

CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

Lecture 12: Taylor Approximation II, 10/1/2021

FINITE DEGREE APPROXIMATIONS

12.1. Here is what we have seen:

The Taylor approximation of a function f at a point c is the polynomial

$$P_n(x) = f(c) + f'(c)(x - c) + f''(c)\frac{(x - c)^2}{2} + \cdots + f^{(n)}(c)\frac{(x - c)^n}{n!}$$

In sum notation this reads

$$P_n(x) = \sum_{k=0}^n f^{(k)}(c)\frac{(x - c)^k}{k!}.$$

12.2.

Example: (Remember homework): Approximate $f(x) = \exp(x)$ as a Taylor polynomial of degree 7 at $c = 0$:

Answer: As $f'(0) = f''(0) = \dots = f^{(n)}(0) = 1$, we have

$$P_7(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \dots$$

12.3.

Example: Write down the degree 4 Taylor polynomial of $f(x) = \exp(3x)$ at $c = 2$. **Answer:** As $f(2) = e^6$, $f'(2) = 3e^6$, $f''(2) = 9e^6$, $f'''(2) = 27e^6$, $f^{(4)}(2) = 81e^6$, we have

$$P_4(x) = e^6(1 + 3x + 9x^2/2! + 27x^3/3! + 81x^4/4!).$$

MATHEMATICA

12.4. It is a good time now to install Mathematica, the computer algebra system. There are instructions on how to install on the website.

```
Taylor[f_, c_, n_] := Function[x, Sum[D[f[y], {y, k}](x - c)^k/k! /. y -> c, {k, 0, n}]];
f = Function[x, Sin[x] + Cos[3 x]];
g = Taylor[f, 0, 5]; g[x]
```

Here is the output

$$1 + x - \frac{9}{2}x^2 + \frac{3}{6}x^3 - \frac{27}{8}x^4 + \frac{5}{120}x^5$$

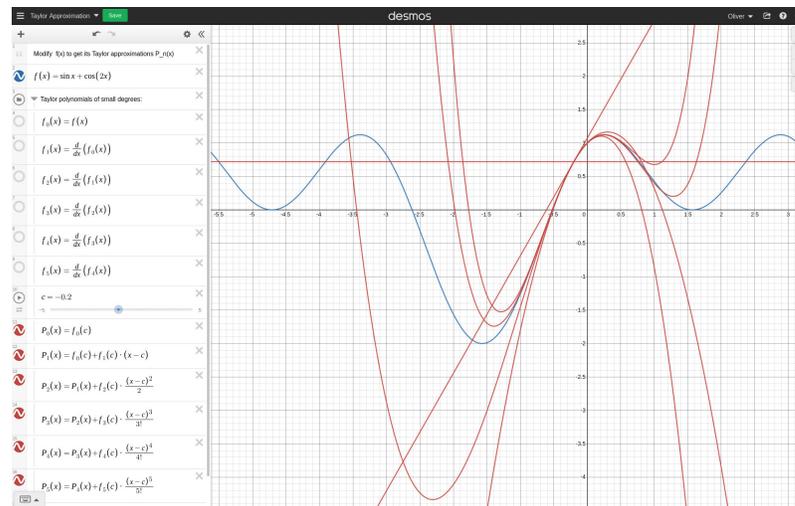
12.5. An here is an example, where the expansion is done at an other point:

```
g = Taylor[f, Pi, 4]; g[x]
```

$$-1 + \pi - x + \frac{9(-\pi + x)^2}{2} + \frac{(-\pi + x)^3}{6} - \frac{27(-\pi + x)^4}{8}$$

DESMOS

12.6. Desmos is a popular online graphing calculator. There are many applets for Taylor series already built. Here is an adaptation to our course:



ON THE RADAR AND REMARKS

- HW 11 is due Wednesday 10/6 after the exam.
- Our first midterm is on October 4th, the upcoming Monday.