CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

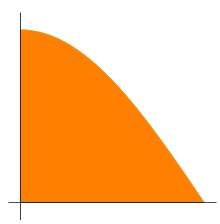
# Lecture 3: Area and Volume, 9/10/2021

## Area

**3.1.** When computing the area of a region, we slice the region and take the limit of the Riemann sums. An important case is if the region is bound between two curves. If a function is non-negative then $\int_a^b f(x)\,dx$ is the area below the graph of $f$ and above the interval $[a,b]$. Remember that this is a limit $\sum_{k=1}^n f(x_k)\Delta x$ with $x_k = a + k\Delta x$ and $\Delta x = (b-a)/n$.

If $f(x) \geq 0$, then $\int_a^b f(x)\,dx$ is the **area** below the graph of $f$

**3.2.** Example: Find the area of the triangular shaped region bound by the cos function and the $x$ and $y$ axes.



**3.3.**

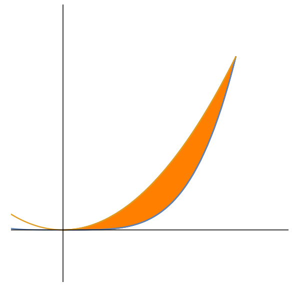
The area of a region enclosed by two graphs $f \leq g$ and bound by $a \leq x \leq b$ is

$$\int_a^b g(x) - f(x)\,dx$$

.

**3.4.** The reason is that this is a limit of a sum of slice areas $(g(x_k) - f(x_k))\Delta x$ with $x_k = a + k\Delta x$ and $\Delta x = (b-a)/n$.

**3.5.** Find the area of the region enclosed by the graphs $f(x) = x^2$ and $f(x) = x^4$.

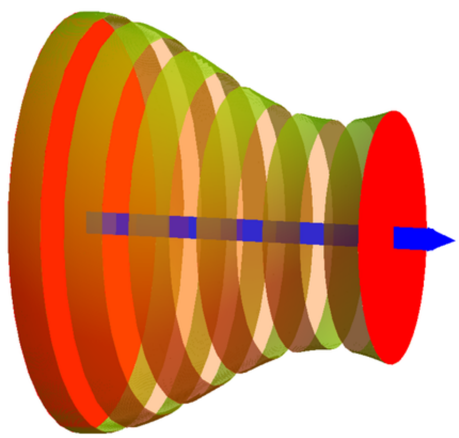


## Volume

**3.6.** The **volume of a solid** can be approximated by a sum of slice volumes If $A(x)$ is the **area of the slice** and the body is enclosed between $a$ and $b$ then

$$V = \int_a^b A(x)\, dx$$

is the **volume** of the body. The integral adds up $A(x)dx$, the volume of the slices.



**3.7.** Rotate the function $f(x) = \sin(x)$ around the $x$-axes. This gives a **lemon**. **Solution:** The area of $\pi \sin^2(x)$. The integral $\int_0^\pi \sin^2(x)\, dx$ is $\pi/2$.

## On the radar

**3.8.**

- Homework PS 02 was due today Friday 9/10.
- The area of an iceberg QRD problem is due today 9/10.
- Homework PS 03 is due Monday 9/13.
- Techniques of integration test is Wednesday 9/15.

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