LECTURE 22

INVERSE TRIG FUNCTIONS

Oliver Knill, Harvard University

October 28 2020

PLAN

1. Poll

2.log review

3. arctan

4. arcsin

5. harvard statue

6. arcsinh

7. CA jam

POLL

what is

$\tan(\arctan(x))$

A 1

B 0

C x

D $\pi/4$

LOG REVIEW

$$y = \log(x)$$

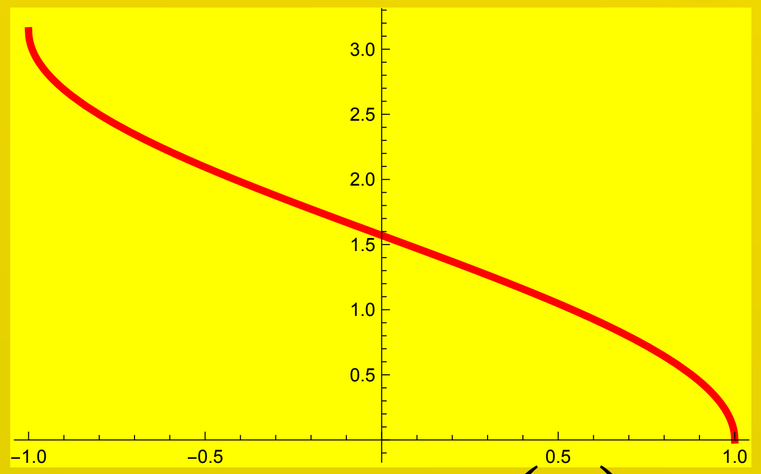
$$\exp(y) = x$$

$$\exp(y)y' = 1$$

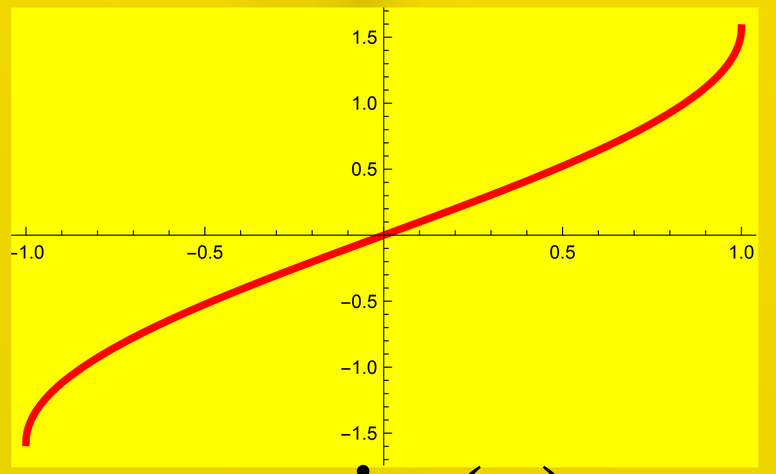
$$xy' = 1$$

$$\log'(x) = \frac{1}{x}$$

INVERSE TRIG



$\arccos(x)$



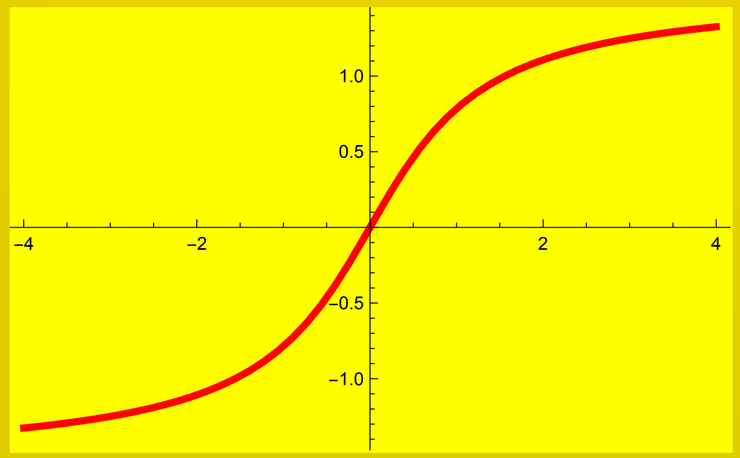
$\arcsin(x)$

Usually:

arccos ->[0,π]

arcsin->[-π/2,π/2]

arctan-> (-π/2,π/2)



$\arctan(x)$

DERIVATIVES

A Write down $f(y(x)) = x$

B Differentiate and keep y'

C Solve for y' and simplify

ARC TAN

$$\tan(\arctan(x)) = x$$

$$\sec^2(y)y' = 1$$

$$\sec^2(y) = 1 + x^2$$

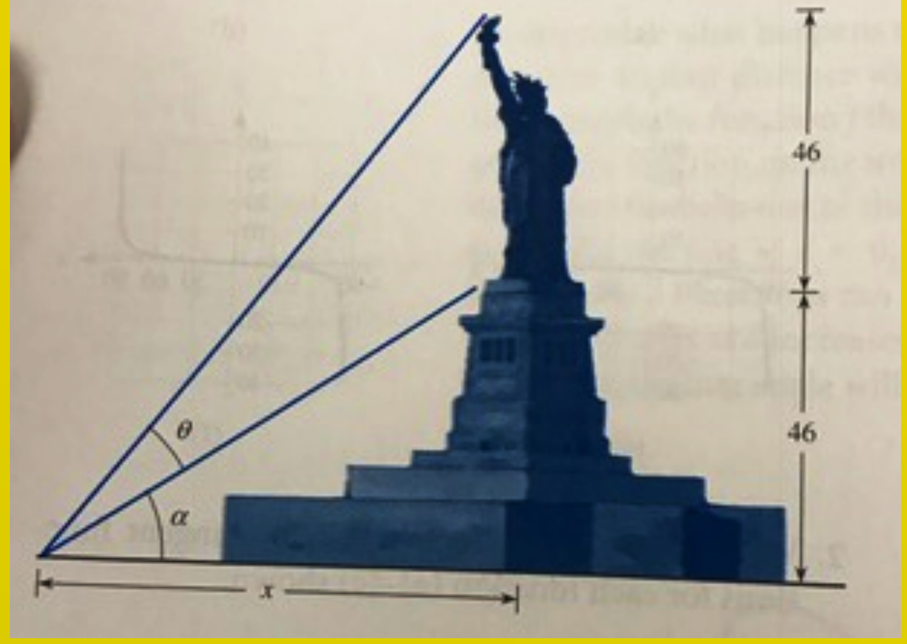
$$\sec^2(y) = 1 + \tan^2(y)$$

$$= 1 + x^2$$

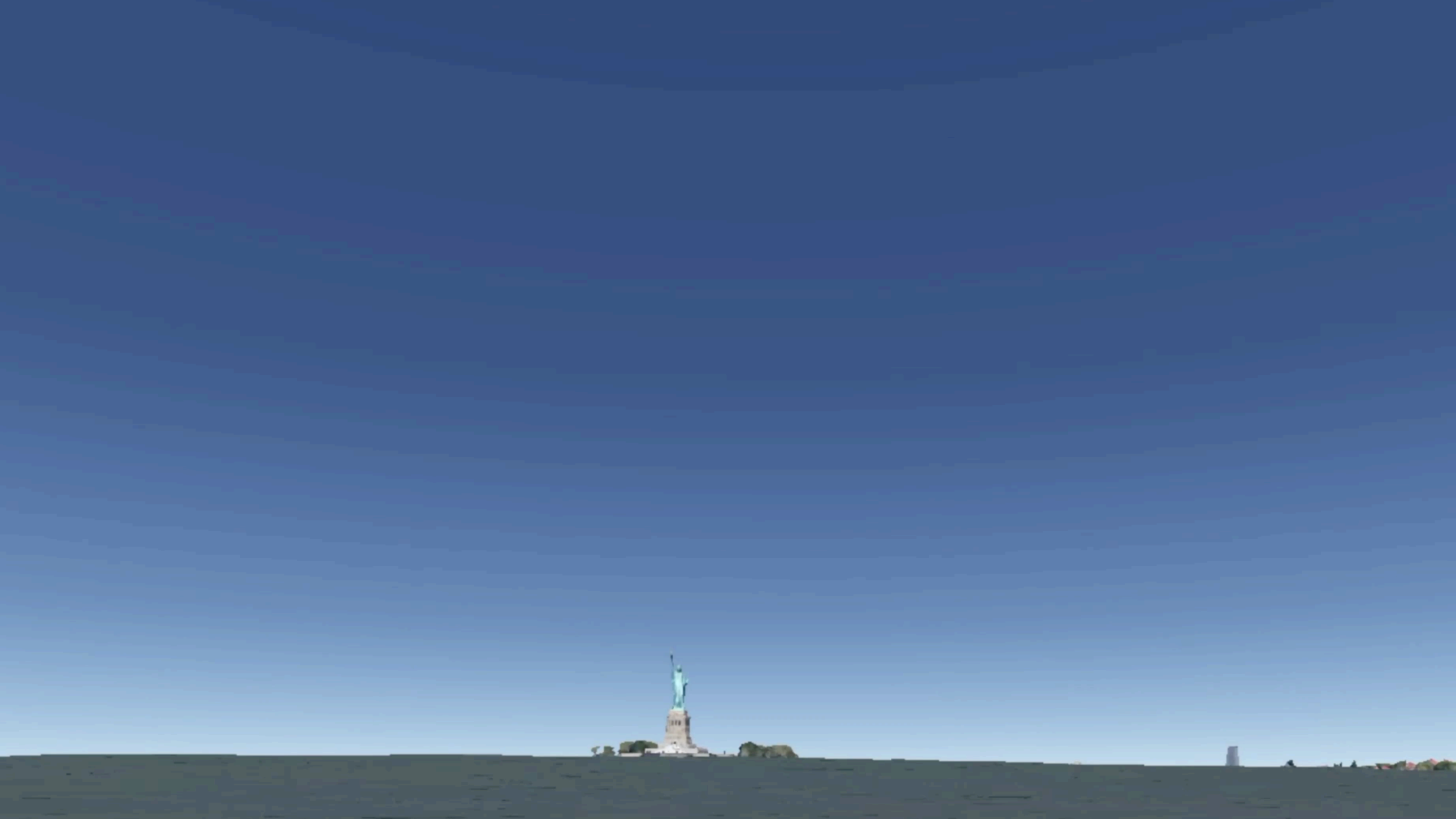
$$\arctan'(x) = \frac{1}{1 + x^2}$$

STATUE OF LIBERTY

4. The Statue of Liberty is 46 meters tall and stands on a base that is also 46 meters tall. Find an expression for the angle subtended by the statue from ground level as a function of distance from the base of the statue. Use this function to estimate graphically the distance when the angle is maximum. Approximately what is this maximum angle?



A classical problem.

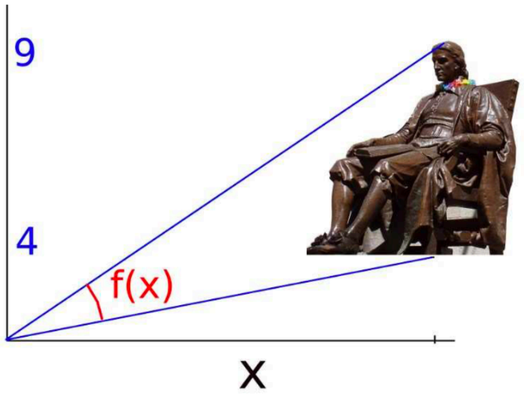


TO JOHN HARVARD

Lecture 12: Worksheet

Extrema with boundaries

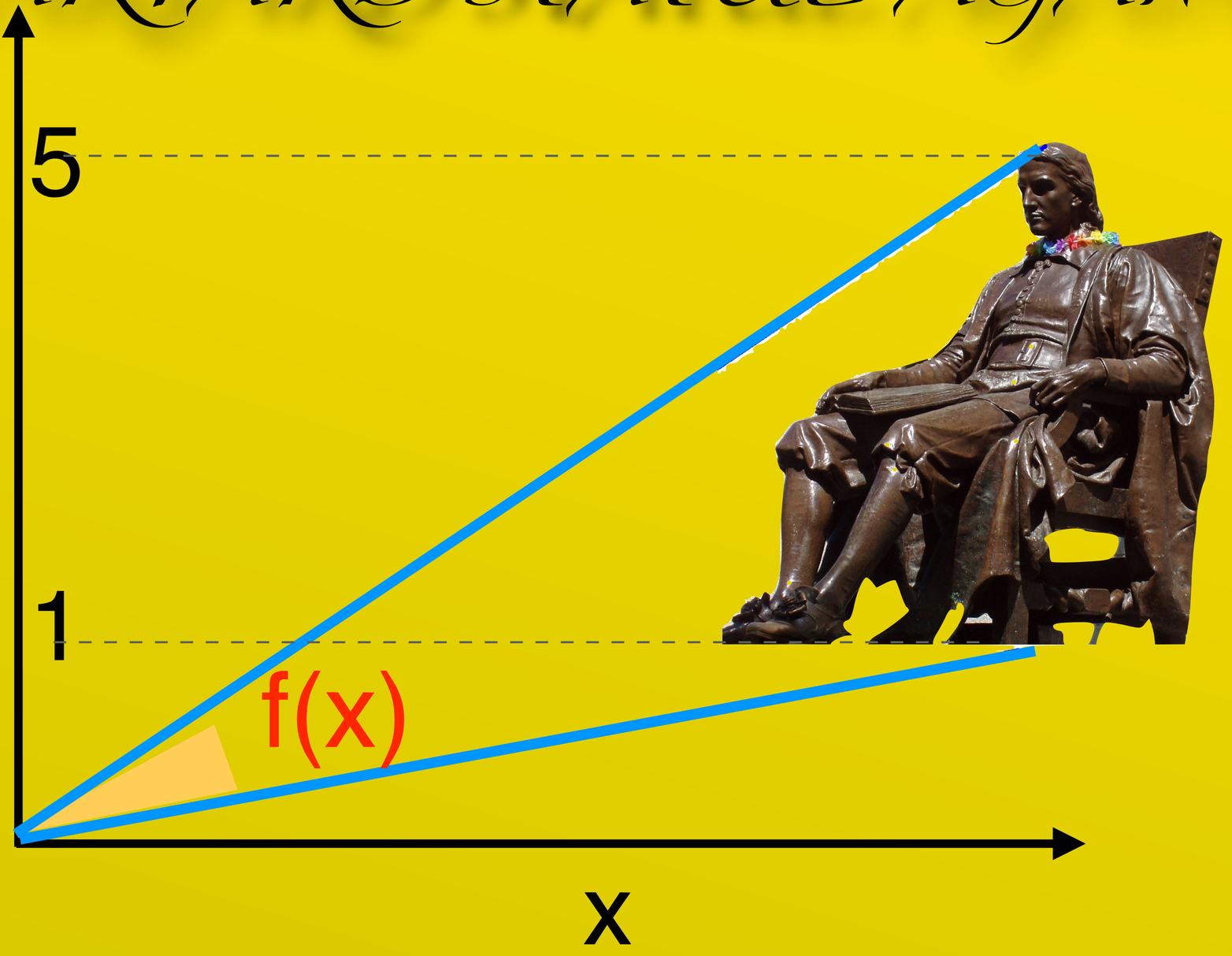
The following famous problem is usually asked with the Statue of liberty. At Harvard, we of course want to use the John Harvard Statue. It is a common situation. You want to look at a statue. If you are too close below it, the viewing angle becomes small. If you are far away, the viewing angle decreases again. There is an optimal distance where the viewing angle is maximal.




October 24, 2020

HARVARD STATUE AGAIN

From which distance x do you see the Harvard Statue best?



ARC CSC

$$\csc(\operatorname{arccsc}(x)) = x$$

$$\csc'(y)y' = 1$$

$$y'(x) = 1/\csc'(y)$$

Use $\csc'(\operatorname{arccsc}(x)) = -x\sqrt{x^2-1}$

$$y'(x) = \frac{1}{-x\sqrt{x^2-1}}$$

JAM

ARC COT

Compute the derivative of

arccot(x)

by following the computation of arctan(x)

ARC SIN

$$\sin(y) = x$$

$$\cos(y)y' = 1$$

$$\sqrt{1 - x^2}y'(x) = 1$$

$$\arcsin'(x) = \frac{1}{\sqrt{1 - x^2}}$$

CAJAM

Why is $x = -\pi/2$ not the the usual solution to

$$\arccos(\cos(x)) = -\frac{\pi}{2}$$

Explain what the problem is by drawing the graphs of cos(x) and arccos(x)

THE END