



# *Lecture 1*

Welcome to Math  
1B, Fall 2021

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SC 432

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8/30/2021 near Mather house

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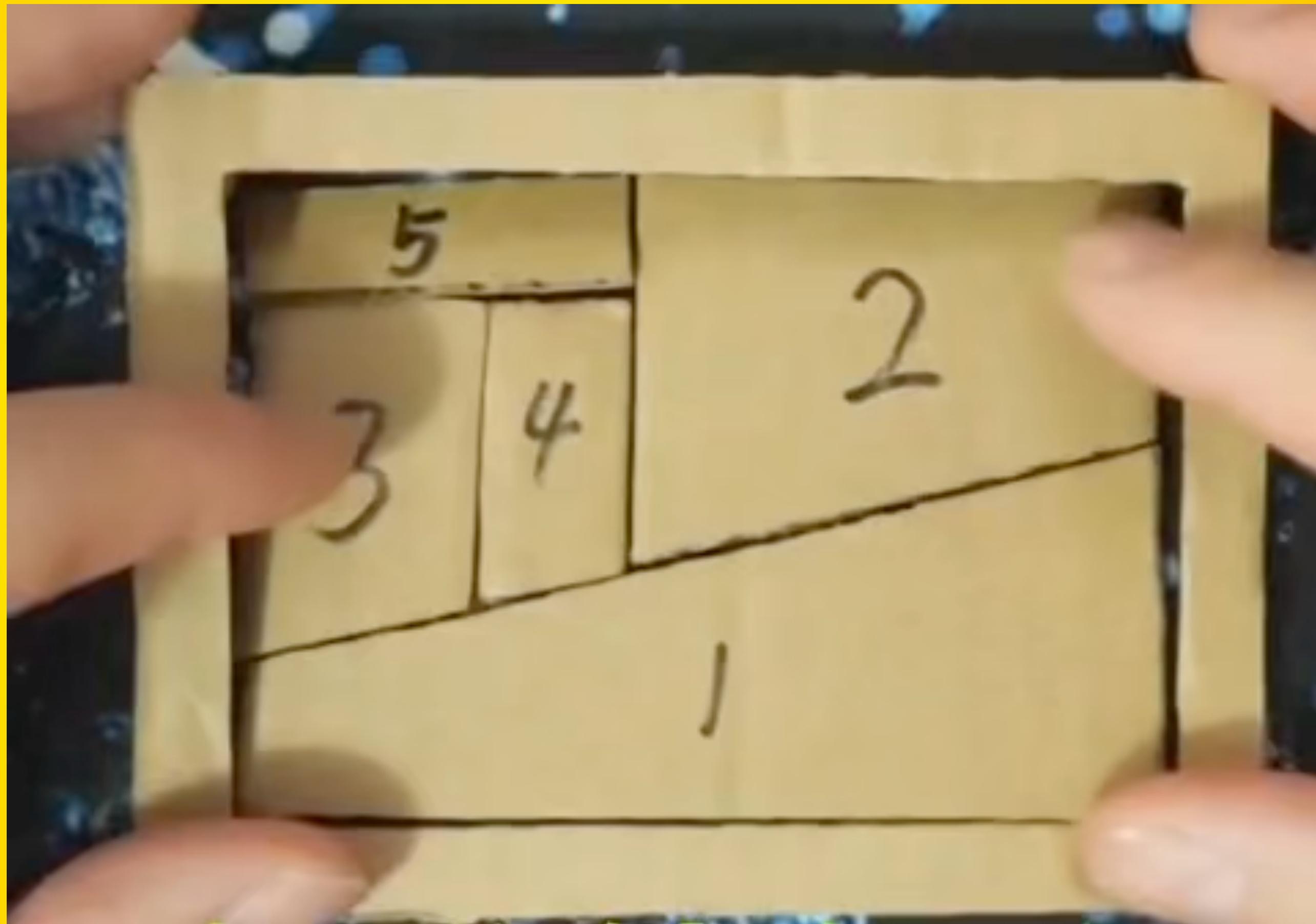
1) Reminders. Start working on the QRD assignment.  
Problem set 1 is due on Wednesday after labor day.

2) Density and approximation  
Problem 1 in worksheet

3) Get upper and lower bounds through slicing  
Problem 2 in worksheet

4) Technique test  
Integration techniques and do some examples

Slicing



*Part 1*

Reminders

# Homework

due 9/8

## PROBLEM SET 01 – DENSITY AND APPROXIMATION

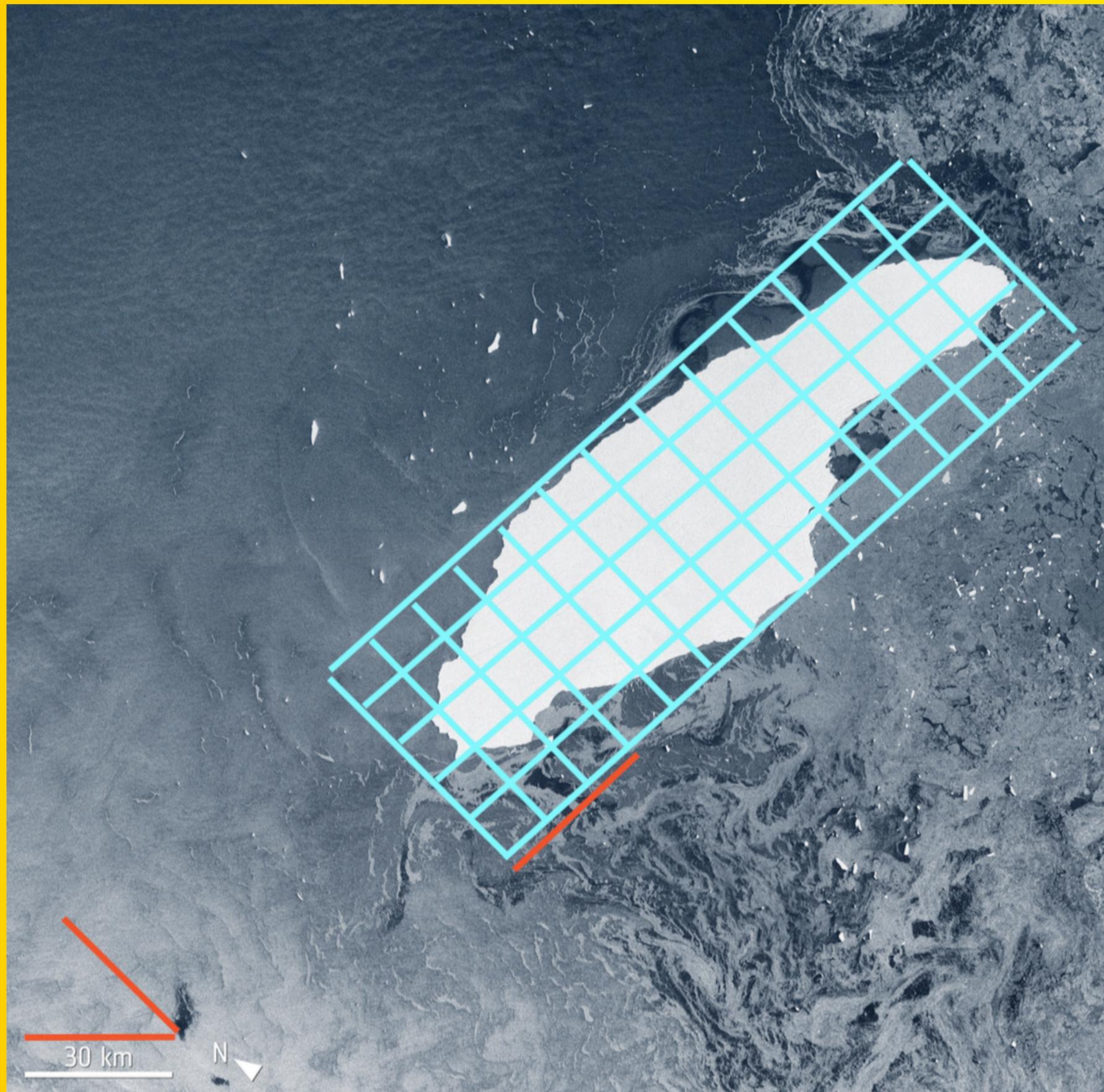
*About homework in Math 1b:*

- Homework is intended to help you master the basic ideas from class and learn to apply them to new situations; therefore, we intentionally give you problems which do *not* look like those you've seen in class. For example, in this problem set, #1 is meant to help you cement the basic ideas from class. #2 leads you through the process of taking the basic idea of #1 and applying it to a more complicated problem. #3 and #4 look at additional applications. Problems 5 and 6 are the two weekly problems that you got last week with Problem Set 00, and are now due.
- When writing up your homework, we expect you to explain your thought process; your final answer is less important than the reasoning you used to reach it. As a rule of thumb, if you're not using any words when writing up the problems, you probably aren't giving enough explanation. We also expect you to communicate your thoughts clearly, which includes using correct mathematical notation.

When writing out your answers, it may help to imagine that you are writing to a fellow student to explain how to solve the problem. This will help you give the right level of detail.

- You are encouraged to discuss homework with other students in the course. However, all submitted work should be written up individually, and you should acknowledge any help you receive.





due 9/10

from QRD

# Information for Techniques of Integration test

# 9/15

- Wednesday, 9/15 from 6-7:30.
- Test will be remote. Download from Gradescope in a 15 minute window starting 6pm and uploading within 90 minutes of your download time. We will give more instructions closer to the test date.
- People with AEO letters should contact Cliff about getting extra time
- People with conflicts should contact Cliff about starting earlier or later.
- There will be make-up Techniques test in mid-October where people can recoup any and all lost points.
- No aids of any kind (electronic, notes, books, people, ...) can be used during the test. Just you and your brain.
- Study material is on the Exam Information (Techniques test) page of the Math 1b website. This includes practice problems with solutions, practice exams with solutions and videos.
- The sections on Monday, 9/13 will be reviewing for the test.

# *Part 2*

**Density and approximation**

# *Density*

$$\int_a^b \rho(x) dx$$

definite  
integral

$$\sum_{k=1}^n \rho(x_k) \Delta x_k$$

Slicing

# *Key notions*

$$\int_a^b \rho(x) dx$$

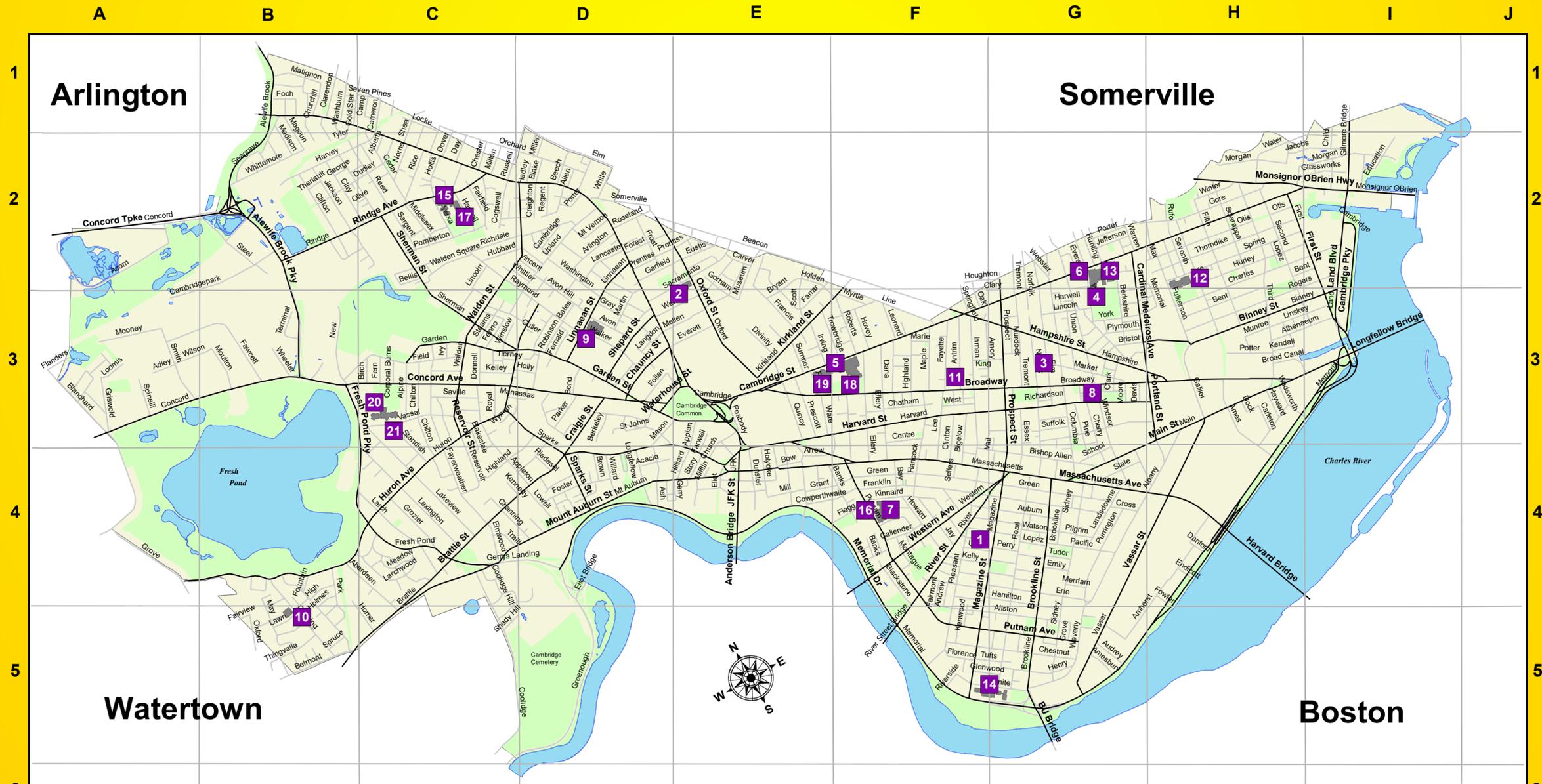
**definite  
integral**

$$\sum_{k=1}^n \rho(x_k) \Delta x_k$$

**Slicing**

# Worksheet : Cambridge

1. According to Wikipedia, the population density in Cambridge in the year 2010 was 7,000 people per square kilometer<sup>(1)</sup>. It also mentions that the land area of Cambridge is about 17 square kilometers. What was the population of Cambridge in the year 2010?

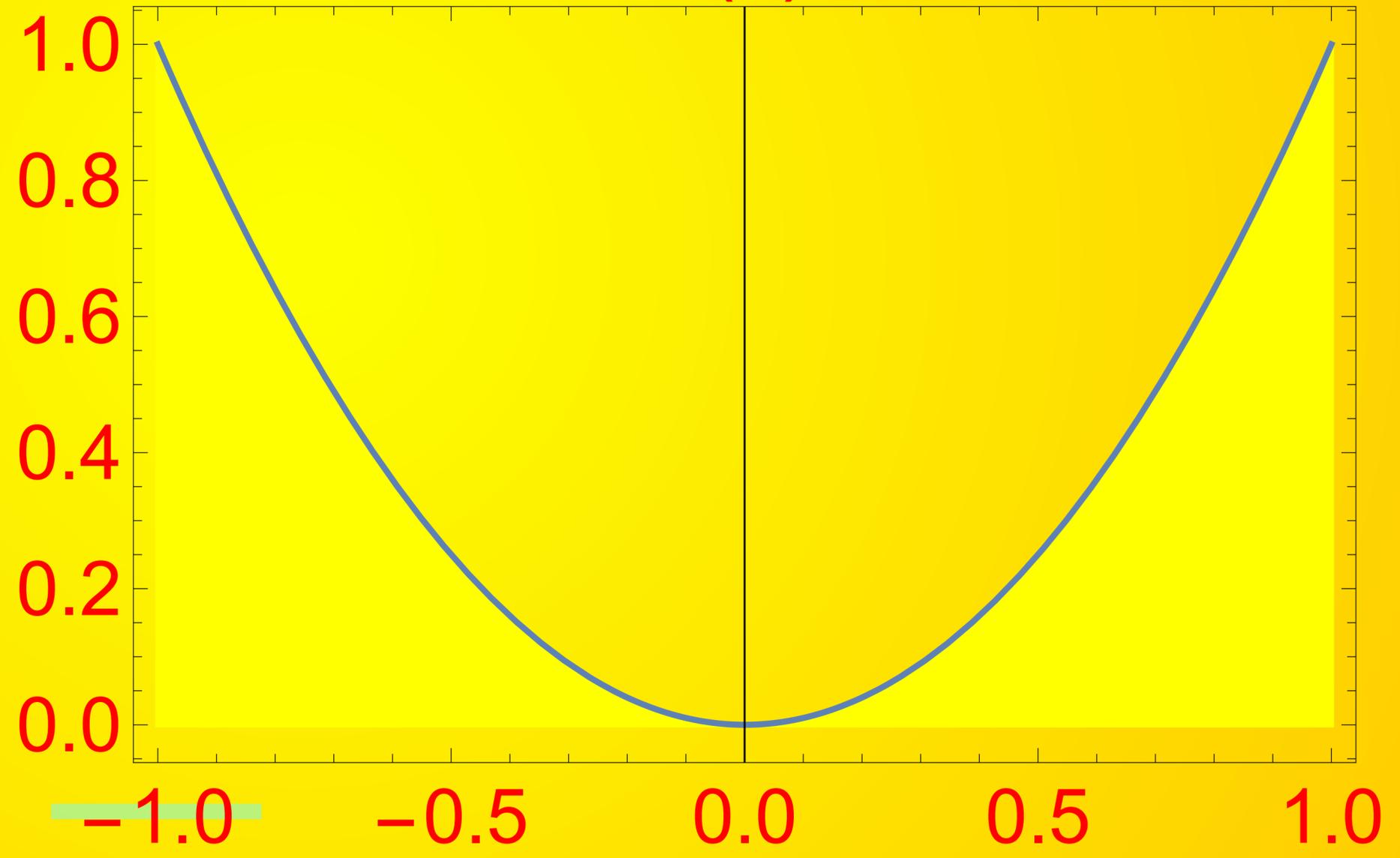


# Example 1

$f(x)$

$$\int_{-1}^1 x^2 dx$$

Can you give a simple upper and lower bound for the total population on the interval with density  $x^2$

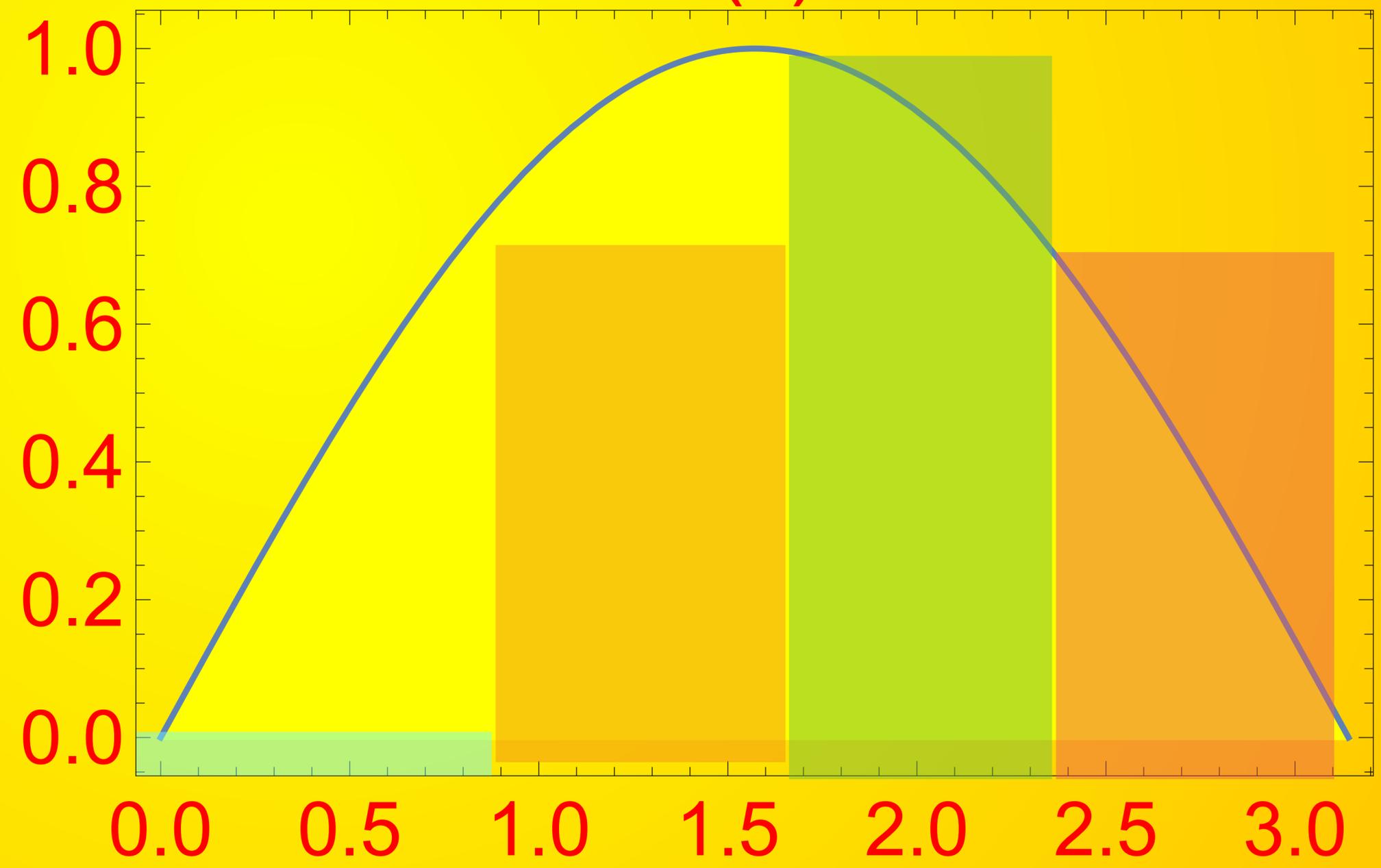


$$\int_0^{\pi} \sin(x) dx$$

# Example 2

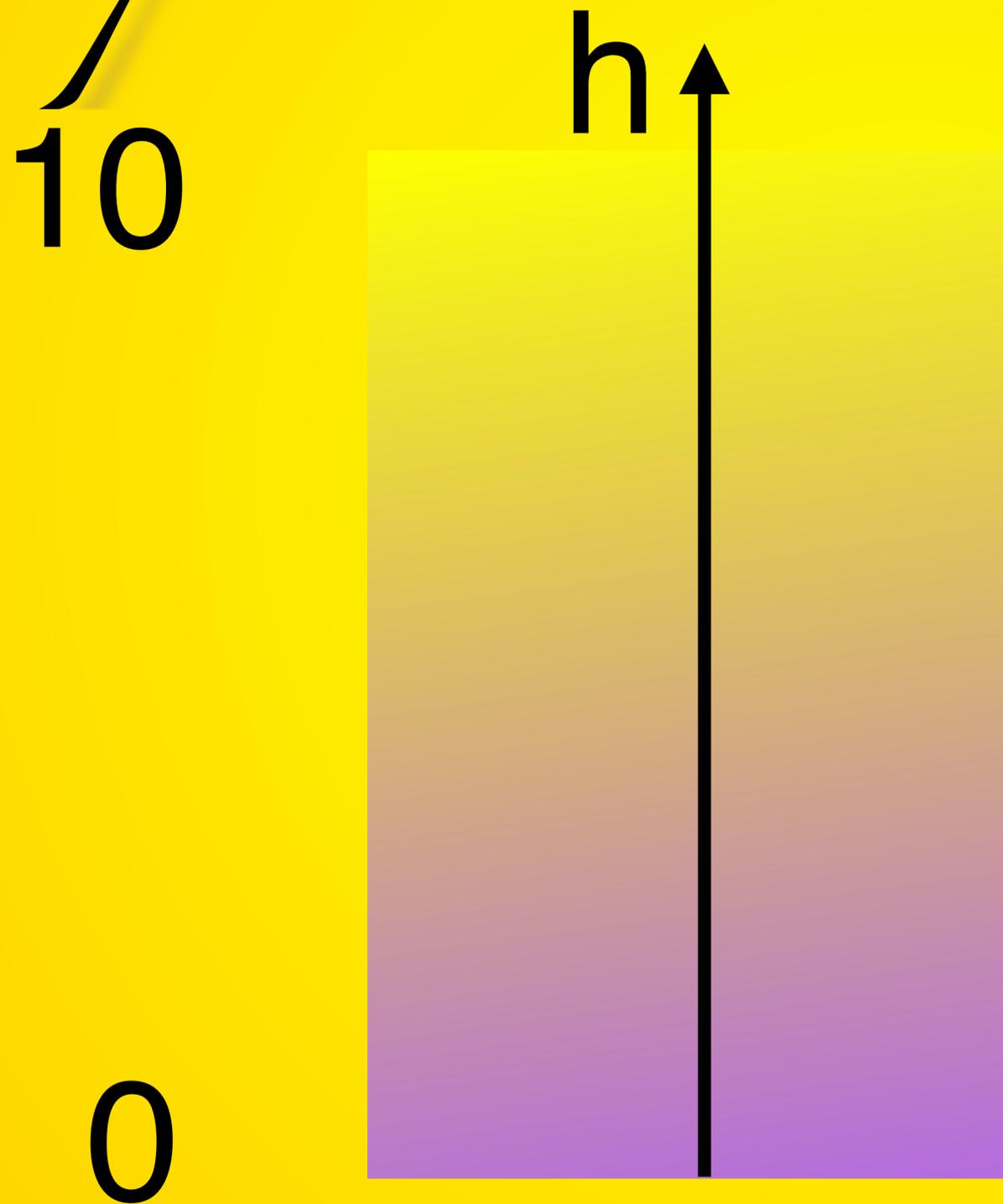
f(x)

Can you  
give a simple  
upper and lower  
bound for  
the total population  
choose  $\Delta x = \pi/4$



# Example 3

- Blueberry juice has density  $\rho$



$$\int_0^{10} \rho(h) dh$$

$$\rho(h) = 100 - 3h$$

# *Estimations*

Slice the interval into  $n$  pieces.

find an upper bound

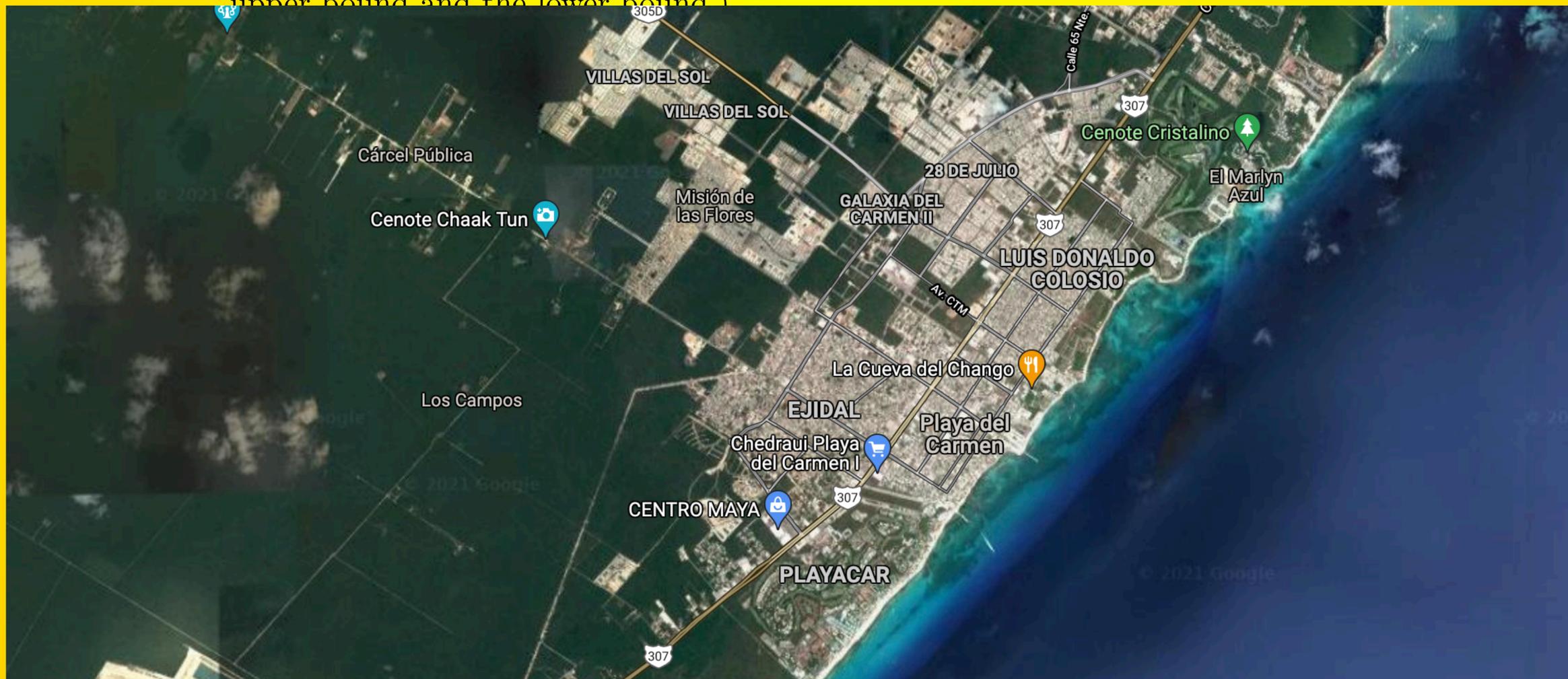
$$\sum_{k=1}^n f(x_k) \Delta x_k$$

find an lower bound

# Worksheet: Playa del Carmen

2. A seaside village, Playa del Carmen, is in the shape of a rectangle with sides of 4 kilometers and 6 kilometers.<sup>(2)</sup> The sea lies along a 6-kilometer long side. People prefer to live near the water, so the population density is not constant, but instead given by  $\rho(x) = 100(20 - x^2)$  people per square kilometer, where  $x$  is the distance from the seaside (so  $0 \leq x \leq 4$  when measuring in kilometers).<sup>(3)</sup>

(a) **Without using any calculus**, can you give rigorous upper and lower bounds for the total population of the village? (Thus, the true population of the village must be confined between the upper bound and the lower bound.)



# *Part 4*

## **Integration Techniques**

*THE END*