

# *LECTURE 26*

$1^\infty$

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# PLAN

1. Poll on indeterminate

2. The indeterminate form  $1^\infty$

4. Euler Number

5. More limits

6. JAM

7. CA time:JAM

*POLL*

A

$0^\infty$

B

$\infty^0$

C

$\infty - \infty$

D

$0/0$

E

$0 - 0$

F

$0^0$

Find the  
determinant form

*MORE*

$0^\infty$

$\infty^0$

$\infty - \infty$

$\infty / \infty$

$0/0$

$\infty/0$

$0 - 0$

$1^\infty$

$0^0$



Because  $1 = e^0$   
we have  $1^{0 \cdot \infty}$

$$f(x) = \arctan(x)(2/\pi)$$

$$g(x) = \log(x)$$

$$f(x)^{g(x)}$$

$$f(x)^{g(x)} \rightarrow 1$$

# IMPORTANT

What is  $\lim_{x \rightarrow 0} \left(1 + \frac{1}{x}\right)^x$

$$\left(1 + \frac{1}{x}\right)^x = e^{x \log(1 + 1/x)}$$

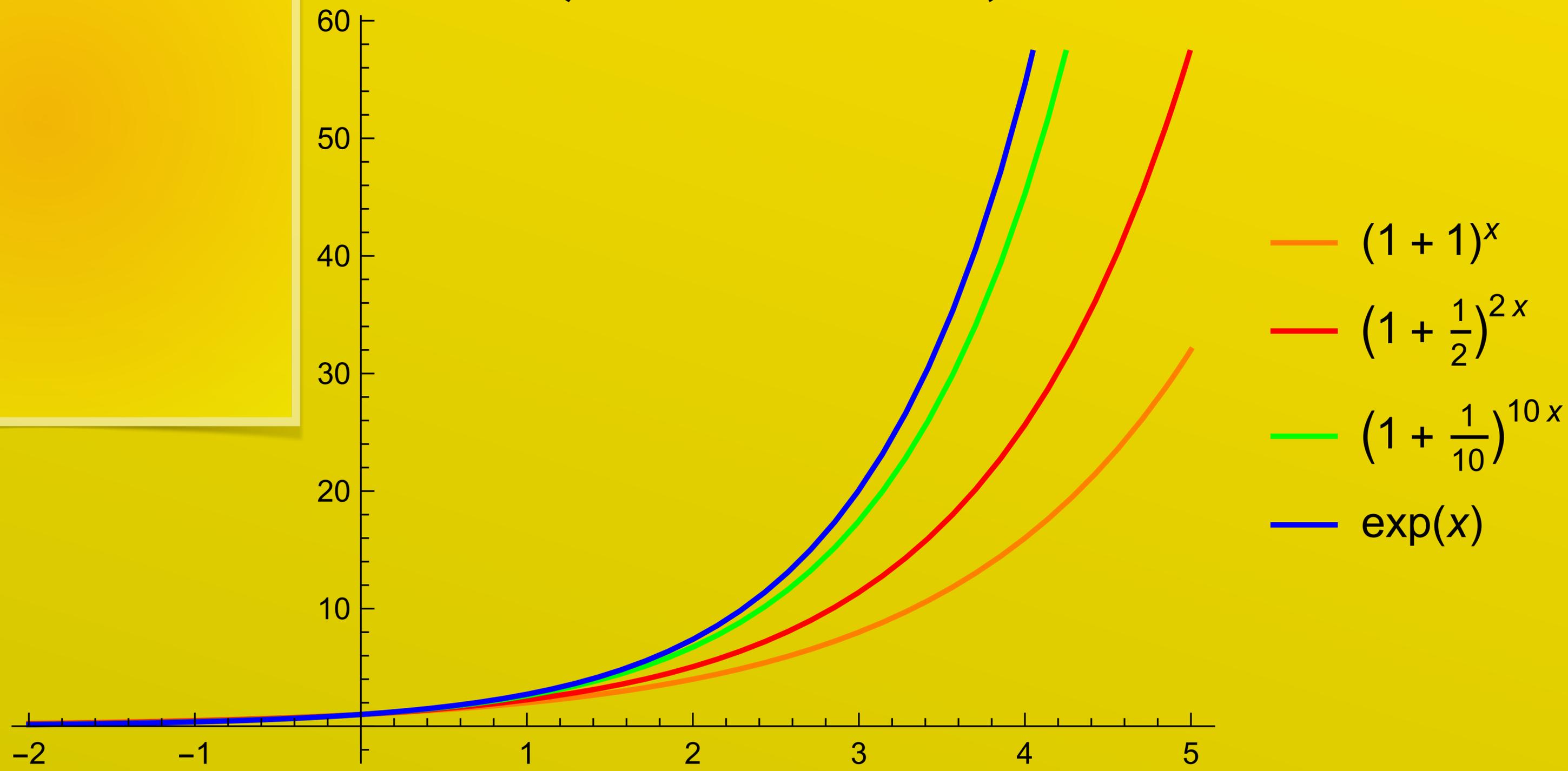
$$x \log\left(1 + \frac{1}{x}\right) = \frac{\log\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} \rightarrow e \quad \lim_{y \rightarrow 0} \frac{\log(1 + y)}{y} = 1$$

$e$

$$\left(1 + \frac{1}{n}\right)^n$$

$$\left(\frac{100001}{100000}\right)^{100000} = 2.71827$$

$$(1 + h)^{x/h}$$



# *AVERAGE RATE OF CHANGE*

$$f(x) = (1 + h)^{x/h}$$

$$\frac{f(x + h) - f(x)}{h} = ?$$

# *COMPOUND INTEREST*

$$(1 + h)^{x/h} \rightarrow e^x$$

Interpretation:  $h$  is interest rate

$n=(x/h)$  number of years



# *COMPOUND INTEREST*



# 1000 \$ PROBLEM



# *APY*

1000 \$ deposit  $i=6\%$  interest

How much after  $n$  years?

Assume we compound

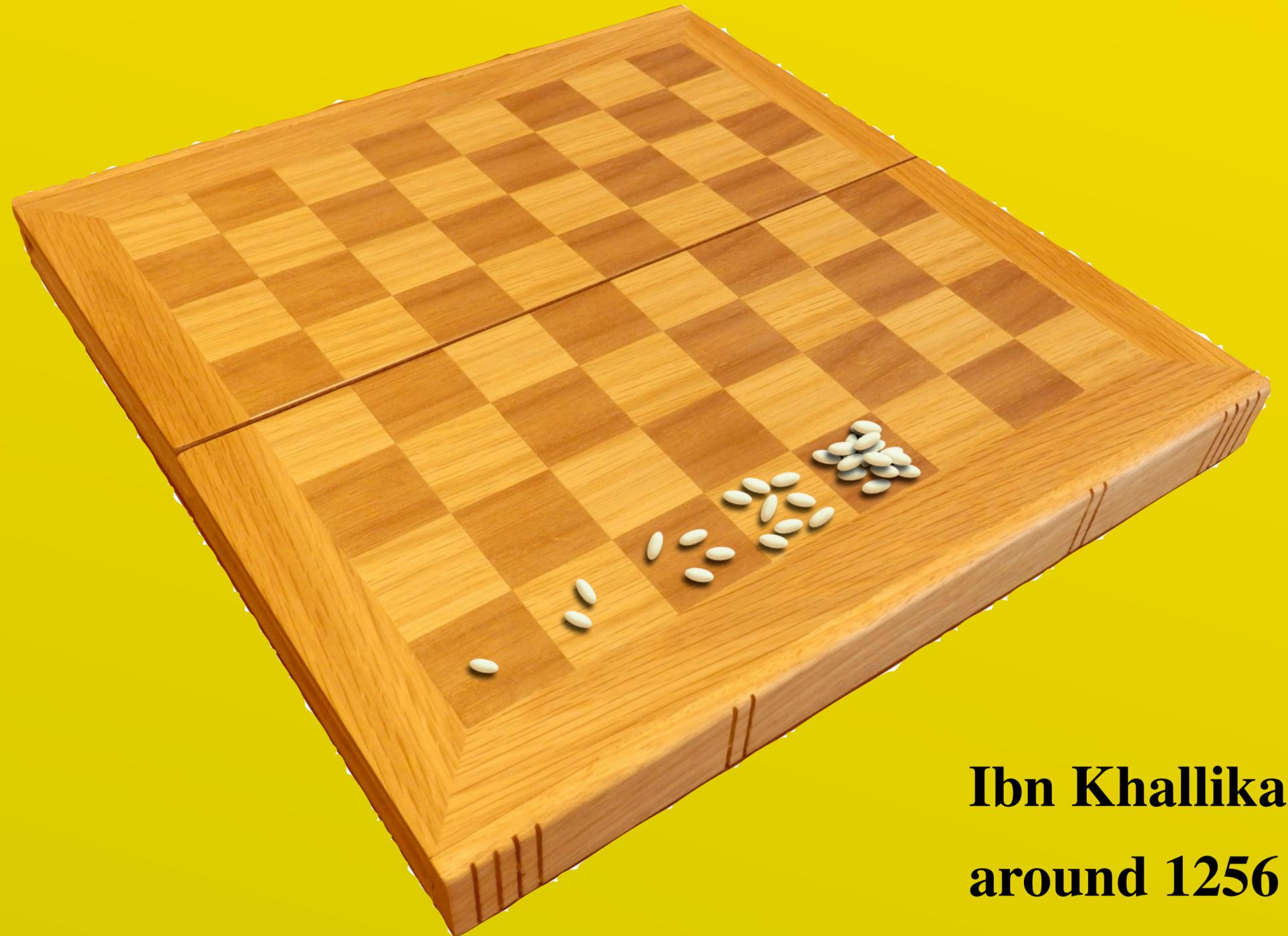
$N=12$  times per year.

Annual  
percentage  
yield

$$APY = \left(1 + \frac{i}{N}\right)^N - 1$$

# *PERSIAN CHESS BOARD*

"Would you rather have a million dollars or a penny on day one, doubled every day until day 30?"



start with 1 grain,  
double in  
each field. How much  
grain do we have?

**Ibn Khallikan  
around 1256**

# *REMINDEES*

Hopital

Simplify

Difference quotient

*JAM*

indeterminant  
forms

A

$$\lim_{x \rightarrow 1} \frac{x}{x-1} - \frac{1}{\log(x)}$$

B

$$\lim_{x \rightarrow \infty} \sqrt{x^2 + x} - x$$

C

$$\lim_{x \rightarrow \pi/2} \frac{\tan x}{\sec x}$$

D

$$\lim_{x \rightarrow 0} (1 - 2x)^{1/\cos(x)}$$

*CA  
JAM*

limits

E

$$\lim_{x \rightarrow 0} (1 - 2x)^{1/\sin(x)}$$

F

$$\lim_{x \rightarrow -\infty} x e^x$$

G

$$\lim_{x \rightarrow \pi/2} \sec(7x) \cos(3x)$$

H

$$\lim_{x \rightarrow \infty} \arctan(x) \log(x)$$

*THE END*