

LECTURE 25
RELATIVE GROWTH
RATES

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PLAN

1. Poll

2. Relative Growth rates

4. Use Hopital

5. Examples

6. JAM

7. CA time:JAM

POLL

A

$$x^3 \log(x)^3$$

B

$$3^x \log(x)$$

C

$$e^x x^3$$

D

$$x^3 / 3^x$$

E

$$3^x / x^3$$

What grows
fastest for

$$x \rightarrow \infty$$

GROWTH

official
common
use is the

Landau little o
notation

$$g = o(f)$$

$$g \ll f$$

means

$$\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = 0$$

Landau
little o
notation

$$g = o(f)$$

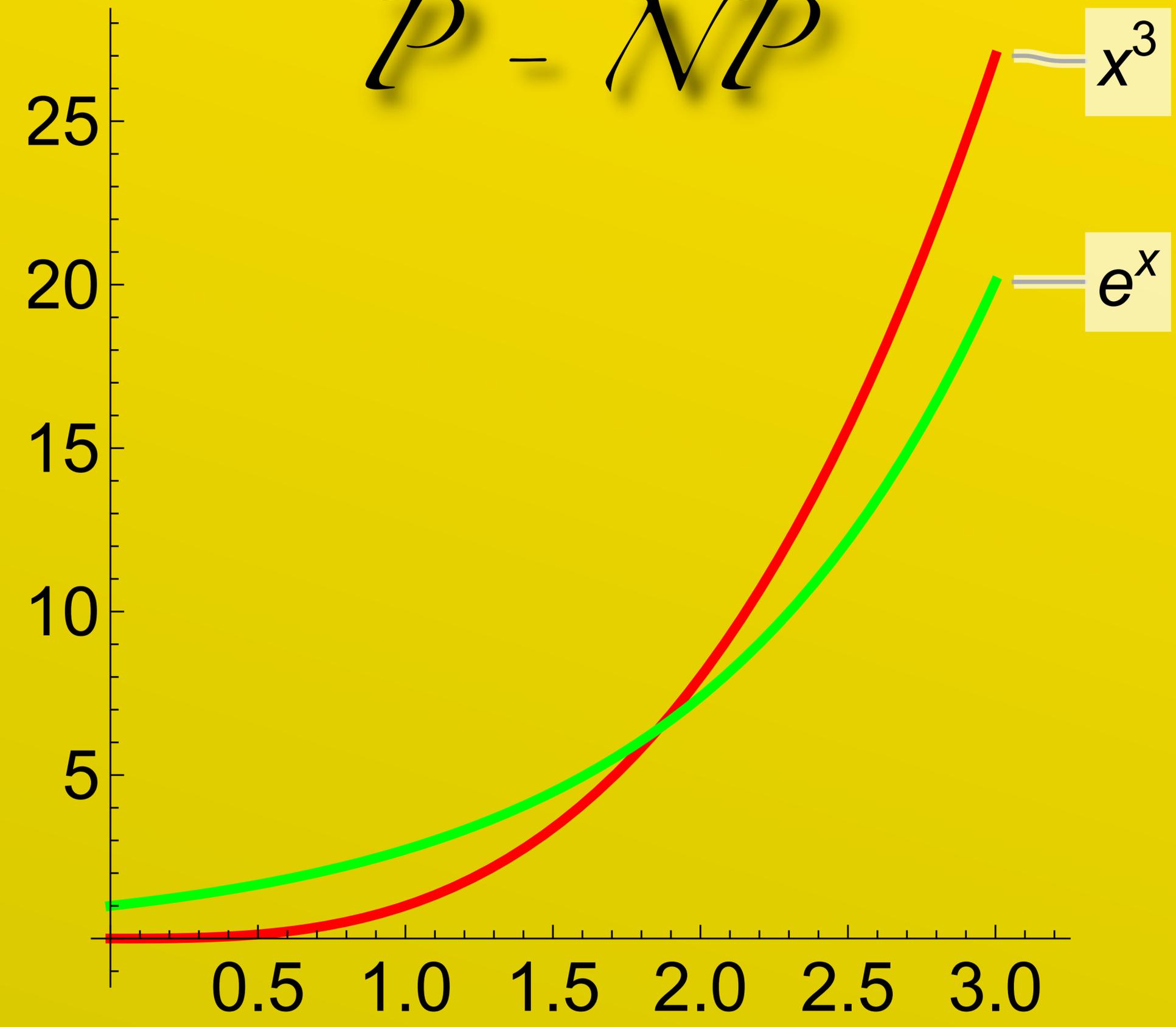
Our soccer team
in a tournament

"LITTLE O"

Oliver



$\mathcal{P} - \mathcal{NP}$



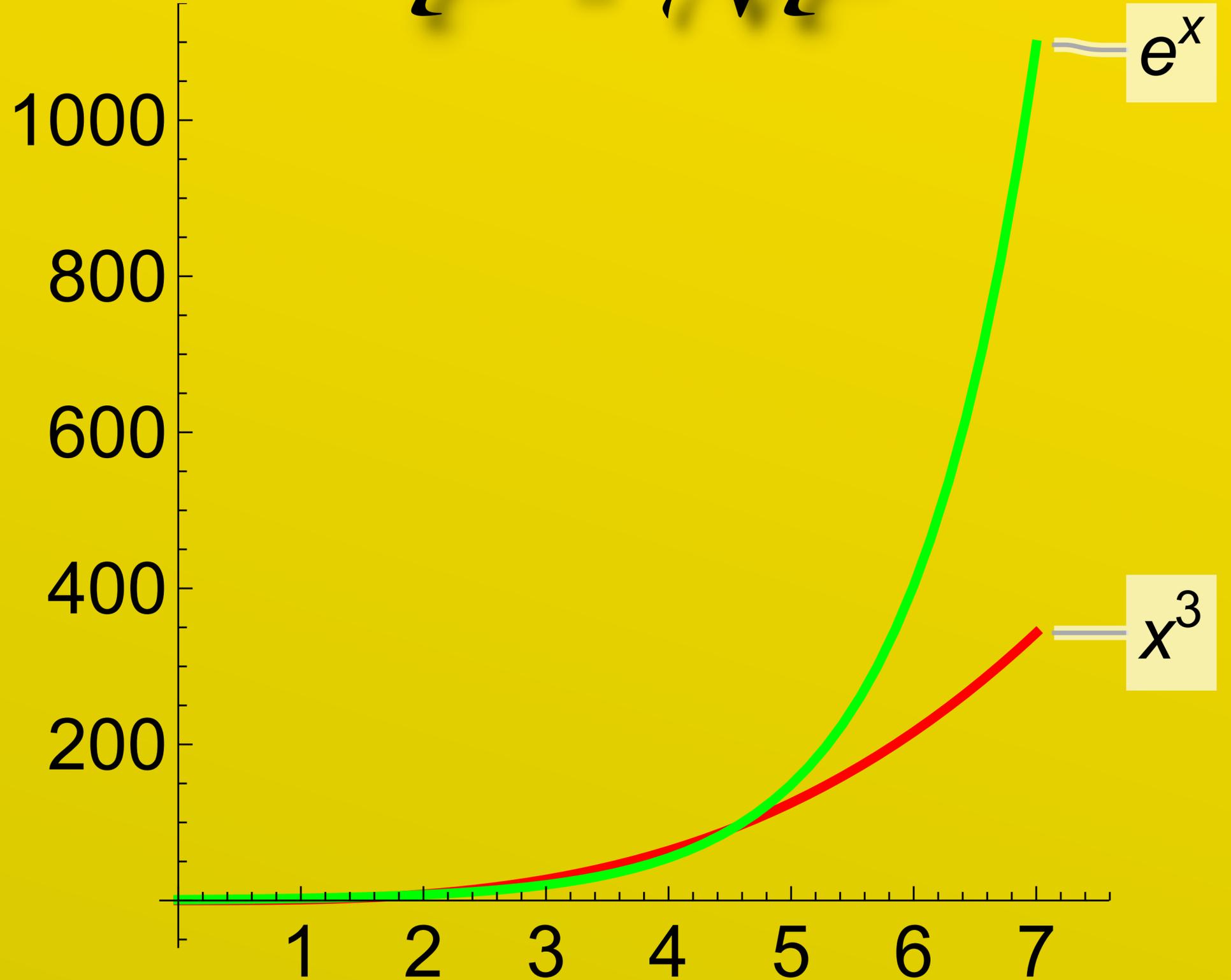
What grows faster?

$e^x \gg x^3$

or

$e^x \ll x^3$

$\mathcal{P} - \mathcal{NP}$



Use Hopital on

$$x^3/e^x$$

to see it!

$\mathcal{P} - \mathcal{NP}$

$$b^x \gg x^b$$

for every positive b !

$$f \gg g$$

means

$$\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = 0$$

P=NP?



Stephen Cook
1971

P - NP PROBLEM

P decision
Problems

NP-decision
Problems

P: can be solved
in polynomial time

NP: can be verified
in polynomial time

P stands for for Polynomial

NP for Nondeterministic Polynomial

Travelling Salesman movie (2012)



LOG-ROOTS

$$\log(x) \ll \sqrt{x}$$

Can you prove this with Hopital?

Clever:

replace x with x^2

$$\log(x^2) \ll x$$

then apply exp on both sides

$$x^2 \ll e^x$$

LOG-ROOTS

Logs grow slower than roots



COMPARE

A

What grows faster?

$$f(x) =$$

$$x + 10^{100} \log(x)$$

$$g(x) =$$

$$x + 10^{100} \sqrt{x}$$

B

$$\lim_{x \rightarrow \infty}$$

$$x + 10^{100} \log(x)$$

$$x + 10^{100} \sqrt{x}$$

C

$f \ll g$ or
 $g \ll f$ or
neither?

JAM: ORDER

order
according
to growth

$$1.1^x$$

$$3^{2x}$$

$$2^{3x}$$

$$x^{10}$$

$$x^{1.1}$$

$$x \log(x)$$

$$\log_2(x)$$

CAJAM

fill in the
colored
bricks
according
to growth

growth



$$e^x$$

$$\log(x)$$

$$\sqrt{x}$$

$$x \log(x)$$

$$x^3$$

THE END