Mars

BRACKETS

Note that the brackets matter:

$$256 = 2^8 = 2^{(2^3)} \neq (2^2)^3 = 4^3 = 64$$

JAM

Find the derivative of

$$x^{(x^x)}$$

CA TIME

Project Work

THE END