8/30/2021 near Mather house

Lecture 3

9/10/2021

Area and Volume

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Finding Volumes by slicing.

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Part 1

Announcements and reminders

Homework 03 – Area and Volume

1. Let $\mathcal{R}$ be the region enclosed by the curves $y = x^2 + 1$ and $y = 4x + 1$. Find the area of $\mathcal{R}$ two ways, once by slicing vertically and once by slicing horizontally. For each method, please draw a picture showing the rectangle you used to approximate the k-th slice.

   Evaluate both integrals to show that you get the same answer either way.

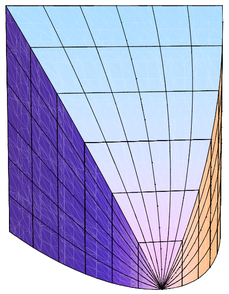
2. Find the area of the region in the first quadrant enclosed by $y = \sqrt{x}$, $y = x - 2$, and the x-axis. (Please evaluate your integral.) You may slice however you like, but please describe your slicing method to the reader.

3. The following definite integrals can be computed exactly without knowing an antiderivative of $\arctan x$. The point is to interpret the given definite integral as signed area and then to either use symmetry or slicing with horizontal strips to arrive at a different definite integral that is easy to evaluate.

   (a) $\displaystyle\int_{-2}^{2} \arctan x \; dx.$

   (b) $\displaystyle\int_{0}^{1} \arctan x \; dx.$ (Make sure your answer agrees with the answer to Problem Set 0, #3(e).)

4. A solid S has a base which is the region enclosed by the curves $y = x^2$ and $y = 4$. The cross-sections of the solid perpendicular to the y-axis are all squares. Write an integral that gives the volume of the solid. (You need not evaluate the integral.)



5. Weekly Problem TI 5 is now due. Here it is:
   Use integration by partial fractions decomposition to integrate the following:

   (a) $\displaystyle\int \frac{x-1}{x^2+2x-15} \; dx.$

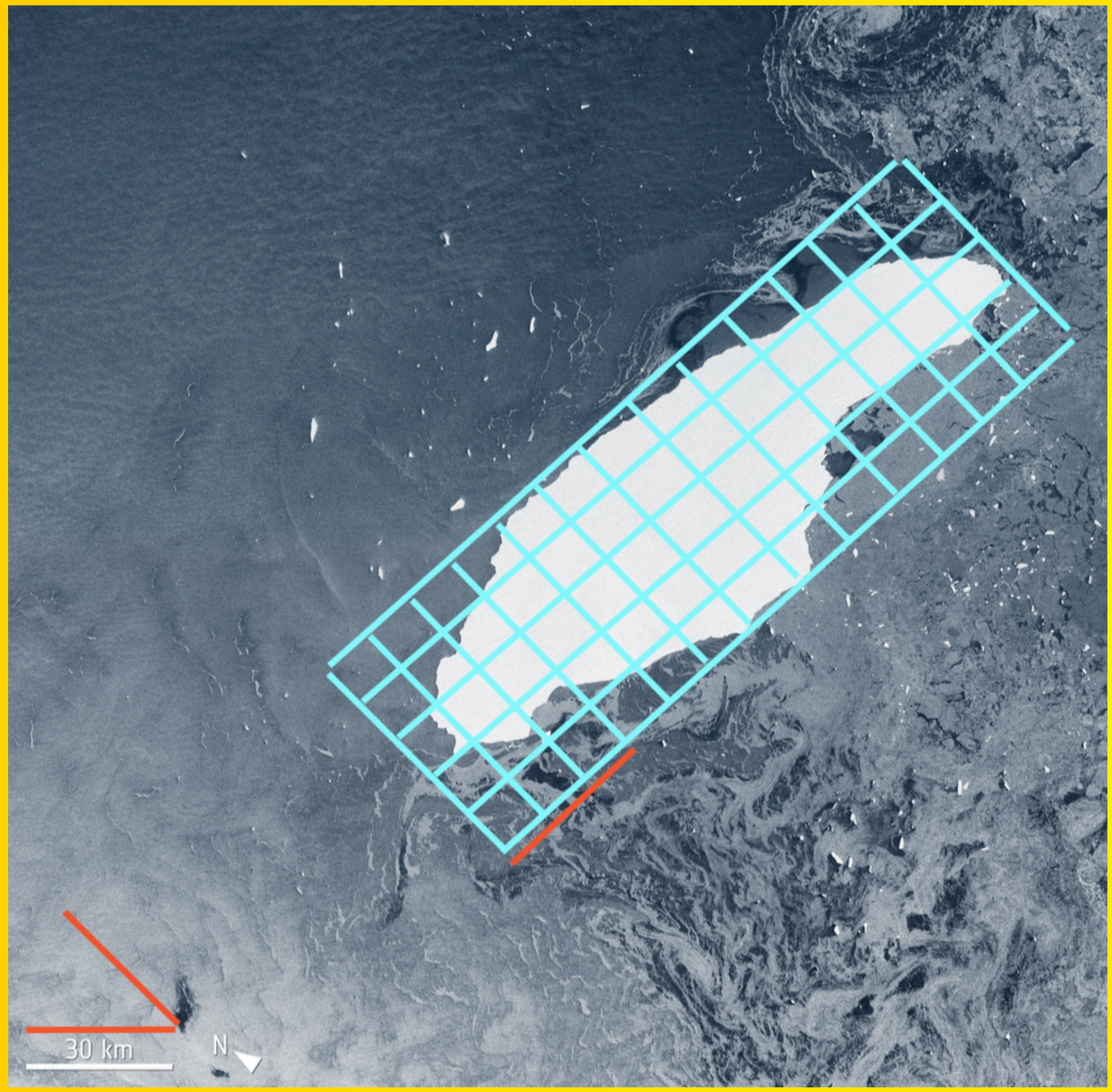
   (b) $\displaystyle\int \frac{x^2-2x-11}{x^2+2x-15} \; dx.$

   Hint: The numerator has the same degree as the denominator.

W Please start looking over the weekly problems! You now know enough to do #1 – #4. (These are different from TI #1 – TI #5.)

Homework due 9/13

5. Weekly Problem TI 5 is now due. Here it is:
Use integration by partial fractions decomposition to integrate the following:



dueToday

from QRD

Information for Techniques of Integration test

9/15

- Wednesday, 9/15 from 6-7:30.
- Test will be remote. Download from Gradescope in a 15 minute window starting 6pm and uploading within 90 minutes of your download time. We will give more instructions closer to the test date.
- People with AEO letters should contact Cliff about getting extra time
- People with conflicts should contact Cliffabout starting earlier or later.
- There will be make-up Techniques test in mid-October where people can recoup any and all lost points.
- No aids of any kind (electronic, notes, books, people, ...) can be used during the test. Just you and your brain.
- Study material is on the Exam Information (Techniques test) page of the Math 1b website. This includes practice problems with solutions, practice exams with solutions and videos.
- The sections on Monday, 9/13 will be reviewing for the test.

Problem Sessions

The Math 1b Problem/review sessions will start this coming Thursday (Sept. 9) and run through the last week of the semester (except for holidays). Here is the schedule:

1. Tuesdays 9-10am in Science Center 309.
2. Thursdays 10:30-11:30am in Science Center 221.
3. Thursdays 12-1pm in Science Center 110
4. Thursdays 4:30-5:30pm in Science Center lecture hall E.

(The schedule and rooms are also on the Math 1b website; click the link on the left hand side-bar that says 'Problem/review session times and rooms'.)

The problem/review sessions for this coming Thursday and this coming Tuesday will focus in part on going over techniques of integration for the upcoming assessment test on September 15. In general, these sessions will review recent course material and homework problems. These are really good opportunities to ask questions about stuff you don't understand. (These are not mandatory; they are here for you to take advantage of. Go to more than one if you want.)

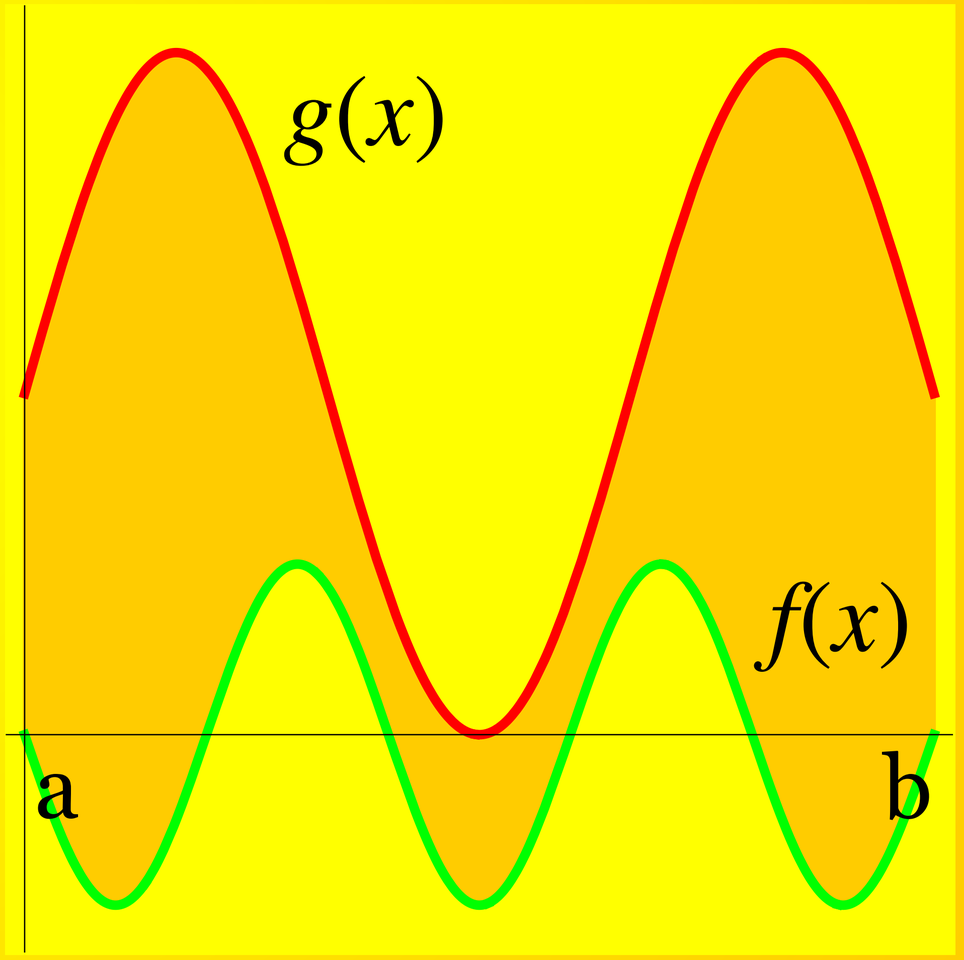
Keep in mind that you can also ask Math 1b questions (and get homework help) at the Math Question Center, room B10 of the Science Center in the evenings, from 7:30-10:30 on Sunday, Tuesday, Wednesday and Thursday; and on Monday from 8:30-10:30; and also go to the TF office hours.

Part 2

Area between curves

Area between curves

$$\int_a^b g(x) - f(x)dx$$

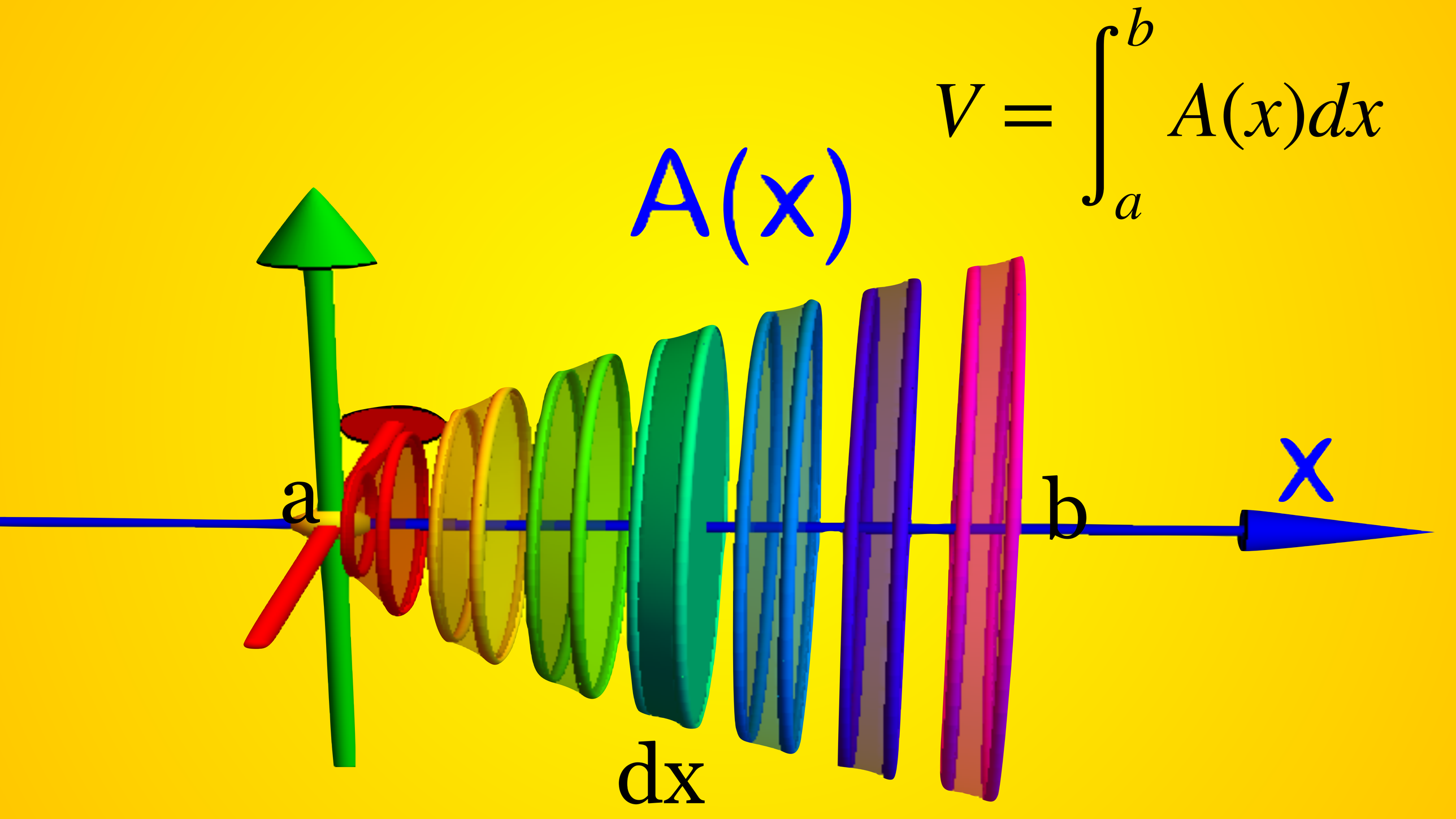


Make your own area problem!

Part 3

Volume computation

$$V = \int_a^b A(x)dx$$

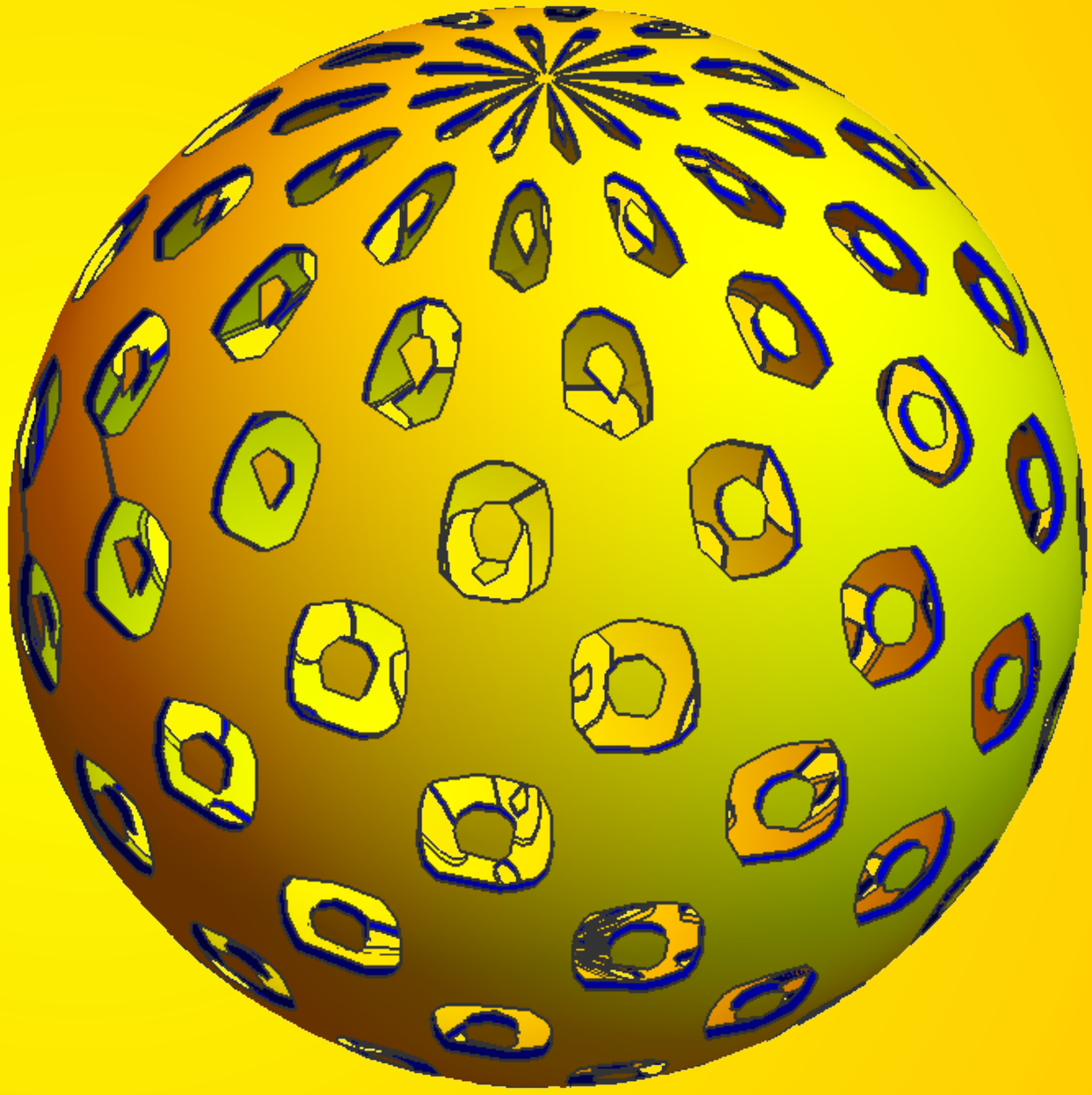


Grape

We have

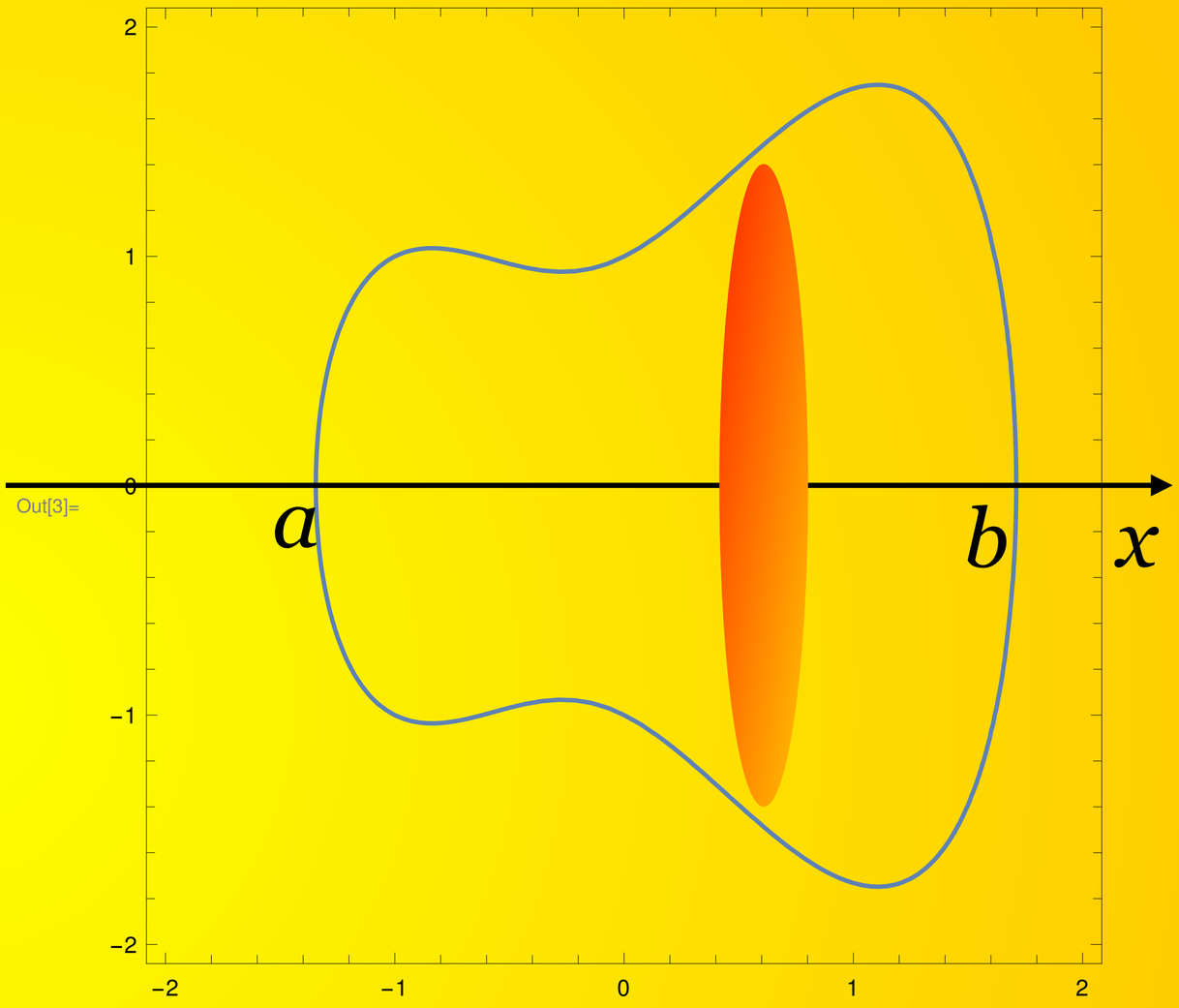
$$y(x) = \sqrt{1 - x^2}$$

What is the volume?



Pear

$$y^2 = x^4 - 2x^2 - x$$



In[4]:=
```
Solve[x^4 - 2 x^2 - x == 0, x]
```

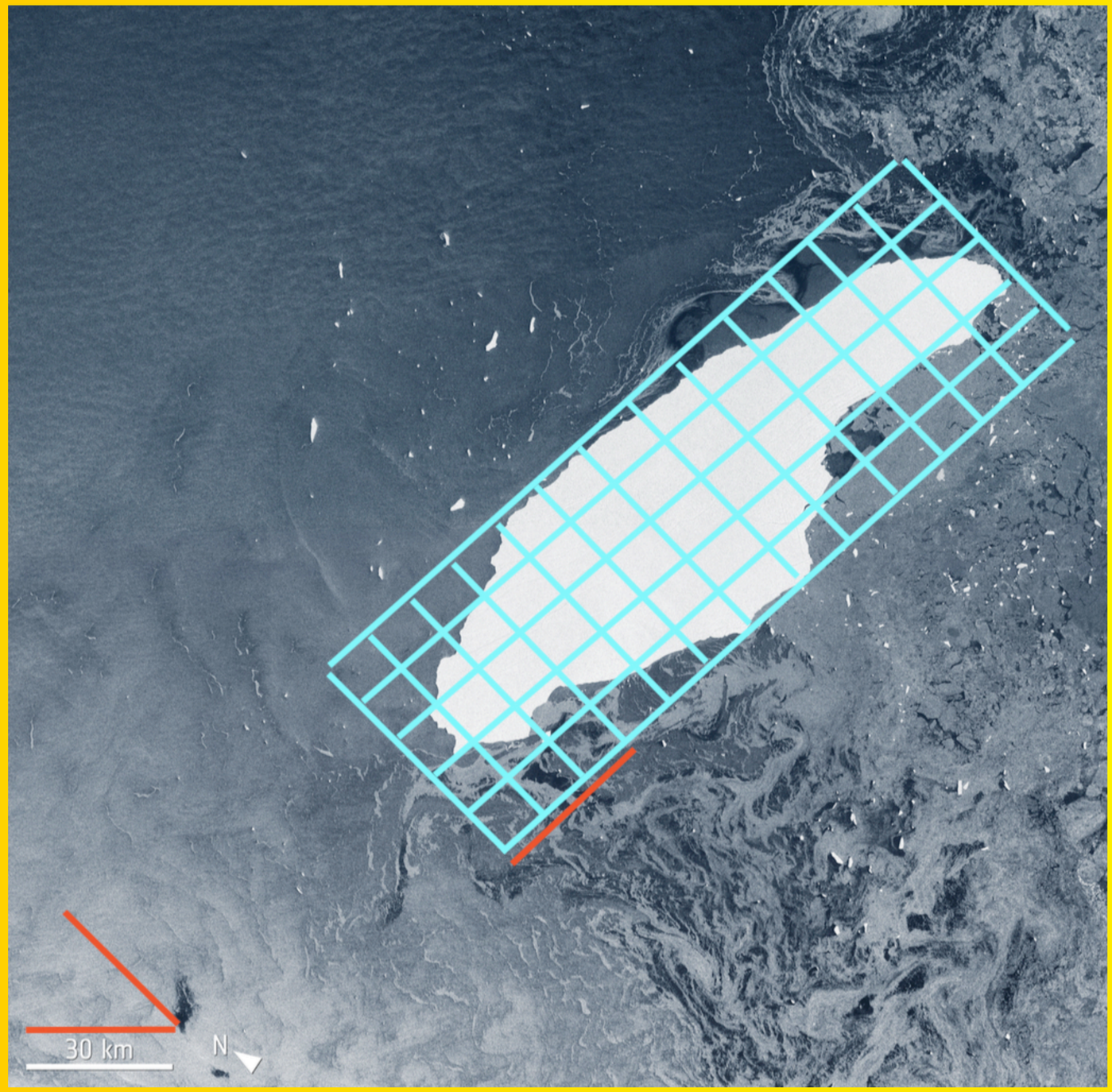
Out[4]=
$$\left\{\{x \to -1\}, \{x \to 0\}, \left\{x \to \frac{1}{2} \times \left(1 - \sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2} \times \left(1 + \sqrt{5}\right)\right\}\right\}$$

In[5]:=
```
N[%]
```

Out[5]=
```
{{x → -1.}, {x → 0.}, {x → -0.618034}, {x → 1.61803}}
```

Part 4

QRD Reminders



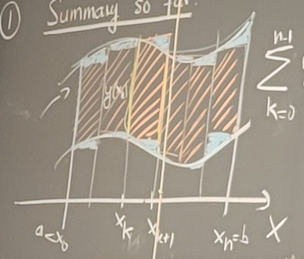
Due Today!

from QRD

Lecture 2

Density and Definite Integral

① Summary so far.



$\sum_{k=0}^{n-1} \delta(x_k) \underbrace{y(x) \cdot \Delta x}_{\text{Area}}$ Left Riemann sum

or $\sum_{k=1}^{n} \delta(x_k)\, y(x_k)\, \Delta x$ Right Riemann sum

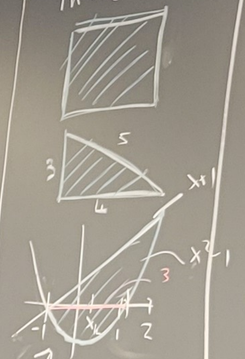
n number of slices.

$\delta(x)$ density

$\Delta x = \frac{(b-a)}{n}$ width of the slice

$y(x_k)$ height

In the homework



$x_K = K\Delta x - 1$

$\Delta x = 3/n$

② Today: take the limit $n \to \infty$

The sum converges to a definite integral

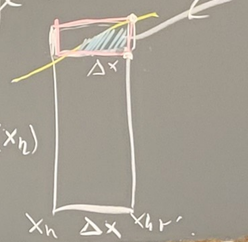
$\sum_{k=1}^{n} \delta(x_k)\, y(x_k)\, \Delta x \xrightarrow[n\to\infty]{} \int_a^b \delta(x)\, y(x)\, dx$

Riemann sum — Definite integral

Why does this converge?

Can we estimate the error:

If the slope of the boundary curves is bounded by M



excess area is smaller or equal than the area $\Delta x \cdot$ slope $\cdot \Delta x$

$\leq M \cdot \Delta x^2$

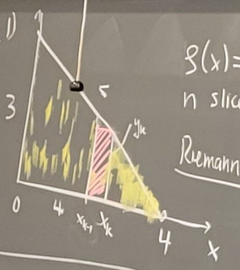
$\leq M(\Delta x)^2 = \frac{M(b-a)^2}{n^2}$

infinitesimal

n slices, each error $\frac{M(b-a)^3}{n^2}$

Total error: $\frac{M(b-a)^2}{n} \to 0$

Examples: 1)



Pizza.

Parmesan cheese

$\delta(x) = x^5$

$y_k = -\frac{3}{4}x_k + 3$

n slices, $\Delta x = \frac{4}{n}$

Riemann sum:

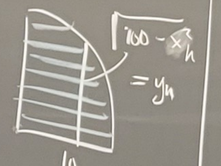
$$\sum_{k=0}^{n-1} \delta(x_k) \underbrace{\left(\frac{3}{4}x_k + 3\right)}_{y_k} \Delta x$$

Definite integral:

Total amount of cheese.

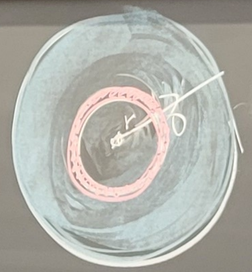
$$\int_0^4 x^5 \left(\frac{3}{4}x + 3\right) dx = \int_0^4 \left(\frac{3}{4}x^6 + 3x^5\right) dx = \left(\frac{3}{4}\frac{x^7}{7} + \frac{3x^6}{6}\right)\Big|_0^4 = \left(\frac{3}{4}\right)\frac{4^7}{7} + \frac{3\cdot 4^6}{6}$$

Example 2)



$\sqrt{100 - x_k^2} = y_k$

Slicing with horizontal pieces



Ink density: $r_k^3 = \delta(r_k)$

new: we are slicing with annular slices. We take n slices.

Δr

$r_k = \frac{10}{n}$

$$\sum_{k=1}^{n} \delta(r_k) \boxed{2\pi r_k \Delta r}$$

$\pi(r_{k+1})^2 - \pi(r_k)^2$

$\pi(r_k + \Delta r)^2 - \pi r_k^2$

$\pi r_k^2 + 2\pi r_k \Delta r + \Delta r^2 - \pi r_k^2$

Can

THE END