CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

Lecture 2: Density and definite integral, 9/8/2021

Density and definite integral

2.1. The most common situation so far is the integral

$$\int_a^b \rho(x) f(x)\, dx\ .$$

which was the limit of the Riemann sum

$$\sum_{k=1}^{n} \rho(x_k) f(x_k) \Delta x$$

with $\Delta x = (b-a)/n, x_k = a + k * \Delta x$.

2.2. The interpretation is that $f(x_k)\Delta x$ is the area of a **slice** and that $\rho(x_k)f(x_k)\Delta x$ is the content of the material included in that slice. We have seen examples, where $\rho(x)$ can be color density, cheese density, population density, mass density.

A philosophical musing about the integral

2.3. You can ignore this part completely but it is a fascinating task to dig deeper what an integral actually is. This part is a maybe rather philosophical musing about what an integral is. Just for the experts: the integral we look at here is a fundamentally different kind of integral than the integral we look at in the fundamental theorem of calculus. This only really become apparent when looking at quantum calculus, calculus on graphs for example.

2.4. Here is a riddle: if we compute the mass density as just described, why does the result not depend on the orientation with respect to which we integrate? Remember that $\int_1^2 f'(x)\, dx$ gives a different result than $\int_2^1 f'(x)\, dx$. By the **fundamental theorem of calculus** for example we have in the first case $f(2) - f(1)$ and in the second case $f(1) - f(2)$. Obviously the integral comes with an orientation.

2.5. Now, if we compute the integral in a density problem, then when looking at the Riemann sum, it does not really matter whether we start summing up from the left or the right. One really only starts to understand what is going on when doing calculus on finite spaces like graphs.

On the radar

2.6.

- Homework PS 02 is due 9/10.
- The area of an iceberg QRD problem is due 9/10.
- Techniques of integration test Wednesday 9/15.

Oliver Knill, knill@math.harvard.edu, Math 1b, Harvard College, Fall 2021