LECTURE 23

MORE PROBLEMS

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PLAN

1. Poll

2.Ernie and Bert

3. Herb Garden

4. Balloon

5. Dogs and Sticks

6. Dog and Sticks JAM

7. CA time: Projects

POLL

If

x y=1

and x'=5

wha is y'?

A 0

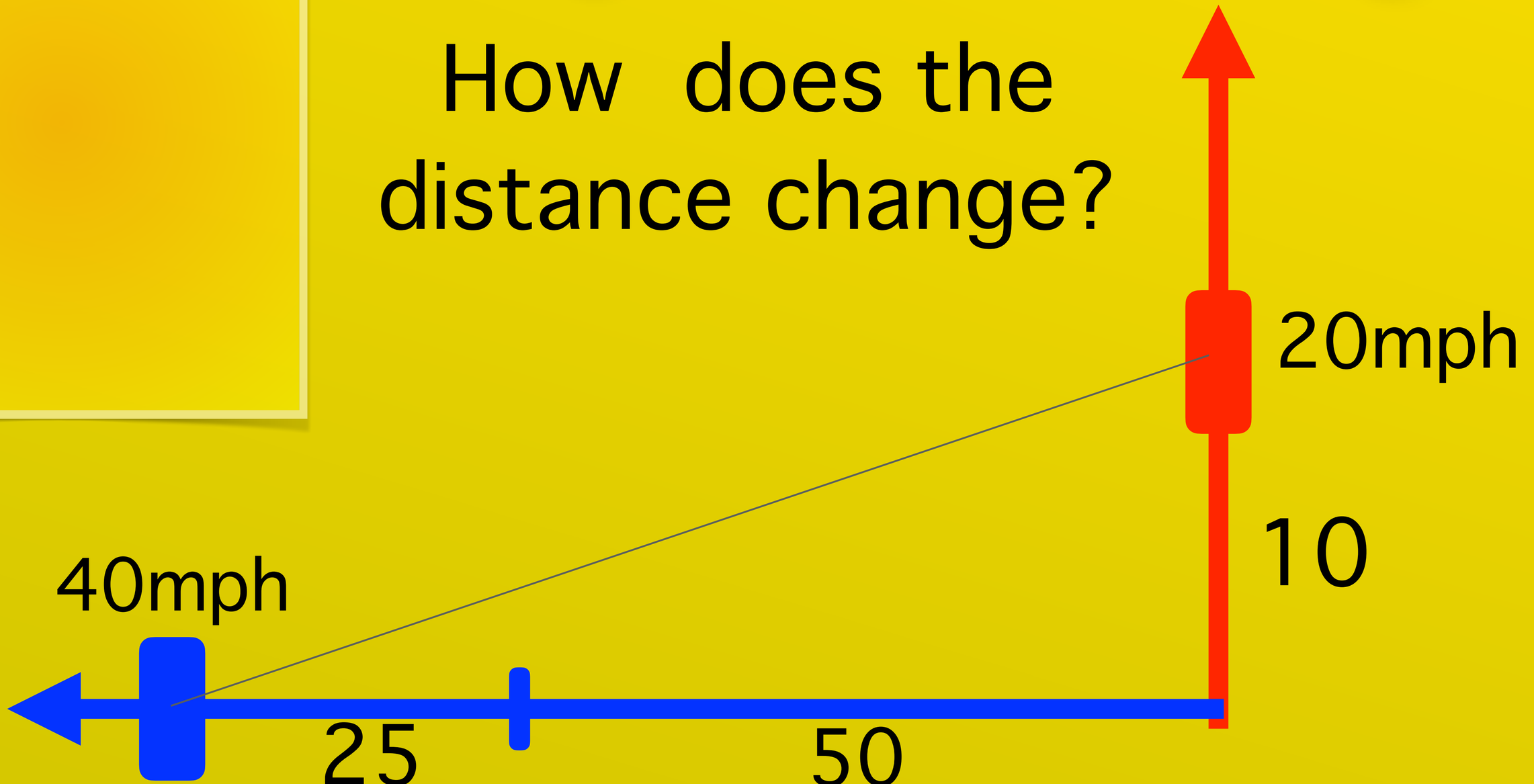
B 5

C -5

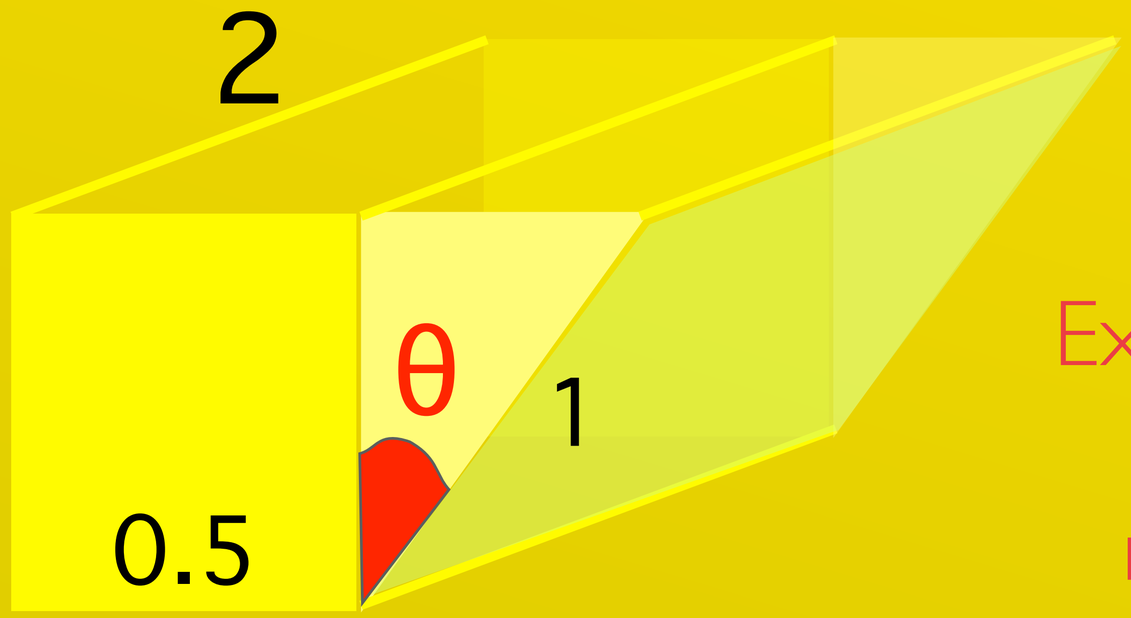
D 1

ERNIE AND BERT

How does the distance change?



HERB GARDEN



Surprise:
Extrema problem,
not a related
rates problem

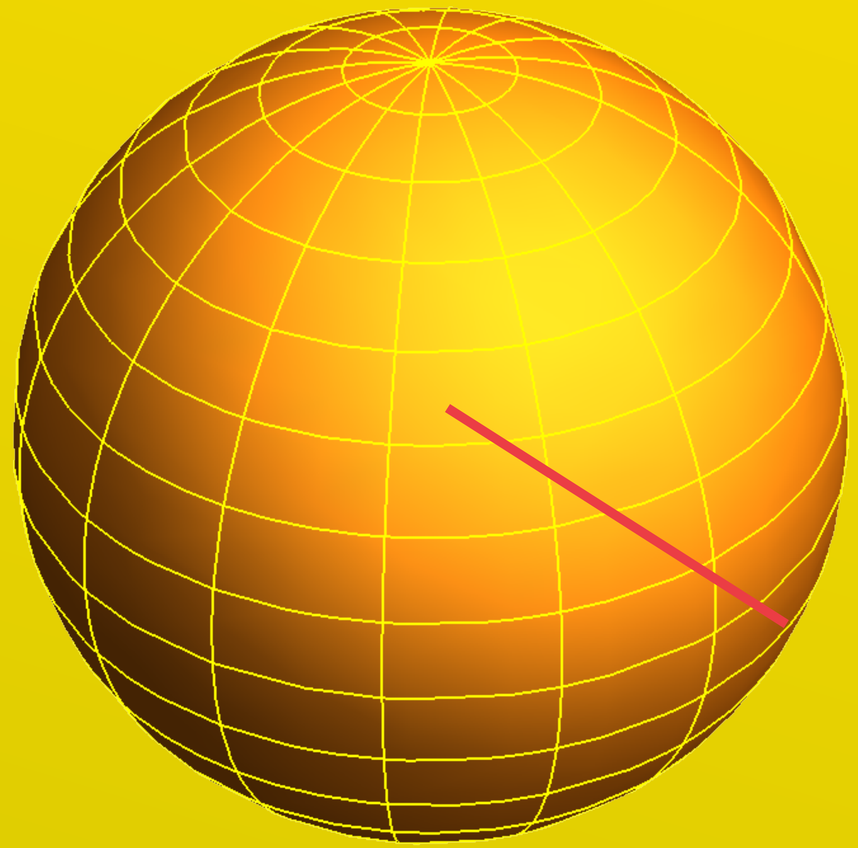
$$V=2\,\frac{\cos(\theta)\sin(\theta)}{2}+2*0.5\cos(\theta)$$

$$=\cos(\theta)\sin(\theta)+\cos(\theta)$$

BALLOON PROBLEM

Aeronauts, 2019

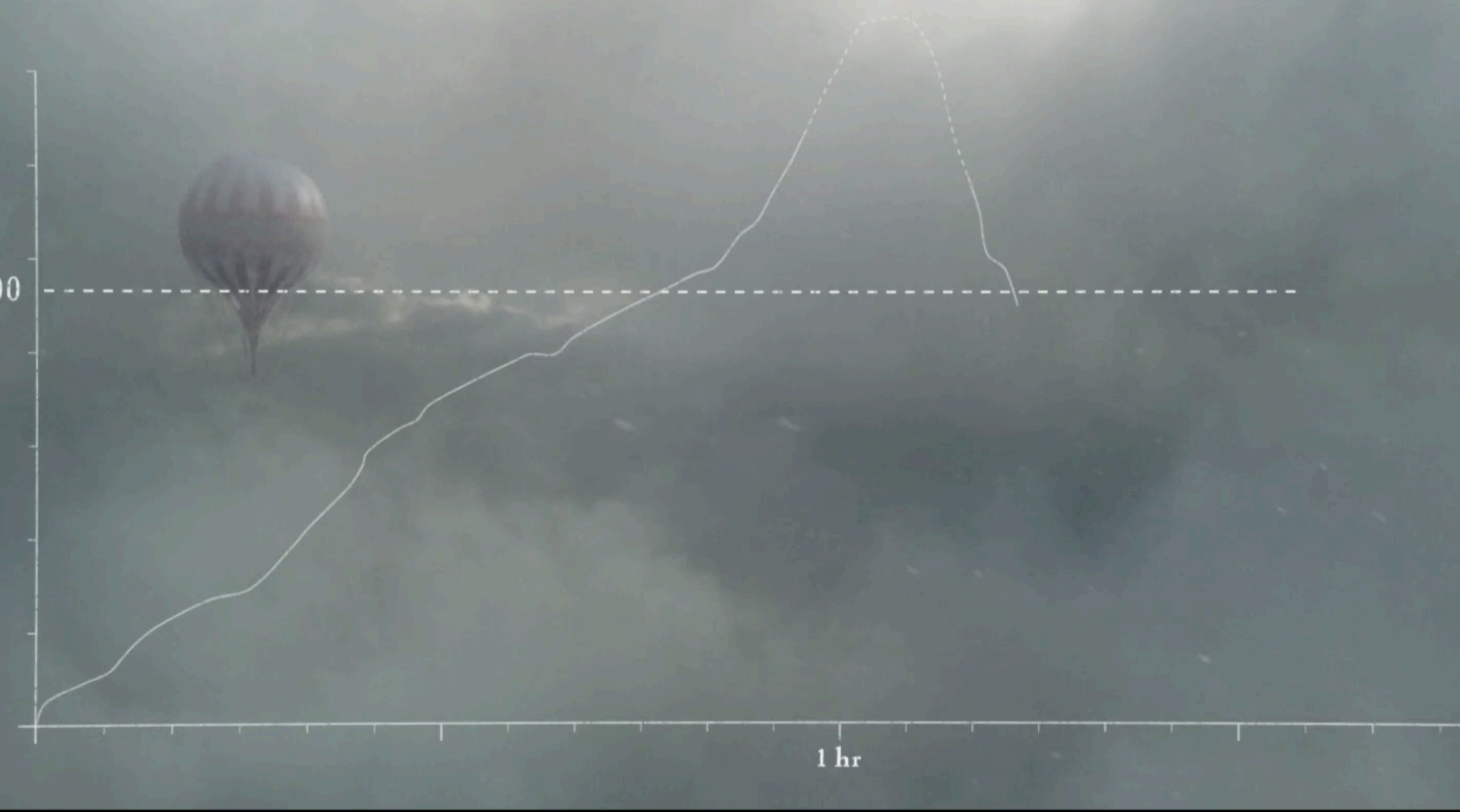
BALLOON



$V' = 1/2$

$r = 10$

What is r'?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1 hr

HYDROPHYLIC SPHERES

Assume that the volume change is proportional to the radius

$$V = 4\pi r^3/3$$

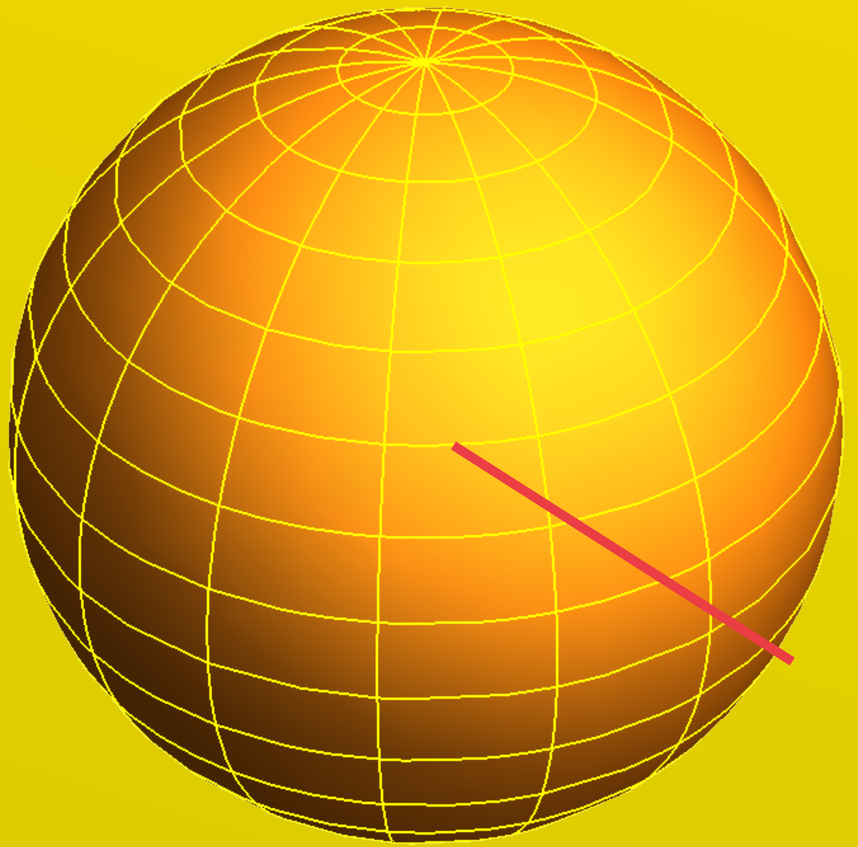
$$V' = 20r$$

What is r' if r=10?

HYDROPHYLIC SPHERES

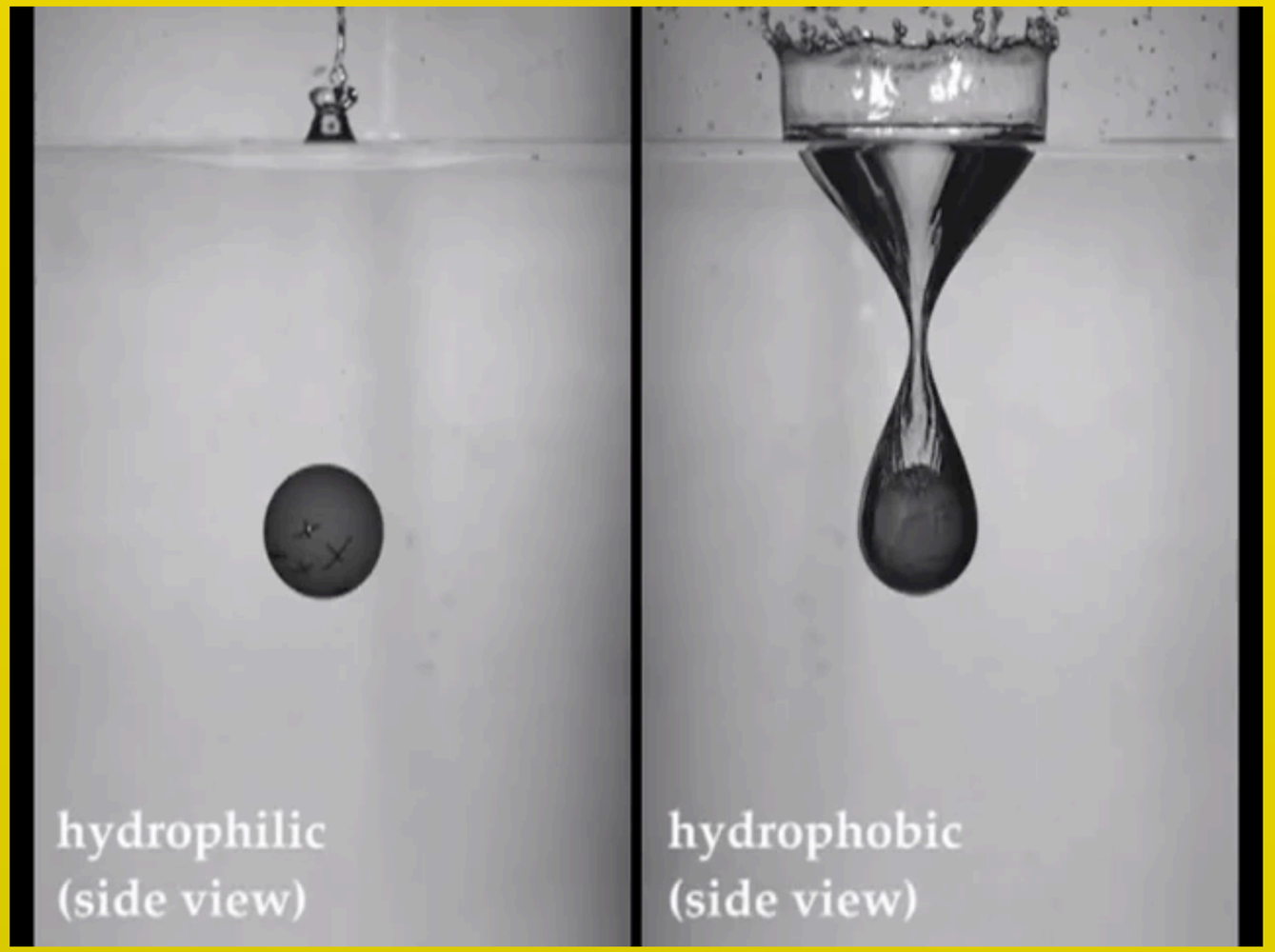
$$V = 4\pi r^3/3$$

$$V' = 20r$$



What is r' if r=10?

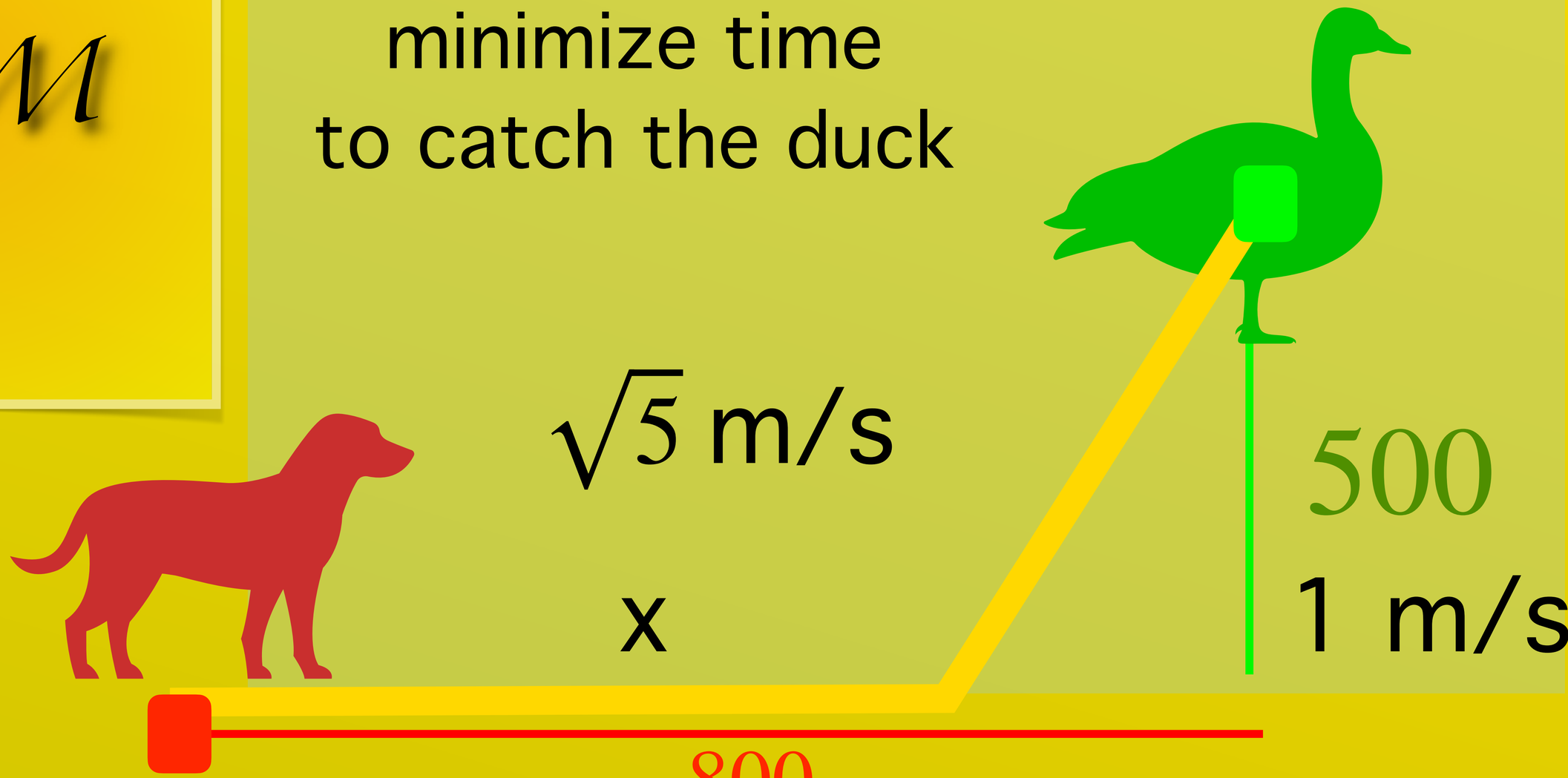
INTERESTING SCIENCE



DOG PROBLEM

JAM

minimize time
to catch the duck



DOG PROBLEM

Most calculus students are familiar with the calculus problem of finding the optimal path from A to B. "Optimal" may mean, for example, minimizing the time of travel, and typically the available paths must transverse two different mediums, involving different rates of speed.

This problem comes to mind whenever I take my Welsh Corgi, Elvis, for an outing to Lake Michigan to play fetch with his favorite tennis ball. Standing on the water's edge (See Figure 1) at A, I throw the ball into the water to B. By the look in Elvis's eyes and his elevated excitement level, it seems clear that his objective is to retrieve it as quickly as possible rather than, say, to minimize his expenditure of energy. Thus I assume that he unconsciously attempts to find a path that minimizes the retrieval time.

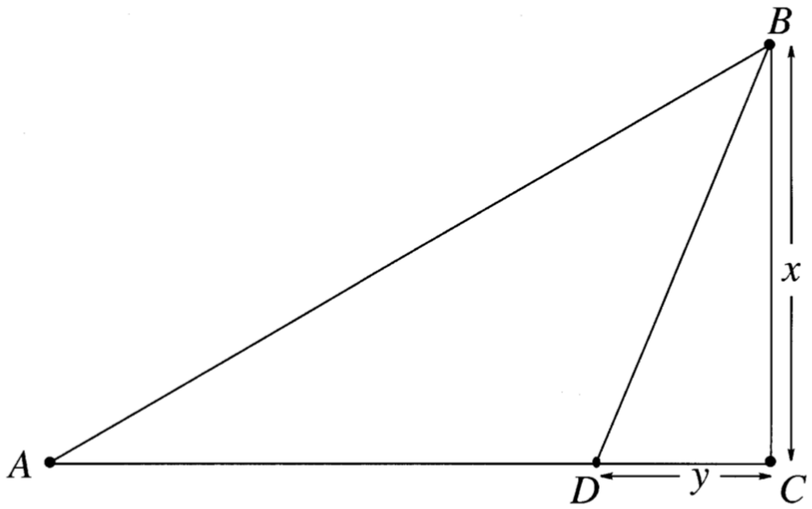


Figure 1. Paths to the ball

Tim Pennings,
2003 College math
journal

DOG PROBLEM

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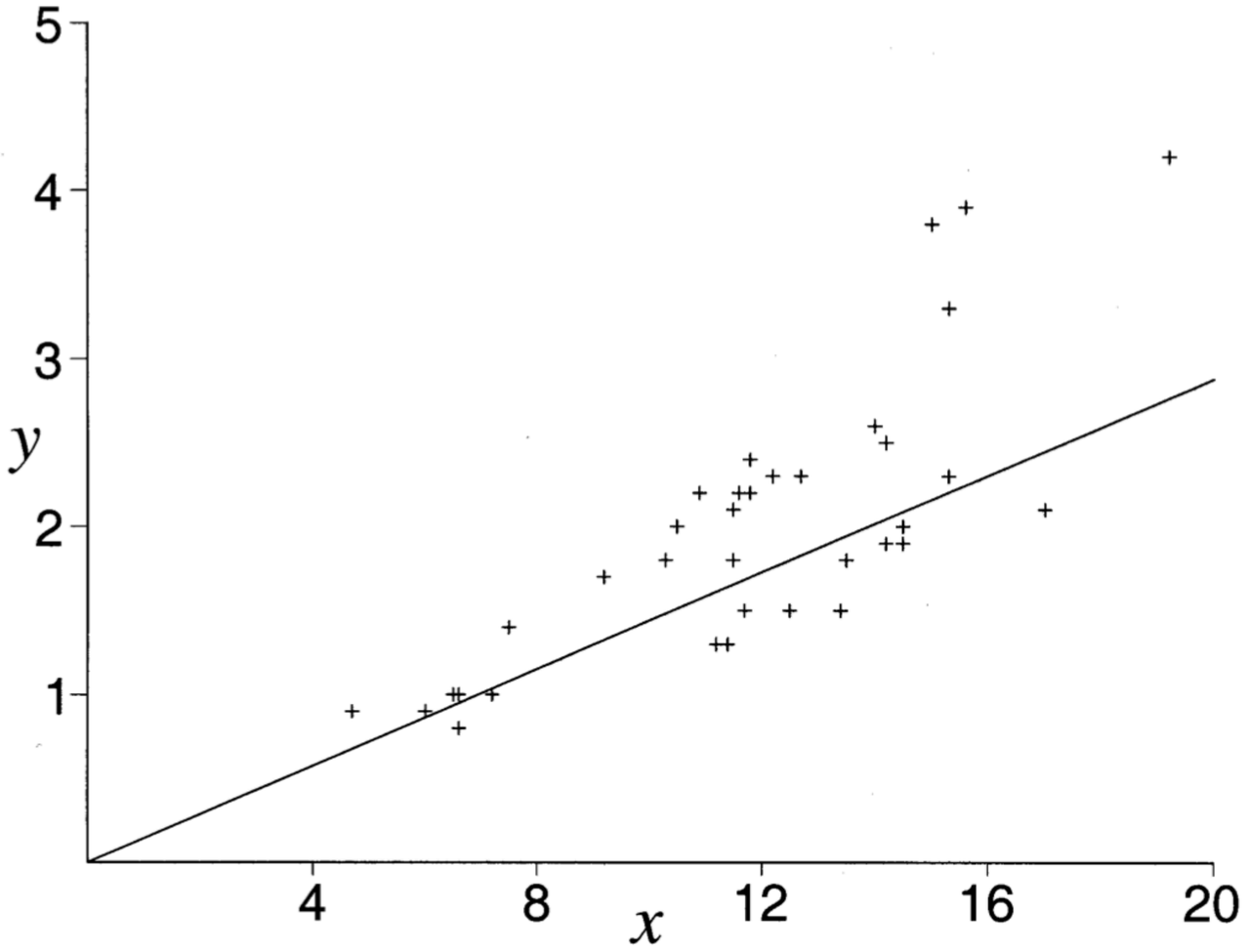

Figure 3. Scatter plot with optimal line

DOG PROBLEM

Second, we confess that although he made good choices, Elvis does not know calculus. In fact, he has trouble differentiating even simple polynomials. More seriously, although he does not do the calculations, Elvis's behavior is an example of the uncanny way in which nature (or Nature) often finds optimal solutions. Consider how soap bubbles minimize surface area, for example. It is fascinating that this optimizing ability seems to extend even to animal behavior. (It could be a consequence of natural selection, which gives a slight but consequential advantage to those animals that exhibit better judgment.)

Finally, for those intrigued by this general study, there are further experiments that are available, other than using your own favorite dog. One might do a similar experiment with a dog running in deep snow versus a cleared sidewalk. Even more interesting, one might test to determine whether the optimal path is found by six-year-old children, junior high aged pupils, or college students. For the sake of their pride, it might be best not to include professors in the study.

Tim Pennings,
2003 College math
journal

CA TIME

Project check in

THE END