

CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

Lecture 35: Overview, 12/01/2021

MAIN TOPICS

This course had three major topics: Integration, Series and Differential equations. There is a good study guide and practice exams on the website.

INTEGRATION

Write down Riemann sums for slicing problems. A typical problem is to get the mass of a body when the density function ρ is given. For improper integrals, the cases $\int_1^\infty x^{-p} dx$ and $\int_0^1 x^{-p} dx$ are important.

In the case of Riemann sum approximations, we have error terms for left and right point series, Trapezoid and midpoint series.

SERIES

Use criteria to determine whether a series is convergent or divergent. Radius of convergence, find anti-derivatives. Find n'th order Taylor approximation and error terms.

Important series to know are:

$$\begin{array}{l} \text{p-series} \quad \sum_{n=1}^{\infty} n^{-p} \\ \text{geometric series} \quad \sum_{k=0}^{\infty} ar^k = a/(1-r) \\ \text{Harmonic series} \quad \sum_{k=1}^{\infty} 1/k \end{array}$$

You also need to know the convergence tests: n'th term test Comparison test Integral comparison Asymptotic comparison Alternating series Ratio test

DIFFERENTIAL EQUATIONS

For differential equations, we have seen slope fields autonomous equations $y' = f(y)$ and their equilibria the roots of f . input output analysis $y' = ry - c$, second order differential equations, systems of equations $x' = f(x, y), y' = g(x, y)$ by finding Nullclines, equilibrium points, the behavior on the nullclines and the global picture.

REMINDERS

The exam is on Thursday, December 9 from 2-5: A-La are Emerson 105, Lb-Sh are Emerson 210, Si-Z are in Emerson 305. The 7-8 problems cover all the material but heavier on the material after second midterm.

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