# LECTURE 32

## BASIC INTEGRALS

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# PLAN

1. Poll

2. Some basic integrals

3. Practicing

4. Lesson problems

5. Jam

6. Jam: More integrals

*POLL*

Which 2 are correct?

A $\int \cos(x)\, dx = \sin(x) + C$

B $\int \sin(x)\, dx = \cos(x) + C$

C $\int \sin(x)\, dx = -\cos(x) + C$

D $\int \cos(x)\, dx = -\sin(x) + C$

# FROM DERIVATIVES TO INTEGRALS

If we know F'=f, then

$$\int f(x)\,dx = F(x) + C$$

# MOST IMPORTANT EXAMPLE

$$\int x^n \, dx = x^{n+1}/(n+1) + C$$

Must Know!

# PRACTICE!

$$\int \sqrt{x}\, dx$$

$$\int x^4 + x^2 + x + 1\, dx$$

$$\int \frac{1}{\sqrt{x}}\, dx$$

# TRIG FUNCTIONS

$$\int \cos(x)\, dx = \sin(x) + C$$

$$\int \sin(x)\, dx = -\cos(x) + C$$

Must Know!

# PRACTICE!

$$\int \cos(3x)\, dx$$

$$\int 5 \sin(2x)\, dx$$

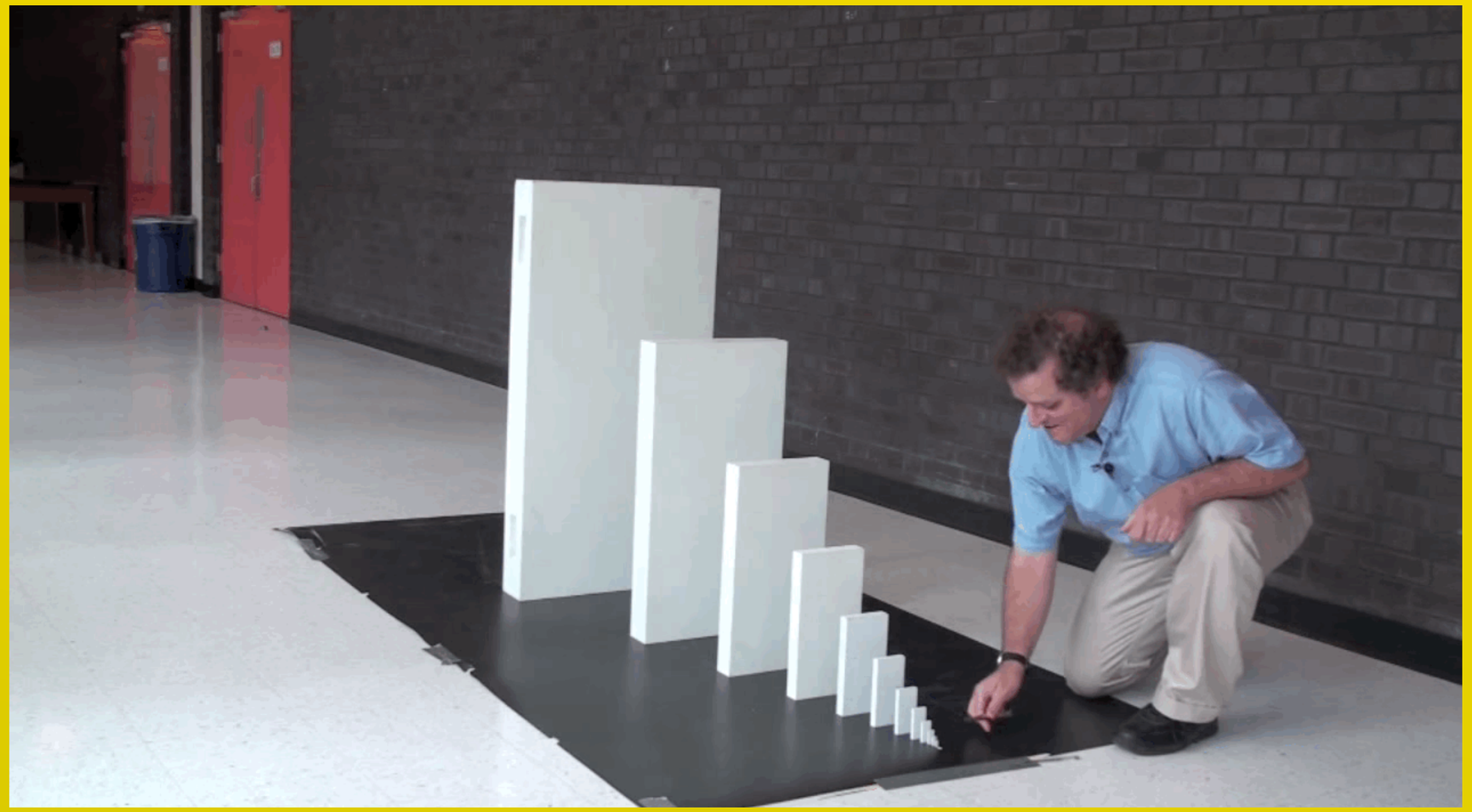
# EXPONENTIAL

$$\int e^x \, dx = e^x + C$$

$$\int e^{ax} \, dx = \frac{e^{ax}}{a} + C$$

Must Know!

# GROWTH



# EXPONENTIAL

$$\int a^x \, dx = a^x / \log(a) + C$$

Must Know!

# PRACTICE!

$$\int e^{3x}\,dx$$

$$\int 5e^{-x}\,dx$$

# RATIONAL FUNCTIONS

$$\int \frac{1}{x}\,dx = \log(x) + C$$

Must Know!

$$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$$

# PRACTICE!

$$\int \frac{1}{x-1}\,dx$$

$$\int \frac{5}{1+x^2}\,dx$$

# RATIONAL FUNCTIONS

Must Know!

$$\int \sec^2(x)dx = \tan(x) + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$$

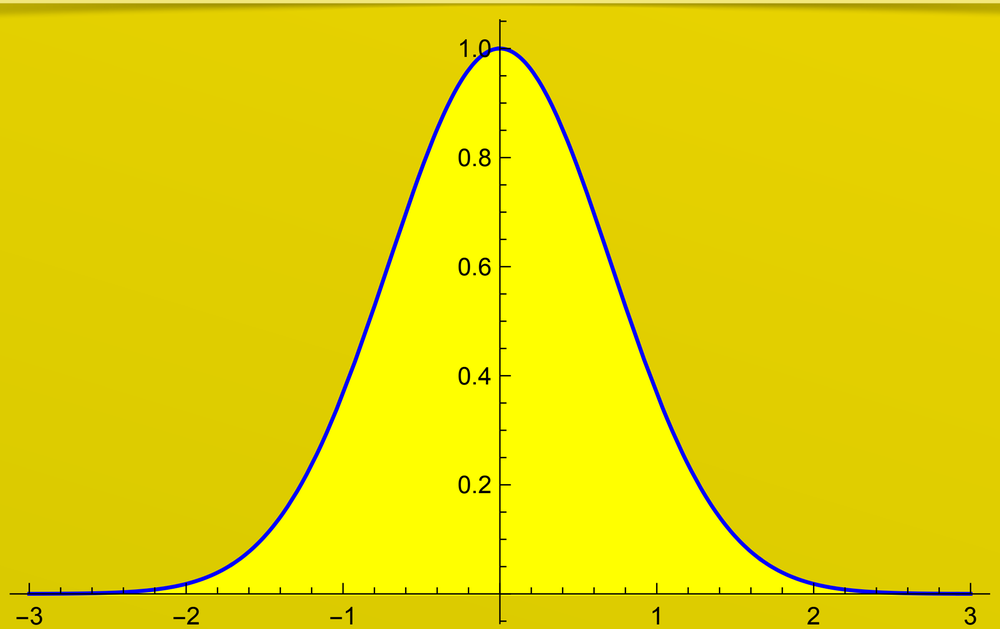
# PRACTICE!

$$\int \frac{5}{\cos^2(x)}\,dx$$

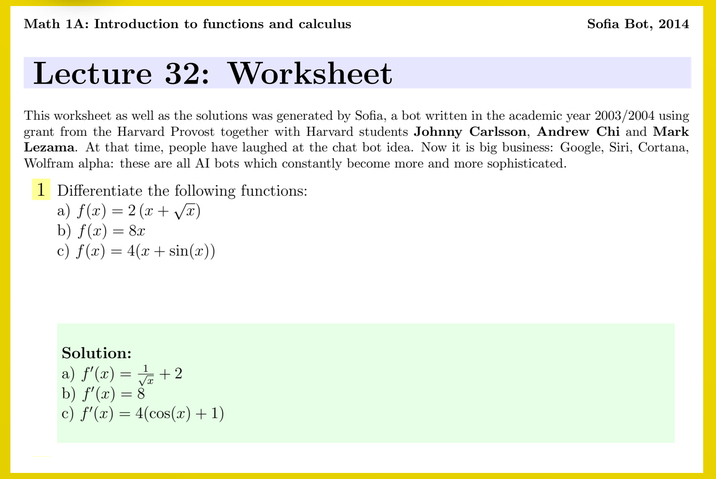
$$\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$$

# CAN WE SOLVE ALL INTEGRALS?

$$\int e^{-x^2}\,dx$$



# HOW TO GENERATE PROBLEMS?



**Math 1A: Introduction to functions and calculus** **Sofia Bot, 2014**

## Lecture 32: Worksheet

This worksheet as well as the solutions was generated by Sofia, a bot written in the academic year 2003/2004 using grant from the Harvard Provost together with Harvard students **Johnny Carlsson**, **Andrew Chi** and **Mark Lezama**. At that time, people have laughed at the chat bot idea. Now it is big business: Google, Siri, Cortana, Wolfram alpha: these are all AI bots which constantly become more and more sophisticated.

1 Differentiate the following functions:
a) $f(x) = 2\,(x + \sqrt{x})$
b) $f(x) = 8x$
c) $f(x) = 4(x + \sin(x))$

**Solution:**
a) $f'(x) = \frac{1}{\sqrt{x}} + 2$
b) $f'(x) = 8$
c) $f'(x) = 4(\cos(x) + 1)$

```
RandomFraction:=Apply[Union,Array[Range[#-1]/#&,Random[Integer,10]+2]][[10]]
Ide[x_]:=x;
Pow[x_]:=x^Random[Integer,{2,5}];
Inv[x_]:=x^Random[Integer,{-3,-1}];
Sca[x_]:=Random[Integer,{-3,3}]*x;
Tra[x_]:=x+Random[Integer,{-4,4}];
FunctionList={Sin,Cos,Log,Exp,Tan,Sqrt,Pow,Inv,Sca,Tra};
RRandomFunction:=FunctionList[[Random[Integer,{1,Length[FunctionList]}]]];
```

# TRUE OR FALSE

JAM

A

F is an antiderivative of f
G is an antiderivative of g
FG is an antiderivative of fg

B

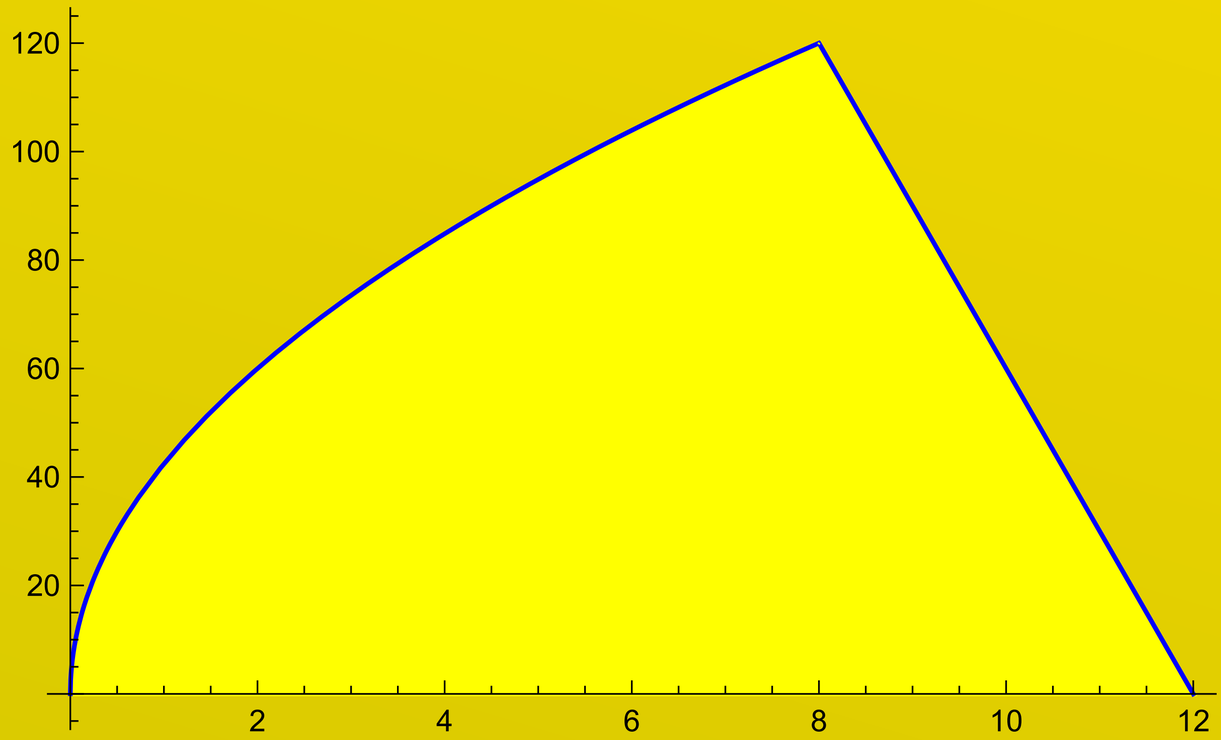
F is an antiderivative of f
G is an antiderivative of g
F+G is an antiderivative of f+g

# BAGLE

# JAM

$$r(t) = 30\sqrt{2t} \qquad 0 \le t \le 8$$

$$r(t) = 30(12 - t) \qquad 8 \le t \le 12$$



# JAM

# BAGLE

1. A busy bagel shop starts making bagels at 5 am each morning. One day, the rate at which bagels are made is

$$r(t) = \begin{cases} 30\sqrt{2t} & 0 \le t \le 8 \\ 30(12-t) & 8 < t \le 12 \end{cases} \text{ bagels per hour,}$$

where $t$ is measured in hours after 5 am. The bagel shop is open 7 am - 5 pm; on this day, customers come at a steady rate of 60 customers per hour between 7 am and 5 pm. Each customer orders one bagel and receives it immediately.

   (a) At what times on this day is the number of bagels the shop has available increasing? At what times is it decreasing?

   (b) At the end of the day, any unsold bagels are donated to a local food pantry. How many bagels are donated at the end of this day?

   (c) What is the largest number of bagels the shop has available at any point during this day?

   (d) Are there times during the day when the shop has exactly 310 bagels available? If so, at how many times?

# MORE PRACTICE

A $\int \frac{5}{7x}\,dx$

B $\int_{-e}^{-1/e} \frac{5}{7x}\,dx$

C $\int e^{\pi} + \pi^{y} + ey\,dy$

D $\int (\pi + x)\sqrt{x}\,dx$

E $\int \frac{e}{x^2} + 2^x 3^x\,dx$

F $\int_{-3}^{0} \sqrt{9 - x^2} + x^2\,dx$

THE END