

*LECTURE 28*

*RIEMANN*

*INTEGRAL*

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# PLAN

1. Poll

2. The integral, three aspects

3. Island population

4. Pep Talk

5. Jam

6. Swimming pool

# *POLL*

A

2

B

0

C

NA

D

4

$$\int_0^2$$



d



What is the value?

*AND ...*

A

2

$\int_2^2$



d



B

0

C



What is the value?

D

NA

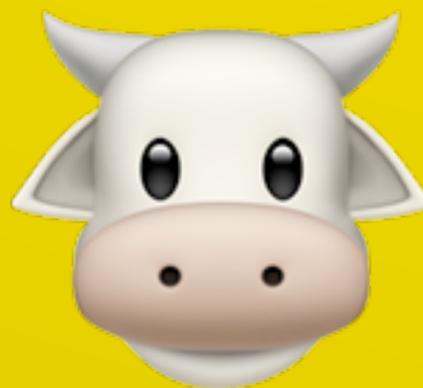
*AND ...*

A

1

$$\int_1^6$$

1 d



B

5

C

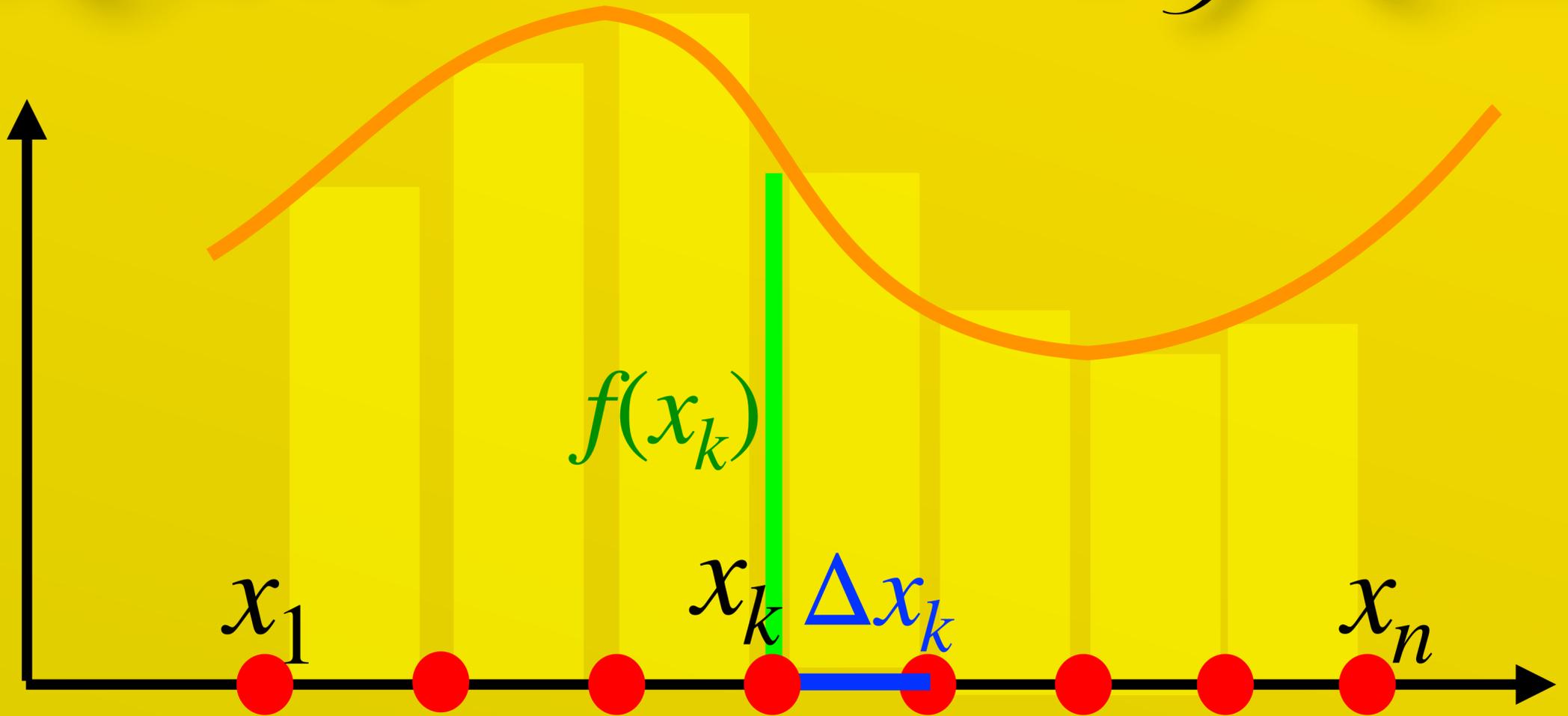


D

7

What is the value?

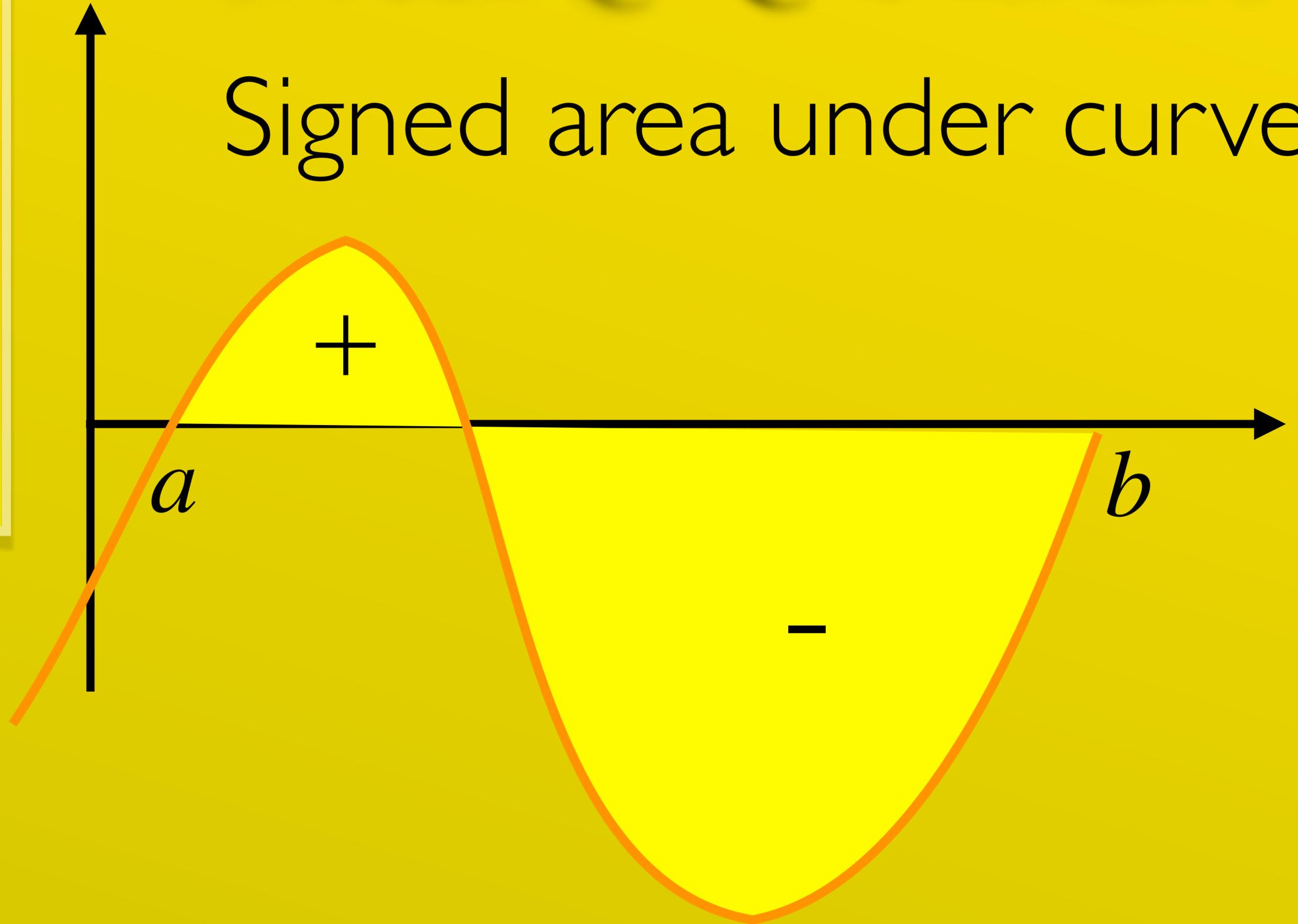
# RIEMANN INTEGRAL



$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{a \leq x_k < b} f(x_k) \Delta x_k$$

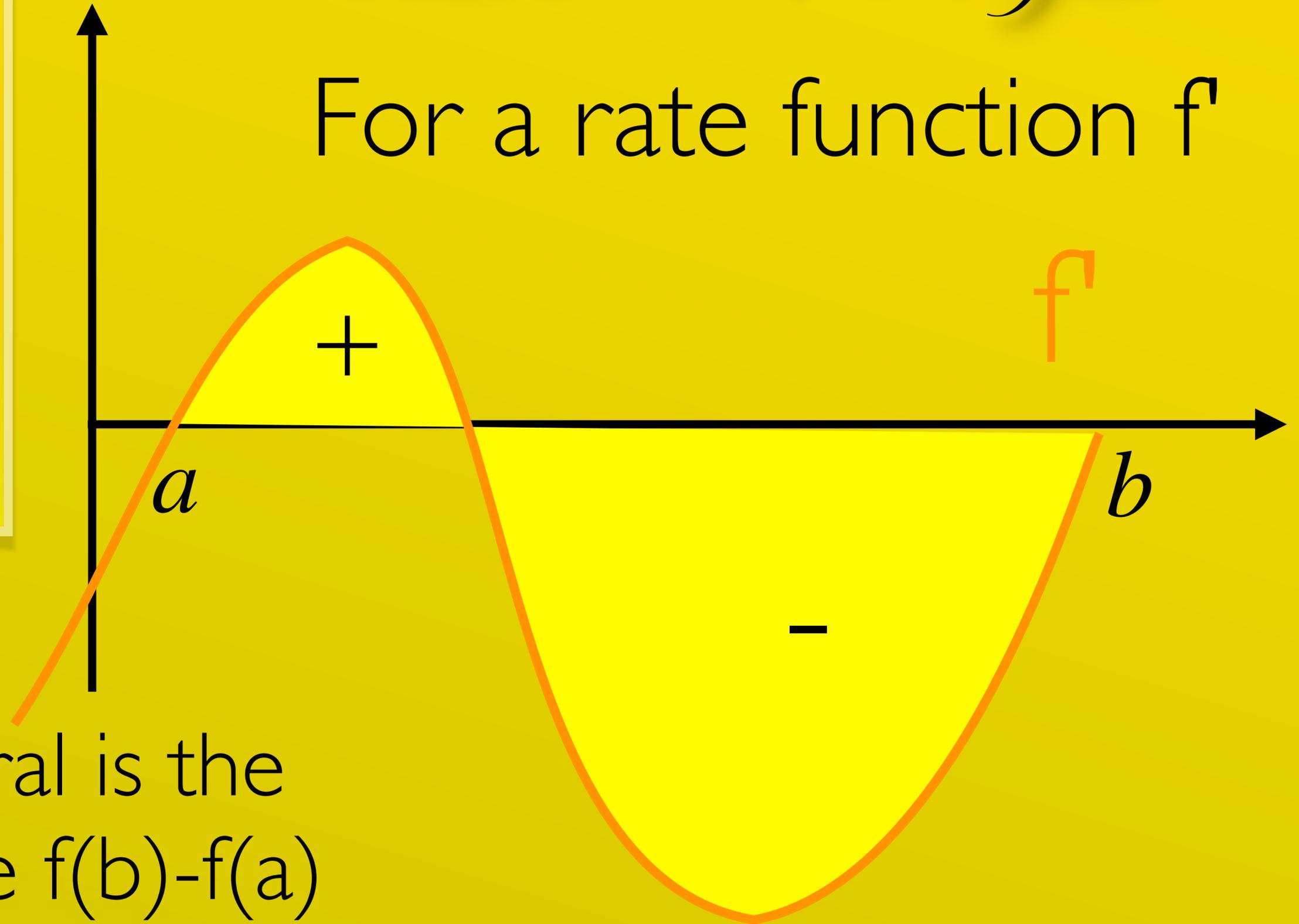
# *INTERPRETATION*

Signed area under curve



# NET CHANGE

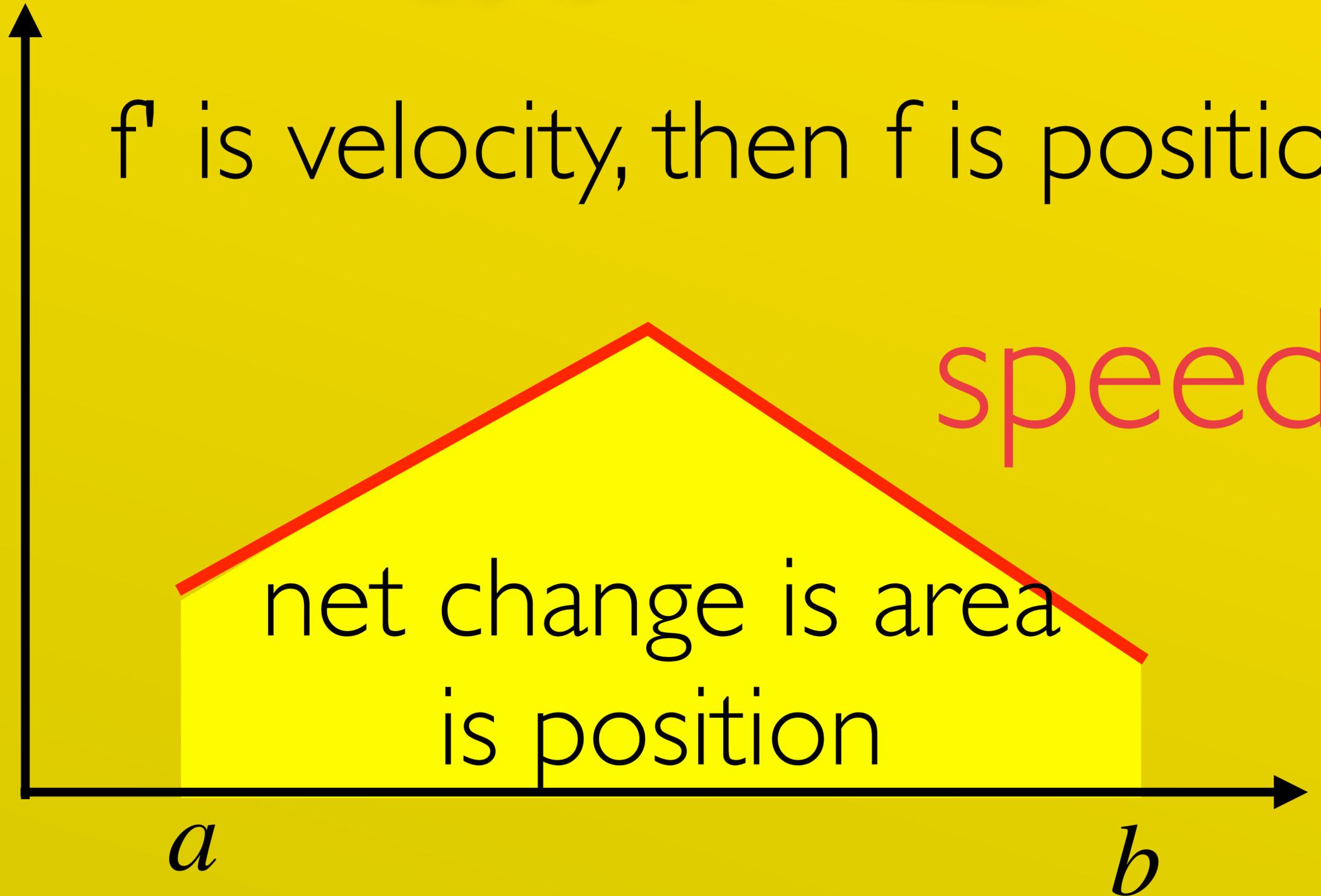
For a rate function  $f'$



the integral is the  
net change  $f(b)-f(a)$

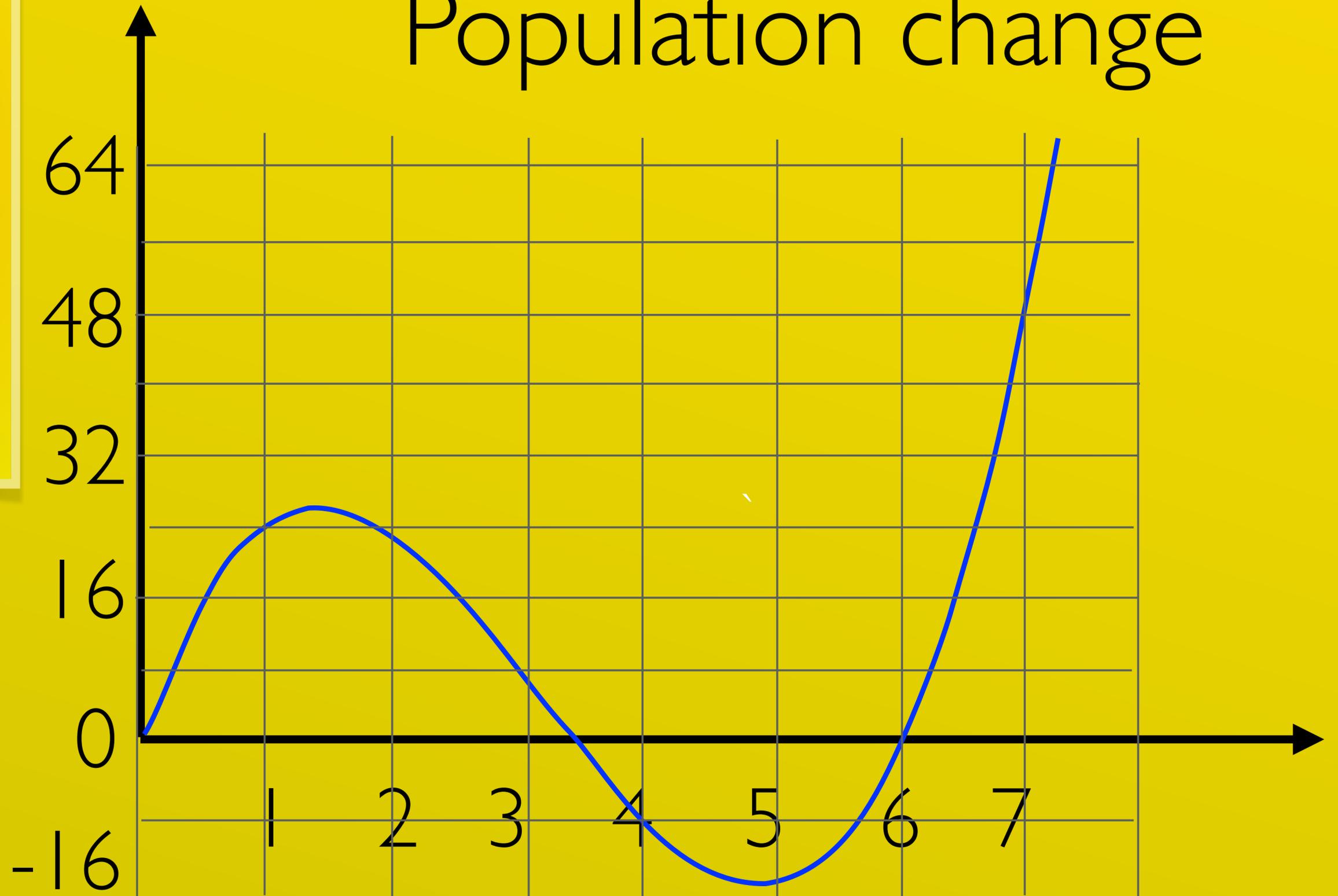
# *EXAMPLE*

$f'$  is velocity, then  $f$  is position



# *ISLAND POPULATION*

Population change



# *WHY CALCULUS?*



Analyse

Predict



*ARCHIMEDES*

# ARCHIMEDES



Nova: Infinite Secrets



*JAM*

$|t| - 1$  is amount of water flowing in gallons/hour.  
Assume at  $t=0$  there is no water.

*WATER FLOWING IN AND OUT OF TANK*

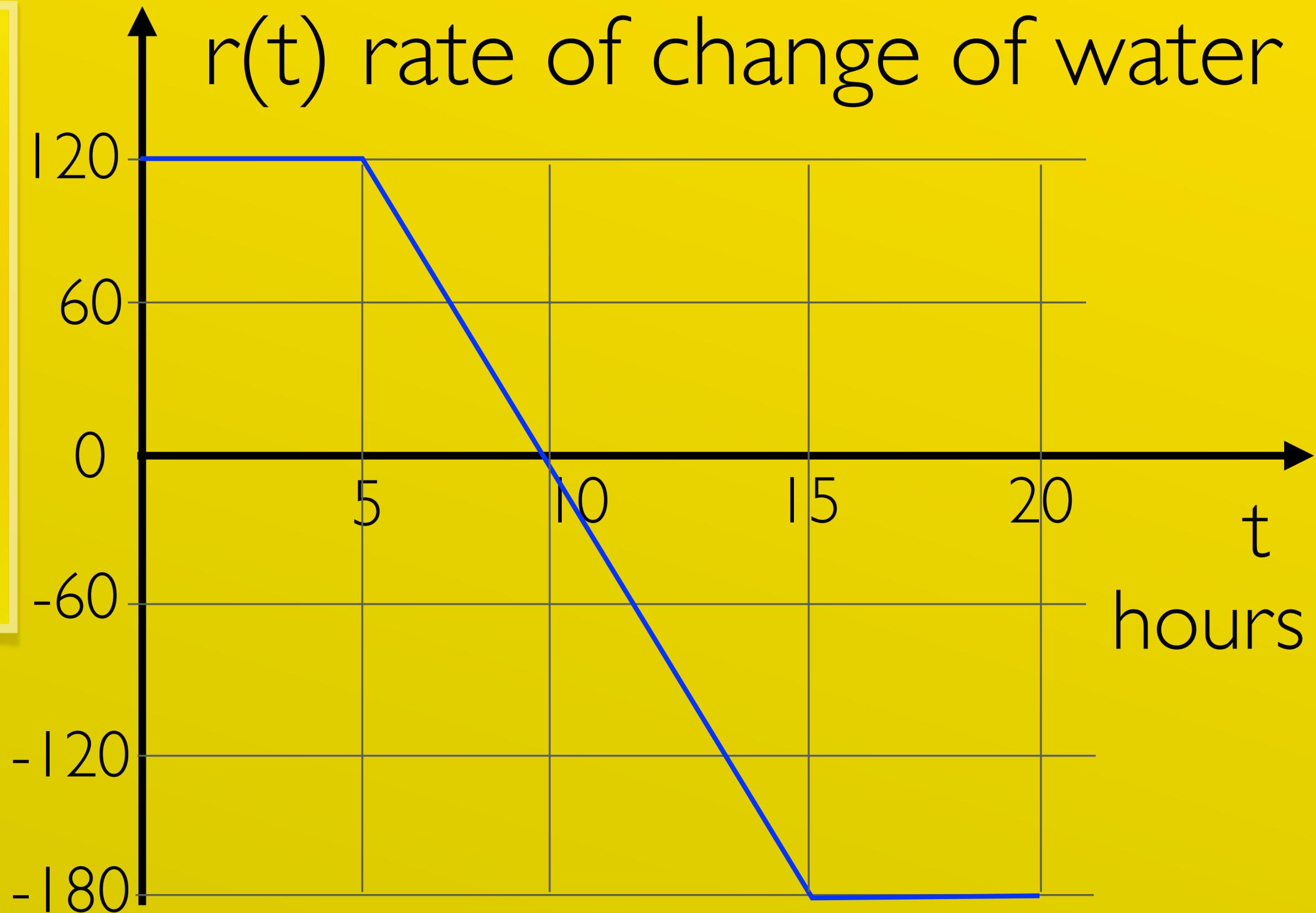
A  $\int_{-1}^2 (|t| - 1) dt$

B How much water at  $t=2$  (2 PM)?

C How much water at  $t=-1$  (11 AM)?

D What is the net change?

*CA  
TAM*



How much water  
is there in the  
pool after 20 hours?

*THE END*