CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

Lecture 6: Three dimensional density, 9/17/2021

The integral

6.1. If $\rho(x)$ is the density and $A(x)$ is the cross section or shell of the solid at x which is given in the range $a \leq x \leq b$, then

$$\int_a^b \rho(x)A(x)\,dx$$

is the total amount of material we measure in the solid. This generalizes the volume, the case when $\rho(x) = 1$ is constant. We call such problems **3-dimensional density problems** We do not have much freedom to compute the total integral as the density defines how we have to slice.

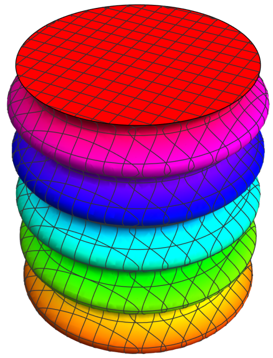
6.2. Example 1):

Assume that a wedding cake radius $10+\sin(h)$ where h ranges in $0 \leq h \leq 10\pi$. Assume the density of the cream at height h is $\rho(h) = 4 + \cos(h)$. What is the total cream of the cake?

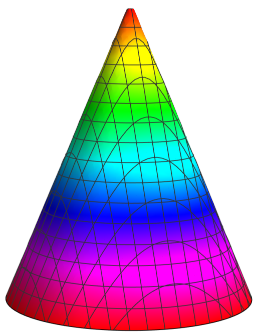
Solution: If we slice the cake horizontally at height $h_k = k\Delta h$ with $\Delta h = 10\pi/n$, we get a disk shape slide of height Δh and area $A(h_k) = \pi(10 + \sin(h_k))^2$. The volume of that slice is $A(h_k)\Delta h$ and the cream content of that slice is $\cos(h_k)\pi(10+\sin(h_k))^2\Delta h$. The total volume is $\sum_{k=1}^n \rho(h_k)\pi(10+\sin(h_k))^2\Delta h$. In the limit when n goes to infinity, this becomes the integral

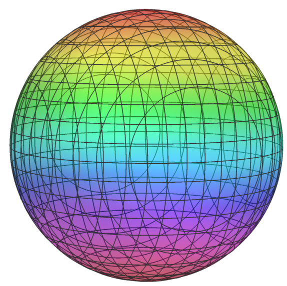
$$\pi \int_0^{10\pi} (4 + \cos(h))(10 + \sin(h))^2 dh\ .$$

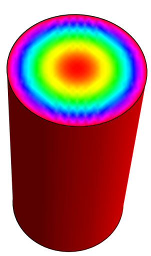
This integral is $4020\pi^2$.

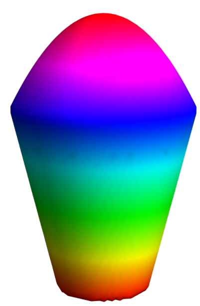


6.3. In class, we will look at various objects. Our heroes are a slush cone, a planet, a candle and a muffin. These examples show that the cross sections can be disks, spherical shells, cylindrical shells or more general two dimensional shells which are rotated around an axis.









On the radar

6.4.

- HW 5: 3 Diemensional density problems is due Monday.