

CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

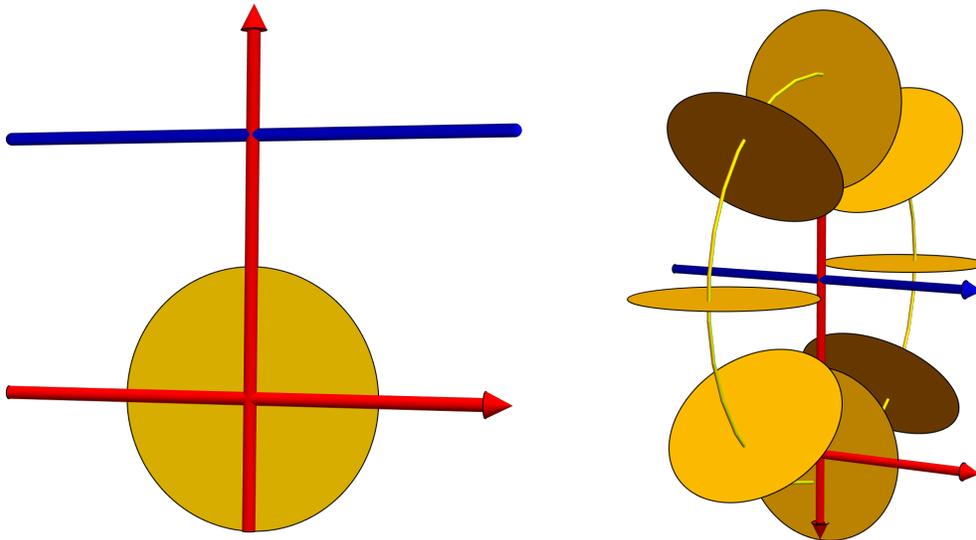
Lecture 5: Volumes of revolution, 9/15/2021

5.1. Given a region G in two dimensions and a line, we can go to three dimensions and rotate the region around that line. It produces a **solid of revolution**. What is the volume of this body? The difficulty of the integrals can depend on how we slice things.

EXAMPLE 1

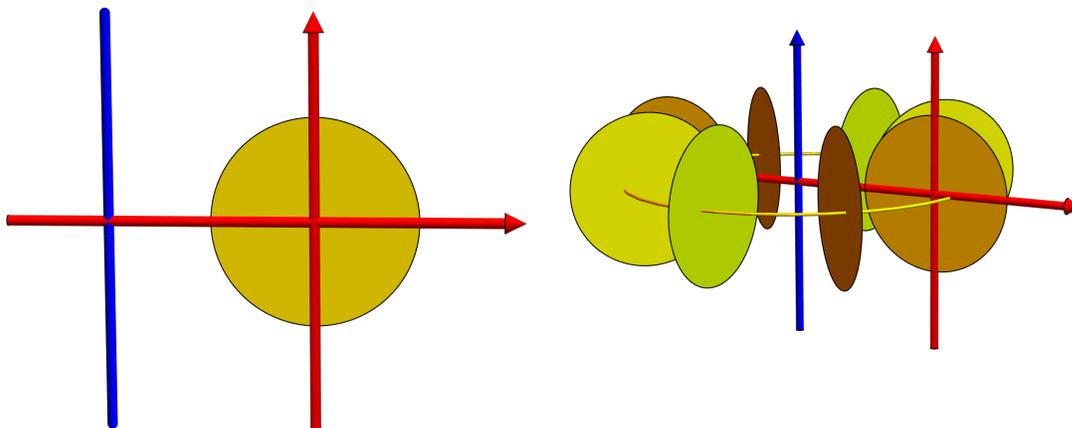
5.2. Take a disk G given by $x^2 + y^2 \leq 1$ and rotate it around the axis $y = 2$. We get a doughnut. To get the volume, slice this up perpendicular to the x -axes. We get cross sections of area $\pi(2 + \sqrt{1 - x^2})^2 - \pi(2 - \sqrt{1 - x^2})^2$. This simplifies to $4\pi\sqrt{1 - x^2}$. If we integrate this from -1 to 1 , we get $4\pi\pi = \boxed{4\pi^2}$.

Can you in the picture bellow label all the axes. Identify the slice at a general position and draw how the slice looks like that is rotated around the axis?



EXAMPLE 2

5.3. Now take the same disc G given by $x^2 + y^2 \leq 1$ and rotate it around the axis $x = -2$. We get the same doughnut. We could slice horizontally (which is equivalent to what we did before). But we let us slice again vertically and get a different setting.



5.4. To get its volume, take a small slice of the region and multiply it with the $2\pi r$, where r is the distance to the axis of rotation. In the above example, with line $x = -2$ and the disk has radius 1, we get $r = 2 + x$ and slice of area $y\Delta x = 2\sqrt{1 - x^2}\Delta x$

$$\int_{-1}^1 \pi(2 + x)2\sqrt{1 - x^2} dx.$$

The expression $\pi(2 + x)$ is the distance to the axis.

5.5. We have $\int_{-1}^1 x\sqrt{1 - x^2} dx = 0$ because of the cancellation on the left and right hand side. The result is again $4\pi \int_{-1}^1 \sqrt{1 - x^2} dx = \boxed{4\pi^2}$.

Can you in the picture label all the axes and indicate where the slice is and how the slice looks like, if you rotate it around the axis?

5.6. P.S. As a historical remark, one should note that in general, a **formula of Pappus** assures that the volume of a solid of revolution is the length of the circle traced by the center of mass of the region times the area of the region. In the case of the doughnut just considered, the center of mass is the center of the disk. It traces a curve of length 4π . The area of the circle is π . We again have $\boxed{4\pi^2}$.

REMINDERS

- HW 5 is due on Friday.
- Techniques of integration test is tonight Wednesday 9/15.