8/30/2021 near Mather house

Lecture 2

9/8/2021

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So, both QRD and Problem set 2 is due on Friday.

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Practice more integration techniques

Part I

Announcements and reminders

Homework due 9/10

Homework 02 – Density and the Definite Integral

1. Evaluate $\lim_{n\to\infty}\sum_{k=1}^{n} \sin x_k \Delta x$, where $x_k = \pi + k\Delta x$ and $\Delta x = \frac{\pi}{3n}$.

When writing up integration application problems (whether for homework or on exams), we always expect you to do the following.

- Explain your basic strategy; for instance, if you are using symmetry to simplify your work, say so.
- Explain clearly how you are slicing the problem (this can be done with a picture or in words).
- Say exactly what interval you are slicing.
- Approximate the desired quantity (in the case of the following problems, amount of cobalt) in the k-th slice.
- If you are asked explicitly for this, write a general Riemann sum approximating the total quantity and take an appropriate limit to arrive at a definite integral. If you are not explicitly asked for a Riemann sum, you are welcome to go directly from your approximation for the k-th slice to the definite integral.

As always, we expect you to use notation correctly when writing up these problems.

Spraying a piece of pottery with cobalt will result in a blue color when the piece is fired. The shade of blue is determined by the density of cobalt; the greater the density of the cobalt, the darker the blue. You can get gradations of blue by applying cobalt glaze with a spray gun and varying the density of the application. Makoto Yabe and Wasma'a Chorbachi are professional potters at the Radcliffe Pottery Studio on Western Avenue, and the next four problems are about glazing pieces of pottery shades of blue.

2. Makoto has made a rectangular sushi platter from a slab of clay 14 inches by 6 inches. He applies cobalt so that the density of the application increases with the distance from one of the long sides of the platter. The density of cobalt glaze is given by $\rho(x)$ mg/square inch where x is the distance (in inches) from one long side of the sushi platter.

   (a) Write a general Riemann sum (i.e., a Riemann sum with n terms) that approximates the amount of cobalt Makoto used.

   (b) Write an integral in terms of $\rho(x)$ that gives the exact amount of cobalt used.

3. Makoto decides to try a more symmetric glaze application on his next sushi platter. The platter is again 14 inches by 6 inches. This time the deepest blue is in a stripe along the long center line of the platter, and the intensity of the blue fades with the distance from this central line. The density of cobalt glaze is given by $\rho(x)$ mg/square inch, where x is the distance (in inches) from the longitudinal center of the sushi platter.

   (a) Write a general Riemann sum that approximates the amount of cobalt Makoto used.

   (b) Write an integral in terms of $\rho(x)$ that gives the exact amount of cobalt used.

4. Wasma'a is glazing a large round plate 16 inches in diameter. She decides to have a deep blue center fading out into pale blue along the rim. She applies cobalt glaze so that its density is given by $\rho(x)$ mg/square inch where x is the distance (in inches) from the center of the plate. Write an integral in terms of $\rho(x)$ that gives the exact amount of cobalt used.

5. For her next round plate (again 16 inches in diameter), Wasma'a decides to have a deep blue line 16 inches long running through the center of the plate. She has the shade of blue fade into paler and paler blue as the distance from this deep blue line increases. She applies cobalt glaze so that its density is given by $\rho(x)$ mg/square inch where x is the distance (in inches) from dark blue diameter of the plate. Write an integral in terms of $\rho(x)$ that gives the exact amount of cobalt used.

Here's a hint to help you check your answers to the previous four problems. If $\rho(x) = x$, then the integrals you found should evaluate to 252, 126, $\frac{1024\pi}{3}$, and $\frac{2048}{3}$. If $\rho(x) = 1$, what should they evaluate to?

6. Please also turn in Weekly Problem TI 3. Here it is:

   When using integration by parts, please be sure to state what u, dv, du, and v are.

   (a) Use integration by parts to do the following integrals.

      i. $\int \sqrt{x}\ln x\,dx$.

      ii. $\int_0^1 x^2 e^{3x}\,dx$.

   (b) Evaluate **three of the the following four** integrals. (One of these indefinite integrals cannot be found using any of the techniques of integration you know! Make sure you pick the three that you *can* do.) Feel free to use both integration by parts and integration by substitution.

      i. $\int \ln(3x+2)\,dx$.

      ii. $\int e^{-x^2}\,dx$.

      iii. $\int e^{\sqrt{x}}\,dx$

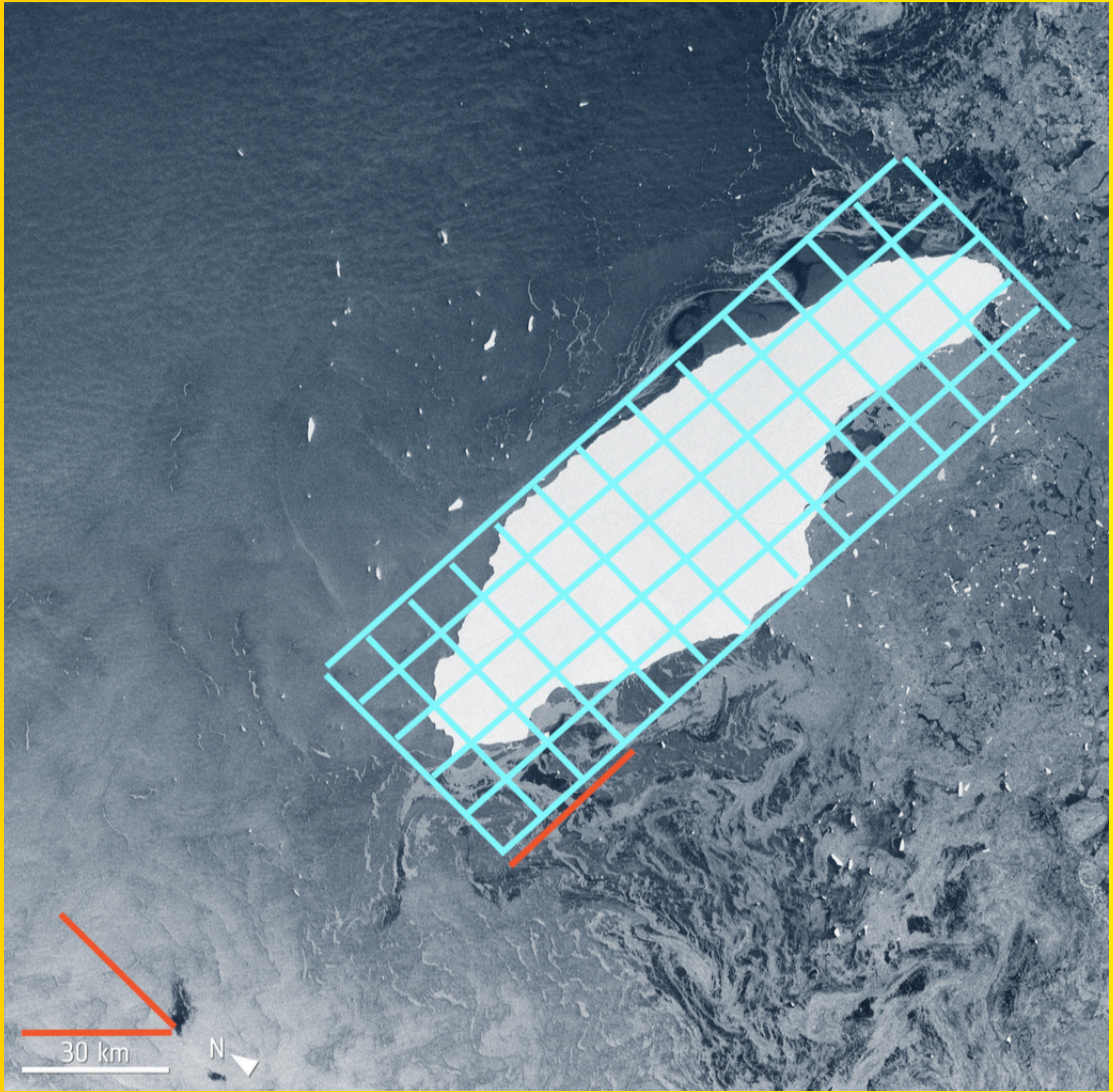
      iv. $\int \ln\sqrt{x}\,dx$.

7. Please also turn in Weekly Problem TI 4. Here it is:

   (a) Prove the formula $\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1}\,dx$, where n is a positive integer.

   (b) Use the formula (a few times) to express $\int (\ln x)^4\,dx$ in a form that doesn't use any integrals.

W. Start Weekly Problems #1 – #3. These will be due with Problem Sets 04 and 05, but you already know everything you need to do them. Even if you don't have a lot of time to spend with them right now, we encourage you to read them and make sure you understand what they are asking. Research



due 9/10

from QRD

Information for Techniques of Integration test

9/15

- Wednesday, 9/15 from 6-7:30.
- Test will be remote. Download from Gradescope in a 15 minute window starting 6pm and uploading within 90 minutes of your download time. We will give more instructions closer to the test date.
- People with AEO letters should contact Cliff about getting extra time
- People with conflicts should contact Cliffabout starting earlier or later.
- There will be make-up Techniques test in mid-October where people can recoup any and all lost points.
- No aids of any kind (electronic, notes, books, people, ...) can be used during the test. Just you and your brain.
- Study material is on the Exam Information (Techniques test) page of the Math 1b website. This includes practice problems with solutions, practice exams with solutions and videos.
- The sections on Monday, 9/13 will be reviewing for the test.

Problem Sessions

The Math 1b Problem/review sessions will start this coming Thursday (Sept. 9) and run through the last week of the semester (except for holidays). Here is the schedule:

1. Tuesdays 9-10am in Science Center 309.
2. Thursdays 10:30-11:30am in Science Center 221.
3. Thursdays 12-1pm in Science Center 110
4. Thursdays 4:30-5:30pm in Science Center lecture hall E.

(The schedule and rooms are also on the Math 1b website; click the link on the left hand side-bar that says 'Problem/review session times and rooms'.)

The problem/review sessions for this coming Thursday and this coming Tuesday will focus in part on going over techniques of integration for the upcoming assessment test on September 15. In general, these sessions will review recent course material and homework problems. These are really good opportunities to ask questions about stuff you don't understand. (These are not mandatory; they are here for you to take advantage of. Go to more than one if you want.)

Keep in mind that you can also ask Math 1b questions (and get homework help) at the Math Question Center, room B10 of the Science Center in the evenings, from 7:30-10:30 on Sunday, Tuesday, Wednesday and Thursday; and on Monday from 8:30-10:30; and also go to the TF office hours.

Part 2

Definite integral

Definite integral

$$\int_a^b \rho(x)\, dx$$

definite integral

$$\sum_{k=1}^{n} \rho(x_k)\Delta x$$

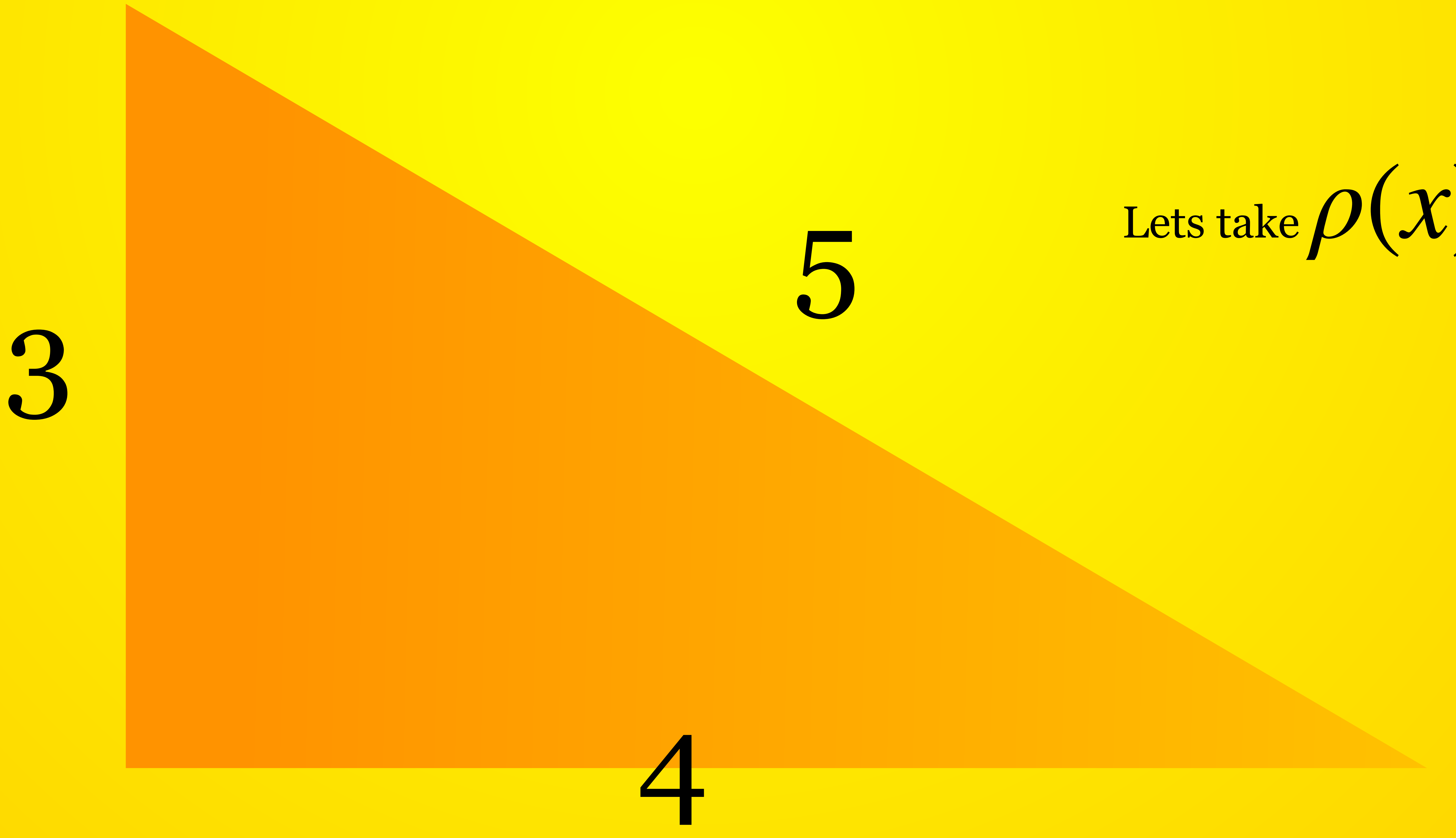
Riemann Sum


Please Use
Other Door


Pizza
Heaven and Hell
Heaven is where:
The chefs are French
The lovers are Italian
The police are British
The mechanics are German
And it is all run by the Swiss
Hell is where:
The chefs are British
The lovers are Swiss
The mechanics are French
The police are German
And it is all run by the Italians

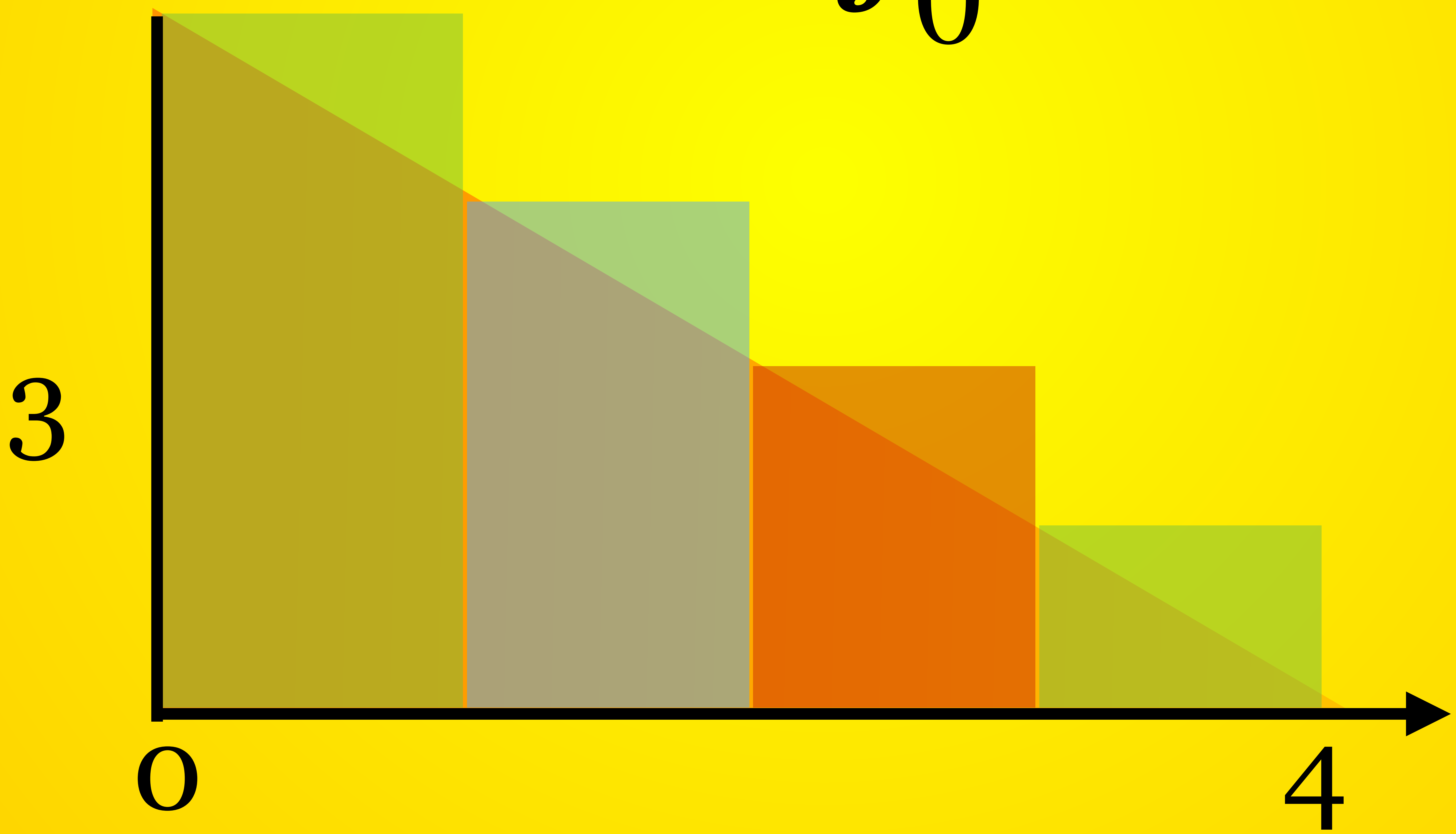
Worksheet : Pizza

1. (Problem Set 1, #3) Pizza **Pinoccio** is known for its "pizza wedges," which are shaped like right triangles with sides of 3, 4, and 5 inches. Parmesan cheese is sprinkled on each wedge so that the density of cheese is given by $\rho(x)$ ounces per square inch, where x measures distance (in inches) from the 3-inch side of the slice. Approximate the amount of Parmesan on each wedge.



Lets take $\rho(x) = x^5$

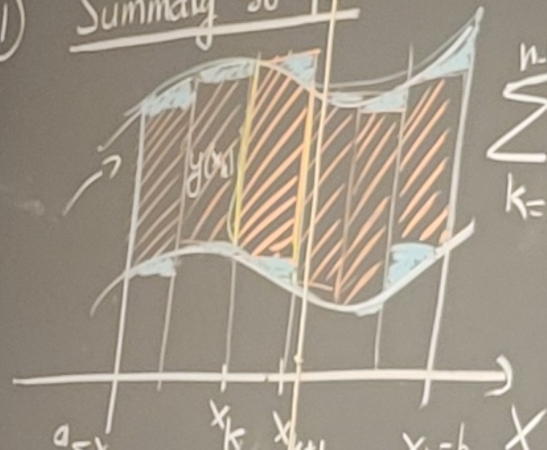
$$\int_0^4 x(3 - x)\, dx$$



Lecture 2

Density and Definite Integral

① Summary so far.



n number of slices.

$\rho(x)$ density

$\Delta x = \frac{(b-a)}{n}$ width of the slice

$y(x_k)$ height

Left Riemann sum

$$\sum_{k=0}^{n-1} \rho(x_k) \boxed{y(x_k)\cdot\Delta x}$$

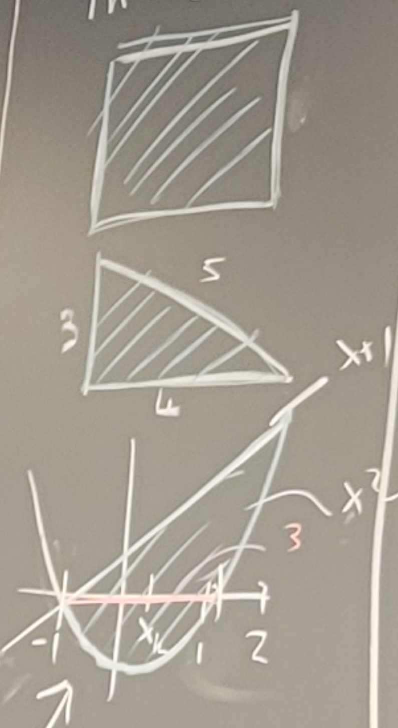
Area

or

Right Riemann sum

$$\sum_{k=1}^{n} \rho(x_k)\, y(x_k)\, \Delta x$$

In the homework



$x_K = K\Delta x - 1$

$\Delta x = 3/n$

② Today: take the limit $n\to\infty$

The sum converges to a definite integral

$$\sum_{k=1}^{n} \rho(x_k)\, y(x_k)\, \Delta x \xrightarrow[n\to\infty]{} \int_a^b \rho(x)\, y(x)\, dx$$

Riemann sum

Definite integral

"infinitesimal"

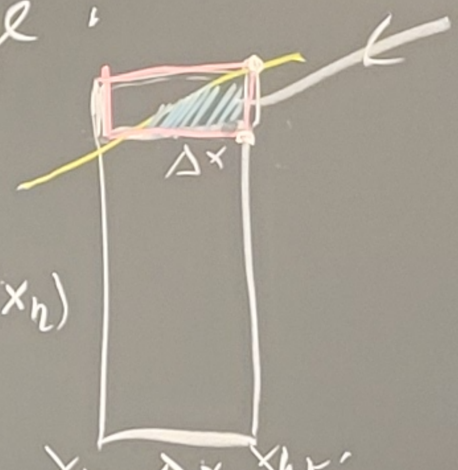
n slices, each error $\frac{M(b-a)^2}{n^2}$

Total error: $M\frac{(b-a)^2}{n} \to 0$

Why does this converge?

Can we estimate the error:

If the slope of the boundary curves is bounded by M

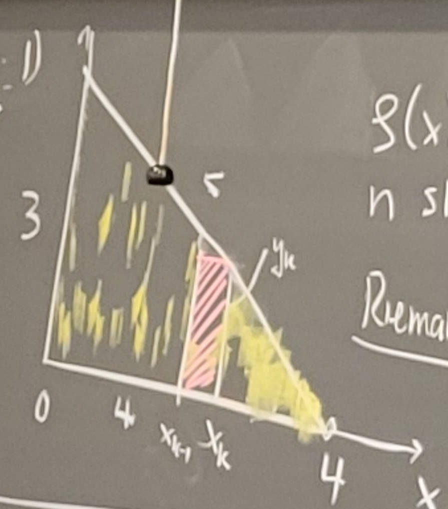


excess area is smaller or equal than the area $\square$ $\le$ slope $\cdot\,\Delta x$ $\le M\cdot\Delta x$

Δx

$\le M(\Delta x)^2 = M\frac{(b-a)^2}{n^2}$

Examples: 1)



Pizza.

Parmesan cheese

$\rho(x) = x^5$

n slices, $\Delta x = \boxed{\frac{4}{n}}$

$y_k = \boxed{-\frac{3}{4}x_k + 3}$

Riemann sum:

$$\sum_{k=0}^{n-1} (x_k)^5 \left(-\frac{3}{4}x_k+3\right) \Delta x$$

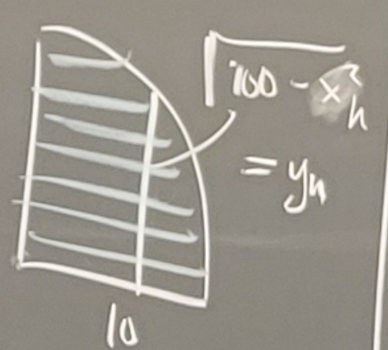
y_k

Definite integral: Total amount of cheese.

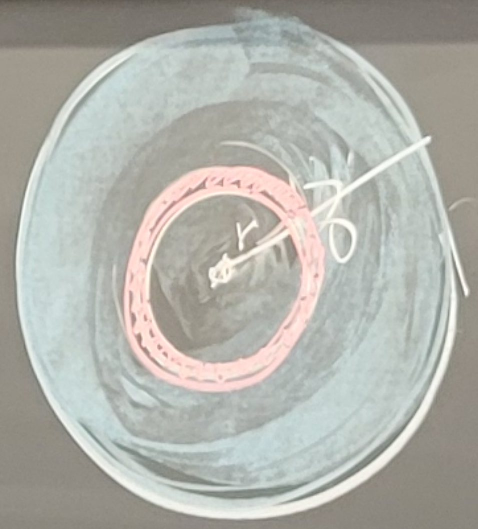
$$\int_0^4 x^5\left(-\frac{3}{4}x+3\right)dx = \int_0^4 -\frac{3}{4}x^6 + 3x^5\, dx = -\frac{3}{4}\frac{x^7}{7} + \frac{3x^6}{6}\Big|_0^4$$

$$= \boxed{\left(-\frac{3}{4}\right)\frac{4^7}{7} + \frac{3\cdot 4^6}{6}}$$

Example 2)



Slicing with horizontal pieces



Ink density: $r_k^3 = \rho(r_k)$

new: we are slicing with annular slices. We take n slices.

Δr

$r_k = \frac{10}{n}$

$$\sum_{k=1}^{n} \rho(r_k) \boxed{2\pi r_k \quad \Delta r}$$

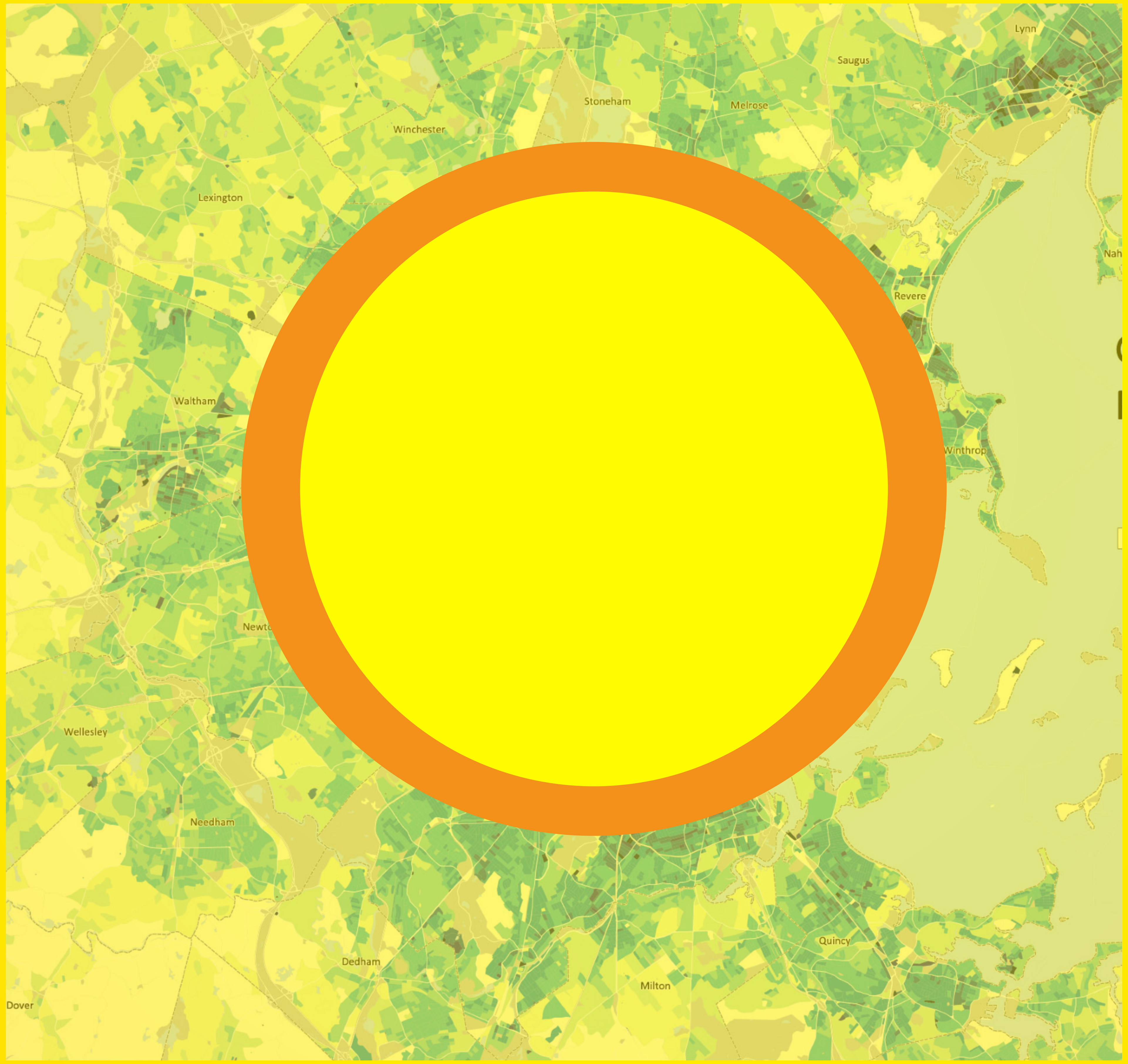
$\pi(r_{k+1})^2 - \pi(r_k^2)$

$\pi(r_k + \Delta r)^2 - \pi r_k^2$

~~πr_k^2~~ $+ \boxed{2\pi r_k \Delta r} + \Delta r^2$ ~~$- \pi r_k^2$~~

Can be neglected

Worksheet: Boston




Greater Boston
Population Density
500/mi²
1,000/mi²
2,500/mi²
5,000/mi²
10,000/mi²
Stoneham
Melrose
Winchester
Lexington
Medford
Malden
Lincoln
Arlington
Nahant
Revere
Belmont
Everett
Chelsea
Waltham
Somerville
Winthrop
Watertown
Cambridge
Weston
Boston
Newton
Brookline
Wellesley
Needham
Natick
Hull
Quincy
Dedham
Milton
Dover
Cohasset


Greater Boston
Population Density (2010)
500/mi²
1,000/mi²
2,500/mi²
5,000/mi²
10,000/mi²
20,000/mi²
30,000/mi²
40,000/mi²
Unpopulated
Wilmington
Reading
Lynnfield
Peabody
Littleton
Carlisle
Harvard
Bedford
Burlington
Woburn
Salem
Marblehead
Boxborough
Acton
Lynn
Saugus
Concord
Stoneham
Melrose
Winchester
Lexington
Medford
Malden
Nahant
Bolton
Stow
Lincoln
Arlington
Revere
Maynard
Everett
Chelsea
Belmont
Hudson
Sudbury
Waltham
Somerville
Winthrop
Watertown
Cambridge
Weston
Wayland
Marlborough
Boston
Newton
Brookline
Southborough
Framingham
Wellesley
Hull
Needham
Natick
Westborough
Ashland
Quincy
Dedham
Milton
Cohasset
Dover
Hopkinton
Sherborn
Westwood
Hingham
Weymouth
Braintree
Scituate
Holliston
Norwood
Medfield
Millis
Canton
Norwell
Upton
Medway
Milford
Walpole
Holbrook
Rockland

Part 4

Integration Techniques

Examples

$$\int \sin^2(x)\,dx \qquad \int \sin^2(x)\cos^2(x)\,dx \qquad \int e^{\sqrt{x}}\,dx$$

$$\int \sin^3(x)\,dx \qquad \int \sin(x)\cos^2(x)\,dx \qquad \int \log(x)\,dx$$

$$\int \sin^4(x)\,dx \qquad \int x\cos(x)\,dx \qquad \int \sqrt{9-x^2}\,dx$$

THE END