

# *LECTURE 31*

## *FUNDAMENTAL THEOREM CALCULUS II*

Oliver Knill, Harvard University

November 20, 2020

# *PLAN*

1. Poll

2. The theorems

3. Anti derivatives vs. indefinite integrals

4. Lesson problems

5. More problems

6. Jam: More integrals

# POLL

Which  
of the  
formulas  
is not  
correct?

A  $\frac{d}{dx} \int_0^x f(x) dx = f(x)$

B  $\frac{d}{dx} \int_1^x f(t) dt = f(x)$

C  $\int_a^b f'(x) dx = f(b) - f(a)$

# POLL

Which  
one  
is the  
indefinite  
integral

A

$$\int x^2 dx = x^3/3 + C$$

B

$$\int x^2 dx = \frac{x^3}{3}$$

C

$$\int_0^1 x^2 dx = 1/3$$

# THE FTC

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

$$\int_a^x f'(t) dt = f(x) - f(a)$$

# THREE NOTIONS

$$\int_a^b f(x) dx$$

Definite  
integral

$$\int f(x) dx = F(x) + C$$

Indefinite  
integral

$F(x)$

is an anti-derivative

# *SPECIAL CASES*

$$\int_a^b 1 \, dx = b - a$$

$$\int_a^a f(x) \, dx = 0$$

You have  $\int_a^a f(x) \, dx$  friends

**CALCULUS**

can be funny if you understand it

# CAJAM COME BACK

$$M(x) = \int_0^x 1000 - 300t^2 dt + C$$

- (Blue Bell revisited) At the Blue Bell ice cream factory, milk is stored in a 40,000 gallon tank. The rate of milk flowing into and out of the tank  $t$  hours after noon one day is  $f(t) = 1000 - 300t^2$  gallons per hour. At 2 pm, there are 15,000 gallons of milk in the tank.
  - Express the net change in the amount of milk between 1 pm and 3 pm as a definite integral.
  - Let  $M(t)$  be the amount of milk in the tank at time  $t$ . What is  $M'(t)$ ? What is  $M(2)$ ? Using these two pieces of information, can you find a formula for  $M(t)$ ?
  - Use your answer to (b) to find the net change in the amount of milk in the tank between 1 pm and 3 pm.



# *LESSON PROBLEMS*

A

$$\int_{-2}^2 4x^3 dx$$

B

$$\int_4^3 x dx$$

# THREE AREAS

A

$$\int_0^1 e^x dx$$

B

$$\int_{-1}^0 e^{-x} dx$$

C

$$\int_0^1 e^{-x} dx$$

Draw them!

# TEMPERATURE

$$r(t) = -2e^{-t}$$

A

Temperature change from  
 $t=0$  to  $t=1$ ?

B

Temperature change from  
 $t=1$  to  $t=2$ ?

C

$T(0)=100, T(1)=?$

# *DEFINITE INTEGRALS*

A  $\int_1^2 \frac{1}{x^2} dx$

B  $\int_0^{\pi/2} \cos(x) dt$

C  $\int_{-2}^2 3x^4 dx$

*WHAT IS THIS?*

What is  $\int d(\text{cabin})/\text{cabin}$



$\log(\text{cabin})$

*WHAT IS THIS?*

$$\int \text{devil} =$$



evil + C

# *INDEFINITE INTEGRALS*

*JAM*

A  $\int \frac{1}{x^2} dx$

B  $\int 1/\cos^2(x) dx$

C  $\int x^{-2/3} dx$

Write the indefinite integral  
and pick a specific  
anti derivative

*CAJAM*

don't  
forget the  
constant for  
the indefinite  
integrals

A  $\int \cos(x) dx$

B  $\int u^3 du$

C  $\int \frac{x}{x^{1/3}} dx$

D Anti derivative of  $4 - \frac{3}{1+x^2}$  with  $F(1)=0$

*THE END*