CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

Lecture 34: More systems, 11/29/2021

Systems

34.1. Last time we analyzed a **competing species** system by finding the **nullclines** and **equilibrium points**, then completed the **phase space**.

$$\begin{aligned} x'(t) &= x(6-2x) - xy \\ y'(t) &= y(4-y) - xy \end{aligned}$$

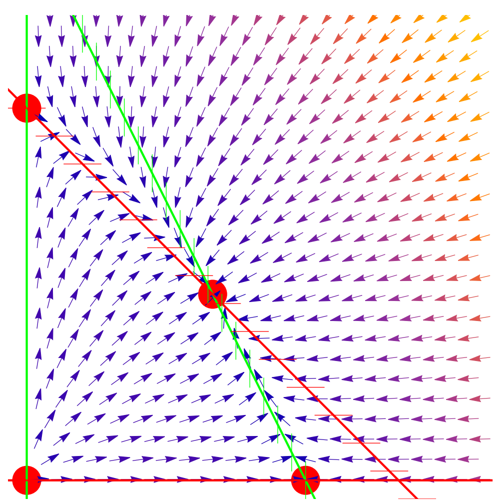

Figure 1. The competing species system. It is the Murray system, we have seen the last time. Both populations have to compromise. The stable equilibrium in the middle is lower for both if one compares with the equilibrium without competition.

34.2. Here is a predator-pray system:

$$\begin{aligned} x'(t) &= x(7-2x) - xy \\ y'(t) &= y(1-y) + xy \end{aligned}$$

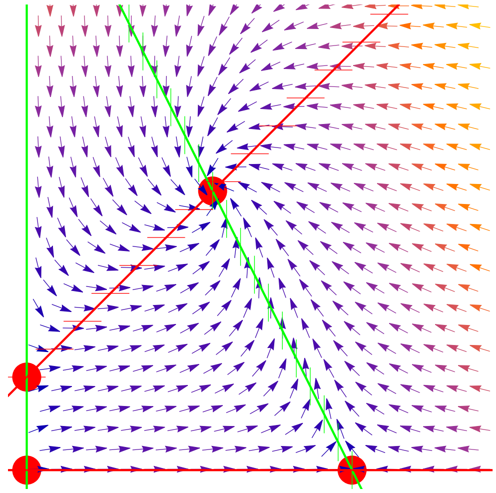

FIGURE 2. A predator-pray system. The stable equilibrium has moved up. The preditor strives better with than without pray.

34.3. The following system is called a **Volterra-Lodka system**. It is remarkable that it is non-linear and still produces circular periodic motion around an equilibrium point. The interpretation is that x is the pray and y is the predator. You see that if there were no predators (y=0), then $x(t)$ would grow exponentially. If there were no prays, then the predator would die out exponentially.

$$\begin{aligned} x'(t) &= 0.4x - 0.2xy \\ y'(t) &= -0.1y + 0.2xy \end{aligned}$$

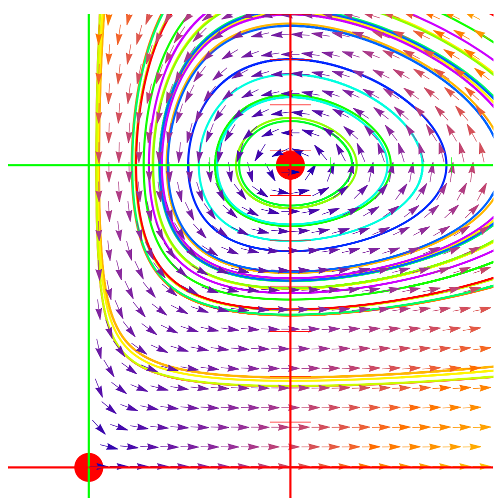

FIGURE 3. A Volterra-Lodka system.

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