LECTURE 30

FUNDAMENTAL THEOREM CALCULUS

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PLAN

1. Poll
2. The theorem
3. Anti derivatives
4. Lesson problems
5. More problems
6. Jam: Ice Cream Problem

POLL

Which of the formulas is not correct?

A $\dfrac{d}{dx}\displaystyle\int_0^x f(t)\,dt = f(x)$

B $\displaystyle\int_0^x f'(t)\,dt = f(x)$

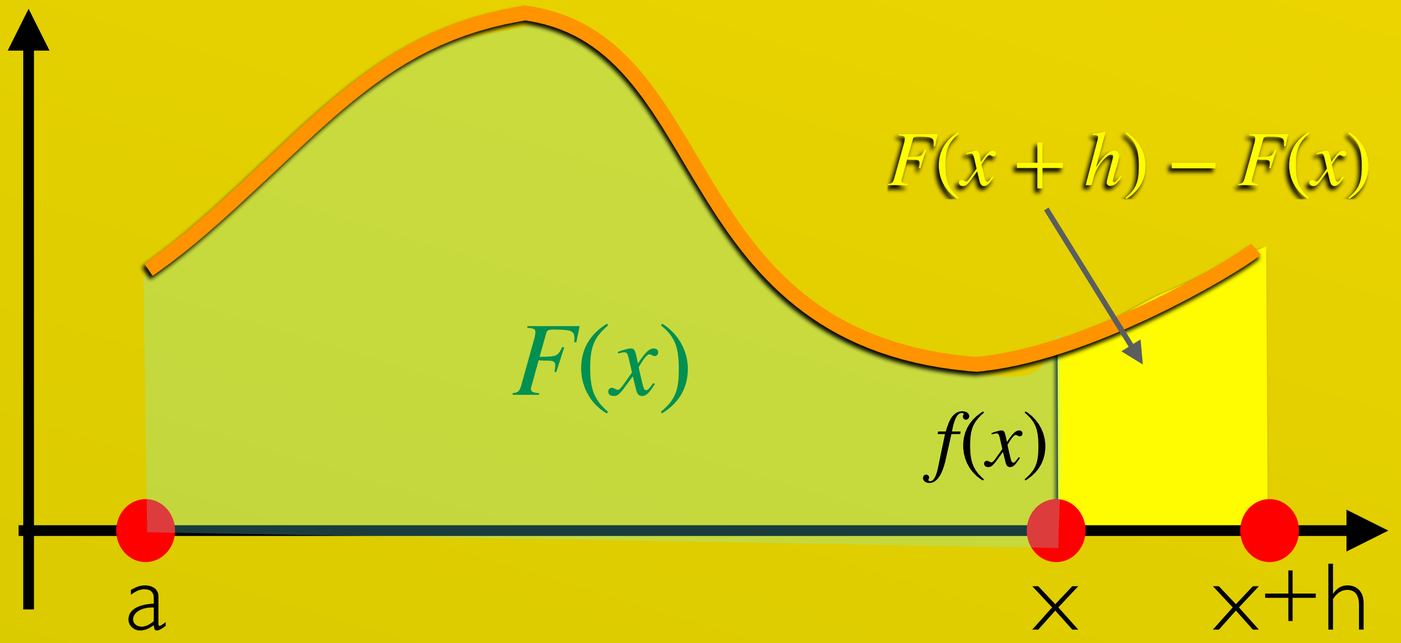
C $\displaystyle\int_0^x f'(t)\,dt = f(x) - f(0)$

THE THEOREM

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Proof:

$$\frac{F(x+h)-F(x)}{h} \sim f(x)$$



AN OTHER VERSION

$$\int_a^b f'(t)\,dt = f(b) - f(a)$$

Total sum of changes add up to net change

IN WORDS

f

Integrate ↓ ↑ Differentiate

F+c

The anti derivative of the derivative is the function up to a constant

The derivative of the anti-derivative is the function.

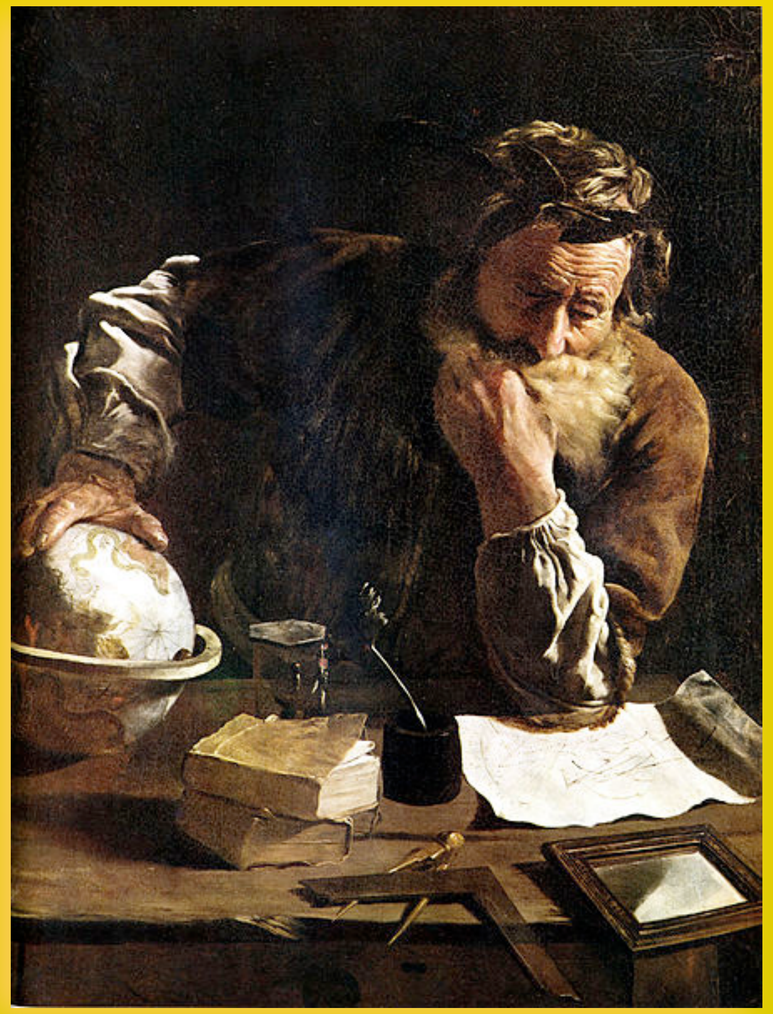
EXAMPLES

A $\int_0^{\pi} \sin(t)\, dt$

B $\int_0^{2} t^3\, dt$

C $\int_1^{3} \frac{1}{x}\, dx$

HISTORY I

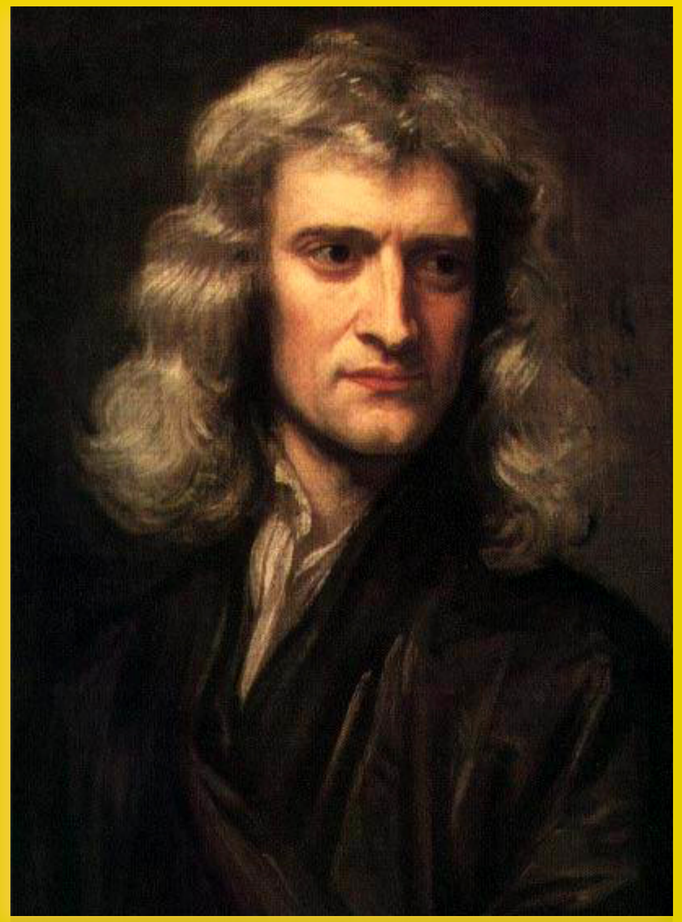


Integration

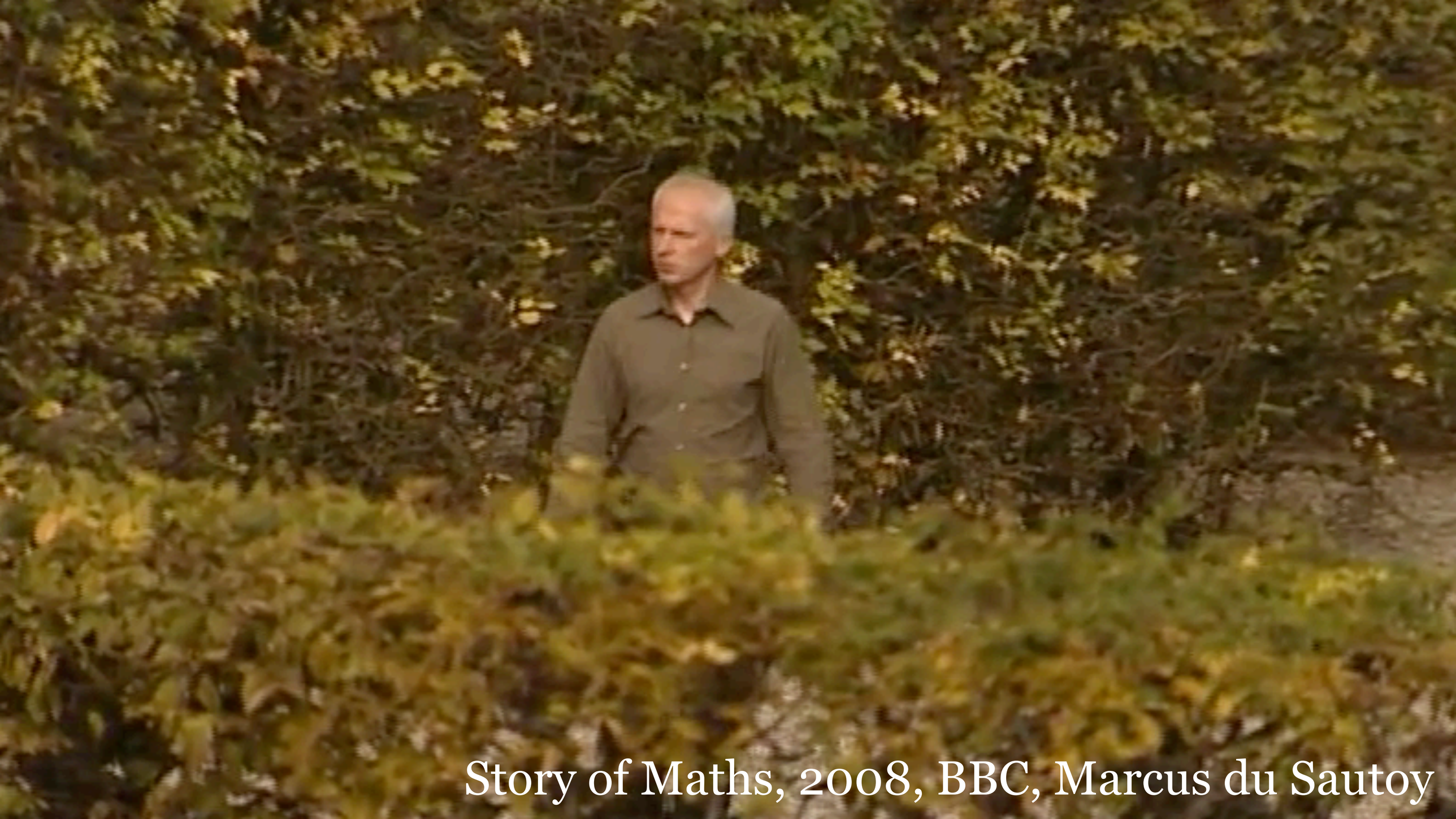
Differentiation

HISTORY II

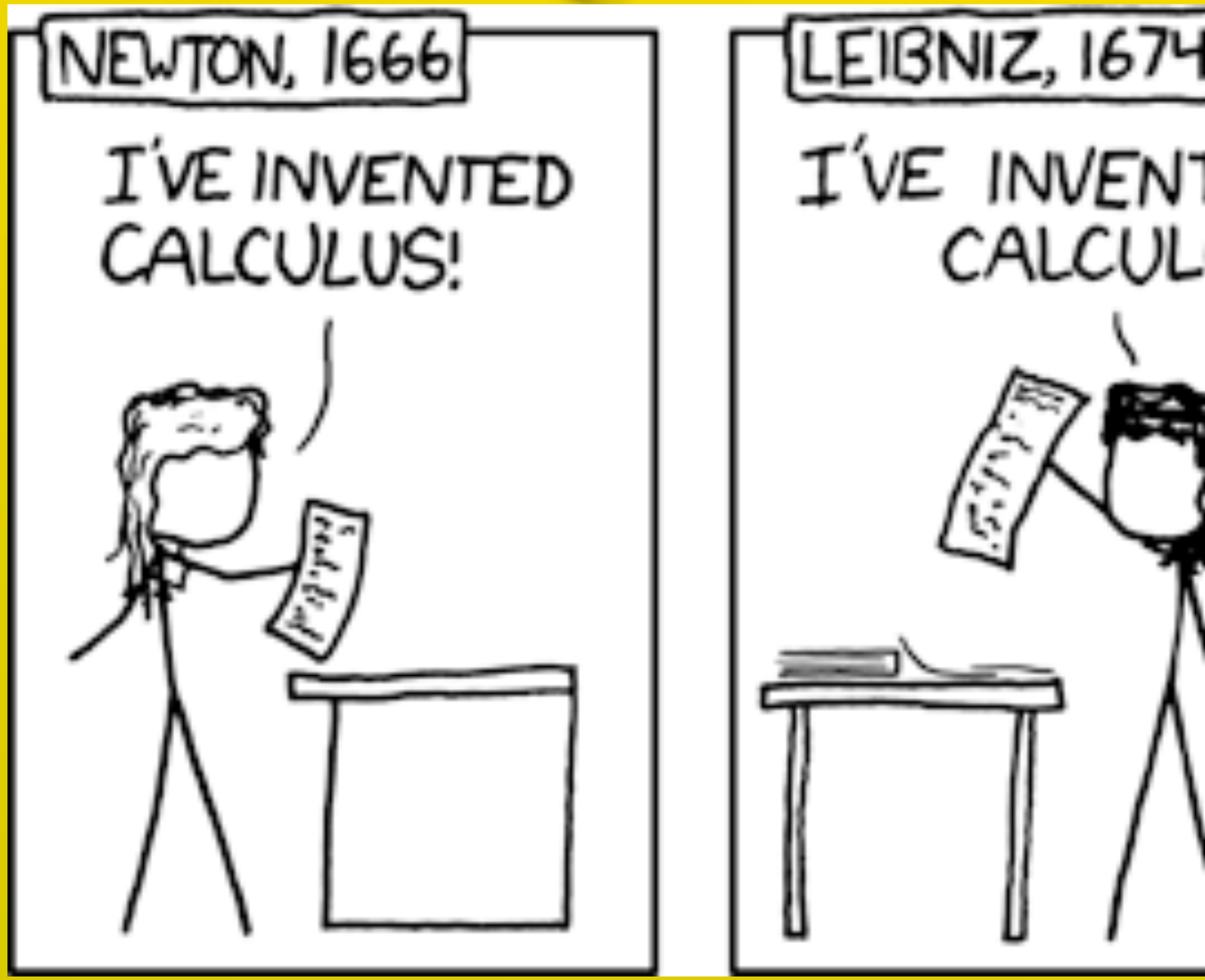
Leibniz



Newton


Story of Maths, 2008, BBC, Marcus du Sautoy

HISTORY


NEWTON, 1666
I'VE INVENTED CALCULUS!
LEIBNIZ, 1674
I'VE INVENT
CALCUL

ISAAC
NEWTON

LESSON PROBLEM 1

Evaluate or or say "not enough info"

We know: $\int_2^5 f(t)\,dt = 7$

A $\int_2^5 f(t) + 4\,dt$

B $\int_2^5 f(t+4)\,dt$

C $\int_5^2 4f(t)\,dt$

D $\int_2^4 f(t) + t dt + \int_4^5 f(t)dt$

LESSON PROBLEM 2

$$G(x) = \int_0^x 2t\,dt \qquad F(x) = \int_3^x 2t\,dt$$

A $G(2), G(-2), F(1), F(-2)$

B $G(x)$ for positive x

C $G(x)$ for negative x

D $F(x)$ for $x \geq 3$, and for $x < 3$

E Graph F,G

F Differentiate F,G

JAM

A $\int_0^1 t^3 + t^5 \, dt$

B $\int_0^2 e^t \, dt$

C $\int_{-1}^1 \frac{1}{1+x^2} \, dx$

CA JAM OF BLUE BELL

$$F(x) = \int_3^x f(t)\,dt$$

f(t) gallons/hour
milk flowing
into tank

At the Blue Bell ice cream factory, milk is stored in a 40,000 gallon tank. Let $f(t)$ be the rate (in gallons per hour) of milk flowing into and out of the tank one day at time t, where t is measured in hours after noon. At 3 pm, there are 15,000 gallons of milk in the tank.

(a) Define $F(x) = \int_3^x f(t)\,dt$. What does $F(7)$ represent?

(b) How much milk is in the tank at 7 pm?

(c) How much milk is in the tank x hours after noon?

(d) Define $G(x) = \int_0^x f(t)\,dt$. What does $G(7)$ represent?

(e) What does $G(x)$ represent?

(f) Let $M(x)$ be the amount of milk in the tank x hours after noon. How would you express "the instantaneous rate at which milk is flowing into/out of the tank at 6 pm" in terms of M? In terms of f? In terms of F?

THE END