LECTURE 33

SUBSTITUTION

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November 30, 2020

PLAN

1. Poll

2. The chain rule

3. Examples

4. Lesson problem review

5. Jam: Pick 2

6. Jam: Project Work

POLL

$$\int \frac{1}{x\log(x)}\,dx =$$

A $\log^2(x)/2 + C$

B $\dfrac{1}{\log(x)x} + C$

C $\log(\log(x)) + C$

D $\log(x) + C$

CHAIN RULE

$$\frac{d}{dx}F(u(x)) = f(u(x))u'(x)$$

$$\int f(u(x))u'(x)\,dx = F(u(x)) + C$$

BEGINNING

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BASIC EXAMPLES

$$\int e^{(x+1)}\,dx$$

$$\int \cos(x^2)2x\,dx$$

$$\int \frac{x}{1+x^2}\,dx$$

PRACTICE

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PRACTICE

$$\int \sqrt{x+2}\, dx$$

$$\int \sin(x^5)x^4 dx$$

$$\int \sin(\sin(x))\cos(x)\, dx$$

HARDER

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HARDER

$$\int \frac{1+x}{1+x^2}\,dx$$

$$\int e^{-4x^2} x\,dx$$

$$\int \frac{1+x}{1-x^2}\,dx$$

did you know?

3 out of 2 people have trouble with fractions.

WHAT NOW?

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LESSON PROBLEMS

(a) $\displaystyle\int_1^4 \frac{1}{3}\sqrt{u}\,du$

(b) $\displaystyle\int \cos(3x)\,dx$

(c) $\displaystyle\int \frac{\sin(\ln x)}{5x}\,dx$

(d) $\displaystyle\int e^{-2x}\cos(e^{-2x})\,dx$

(e) $\displaystyle\int_1^4 x^2\sqrt{1+x^3}\,dx$

(f) $\displaystyle\int \sin x\cos^5 x\,dx$

(g) $\displaystyle\int_{1/e}^{e^3} \frac{4\ln x}{x}\,dx$

(h) $\displaystyle\int \frac{e^x}{2e^x+5}\,dx$

(i) $\displaystyle\int_0^1 \sqrt{5x+4}\,dx$

LETS PLAY (SYGRAJEM)

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1 $10,000 CASH IMMEDIATELY	2 BRAND NEW HOME BUT YOU CAN NEVER SELL IT OR MOVE	3 UNLIMITED FREE MEDICAL CARE FOR LIFE
4 FREE RENT/ MORTGAGE FOR 6 MONTHS	PICK TWO	5 BRAND NEW LUXURY CAR. YOU CAN NEVER SELL OR OWN ANOTHER CAR
6 PERFECT CREDIT SCORE	7 UNLIMITED AIRLINE TICKETS FOR LIFE	8 FREE GROCERIES FOR FIVE YEARS

if finished, which 2 of the above 2 options for you?

JAM

Pick two you can do!

$$\int e^{-x^2} dx$$

$$\int x^2 e^{-x^2} \, dx$$

$$\int x e^{-x^2} \, dx$$

$$\int \log(x)/x \, dx$$

CA WORK

Deep question: what is the difference between figure 5 and figure 8?

Water **2014**, 6, 2717-2731; doi:10.3390/w6092717

OPEN ACCESS

water

ISSN 2073-4441

www.mdpi.com/journal/water

Article

Comparing Simple Flood Reservoir Operation Rules

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Received: 17 November 2013; in revised form: 14 August 2014 / Accepted: 29 August 2014 / Published: 11 September 2014

Abstract: The effectiveness of three simple flood operating rules in reducing the peak flow is compared for four simplified hydrograph shapes. The Minimize Flood Peak rule uses available flood storage capacity to store peak flows from an accurate hydrograph forecast. The less demanding Minimize Flooding Frequency operating rule releases water at or below channel capacity until the flood storage pool is filled and outflows are forced to exceed the channel capacity. The Short Forecast Peak Minimization rule minimizes flood peak over a short foreseeable future with existing flood storage capacity. Four simplified hydrograph shapes (triangular, abrupt wave, flood pulse and broad peak) were used. The Minimize Flood Peak rule reduces peak flows better than alternatives, but is often impractical. The Short Forecast Peak Minimization rule reduces peak flows for a wide range of conditions. The Minimize Flood Frequency rule may be more relevant where damages occur abruptly, as in many leveed systems. All rules reduce peak outflow more efficiently for more steeply rising hydrographs. The approach suggests some general insights for flood operations of reservoirs.

Keywords: reservoir operation; hydrographs; peak flow reduction; flood forecasting

Figure 5. Minimize flood frequency (flooding prevented) for (**a**) Triangular hydrograph and (**b**) Broad peak hydrograph.

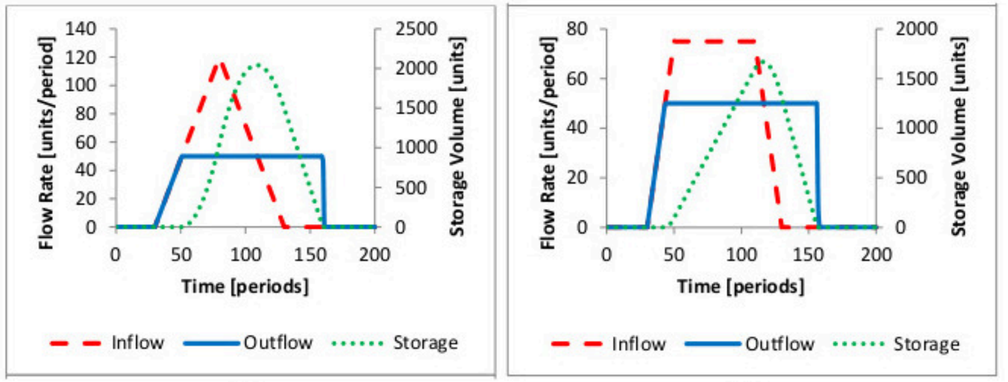
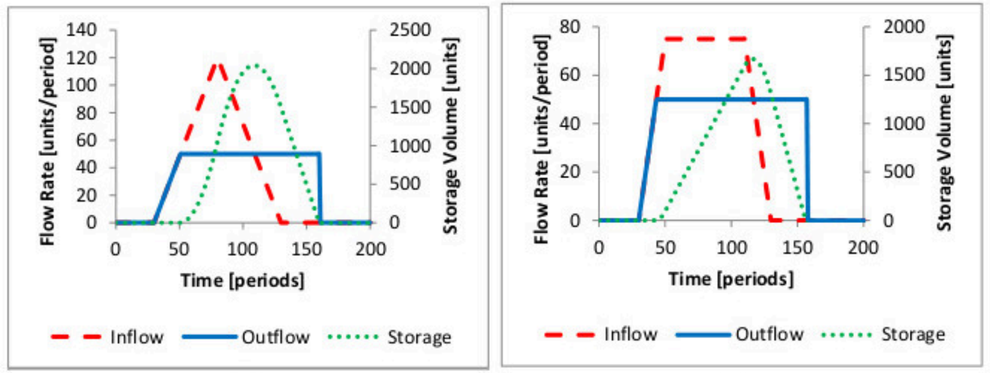


Figure 8. Short forecast peak minimization operation rule (with flooding prevented) for (**a**) Triangular hydrograph and (**b**) Broad peak hydrograph.



THE END