CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

Lecture 33: Phase space analysis, 11/22/2021

Systems

33.1. There is a straightforward algorithm to analyze a phase-portrait of a system of two differential equations. We look at this in an example, then practice it in many cases.

33.2. Let us look at the Murray system:

$$\begin{aligned} x'(t) &= x(6-2x)-xy \\ y'(t) &= y(4-y)-xy \end{aligned}$$

33.3. Factor:

$$\begin{aligned} x'(t) &= x(6-2x-y) \\ y'(t) &= y(4-y-x) \end{aligned}$$

33.4. Nullclines: Draw the x-nullclines, the place where $x'=0$ and the y-nullclines, the place where $y'=0$.

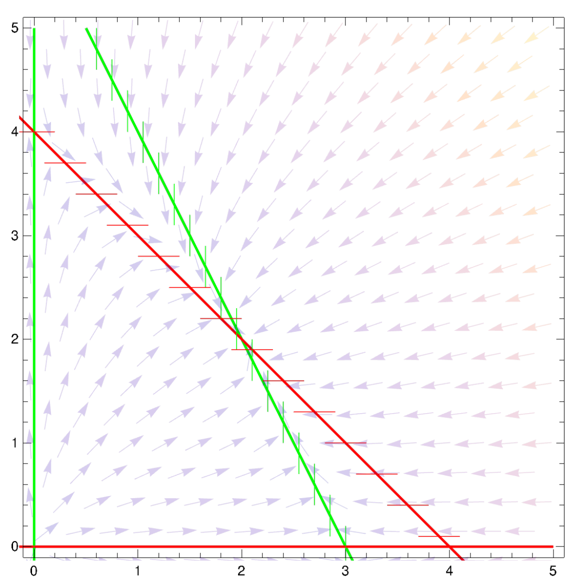

FIGURE 1. Drawing the nullclines. Color them or mark them differently so that you know which one is the x-nullcline and which is the y-nullcline.

33.5. Equilibria: The equilibrium points are where x-nullclines and y-nullclines intersect.

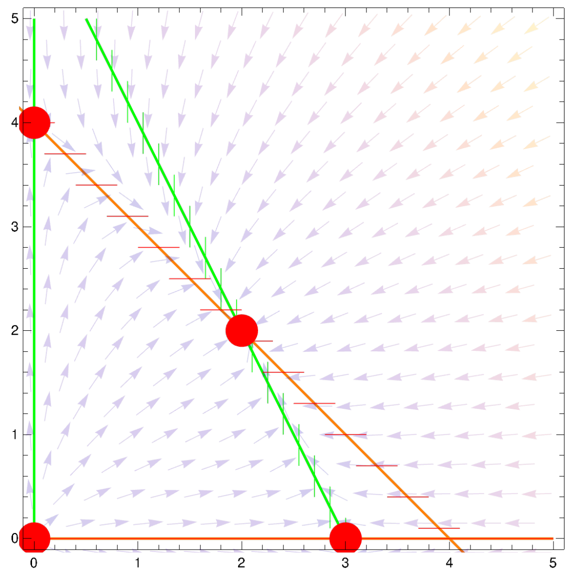

FIGURE 2. Equilibrium points.

33.6. Complete: We know how the orbits move on the null-clines. Complete the picture by filling in the void:

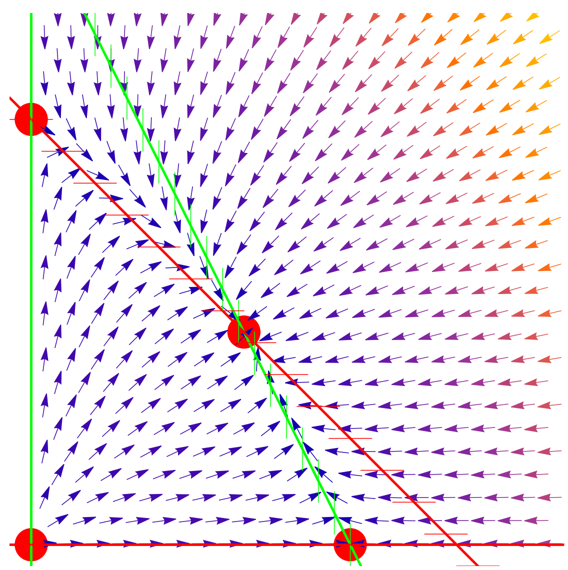

FIGURE 3. The completed phase space picture.

33.7. We have now a complete picture and know that if we start with a population initial condition $x > 0$ and $y > 0$, then we end up at the equilibrium point $(2, 2)$. PS 31 is due Monday after thanksgiving.

OLIVER KNILL, KNILL@MATH.HARVARD.EDU, MATH 1B, HARVARD COLLEGE, FALL 2021