



8/30/2021 near Mather house

Lecture 5

9/15/2021

Revolution Integrals

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Part 1

Reminders

Homework 4 for Friday

PROBLEM SET 04 – VOLUMES OF REVOLUTION

As always, please explain how you are slicing, and show us your approximation for a typical slice; see Problem Set 02 for a reminder of the things you should include when writing up integration application problems.

If you are looking for sample problems and careful solutions, we encourage you to look at the Handouts page of the course Canvas site. The worksheets in particular, contain sample problems (you may have worked on some of these in class) and the worksheet solutions contain detailed solutions.

1. Let \mathcal{R} be the region enclosed by the curve $y = \sqrt{x}$ and the line $y = \frac{1}{2}x$. Find the volume obtained by revolving \mathcal{R} around the vertical line $x = -1$. Do this two ways, once slicing vertically and once slicing horizontally. Evaluate both integrals and show that you get the same answer.
2. Let \mathcal{R} be the region between $y = 1$ and $y = 2$ bounded by $y = 2x + 2$ and $y = \frac{1}{x}$. Write an integral that gives the volume generated when \mathcal{R} is rotated about:
 - (a) the x -axis.
 - (b) the vertical line $x = 1$.
 - (c) the horizontal line $y = 2$.
 - (d) the vertical line $x = -2$.

Hint: One method of slicing is far less work than the other!

You need not evaluate the integrals. However, if you'd like to check your answers, go ahead and evaluate; you should get $\frac{8\pi}{3}$, $\pi\left(\frac{1}{12} + 2\ln 2\right)$, $\pi\left(4\ln 2 - \frac{5}{3}\right)$, and $\pi\left(\frac{17}{12} + 4\ln 2\right)$.

3. A parfait cup is formed by revolving the curve $y = x^3, 0 \leq x \leq 2$, about the y -axis. The parfait cup is filled to the brim with hot chocolate. If you drink exactly half the hot chocolate in the cup, what height will the liquid be when you stop drinking?
4. (Exploratory – you will get full credit for these questions as long as you make a serious attempt and you turn in evidence of your thought process.)

- (a) Imagine you're in charge of Math 1b, and you need to get copies of the Techniques Test printed.

At the course meeting, the TFs for the four sections⁽¹⁾ tell you:

“My section has about 20 students.”

“Mine's got a little under 30.”

“I also have around 30.”

“I've got about 20.”

How many copies of the test should you print to be sure that you'll have enough? WRITE your answer down now and don't change it later!

- (b) Write a sentence or two describing the strategy you used to determine how many tests to print.

⁽¹⁾Yes, there are seven sections. But we're pretending, so let's make it a little easier.

Information for Techniques of Integration test

9/15

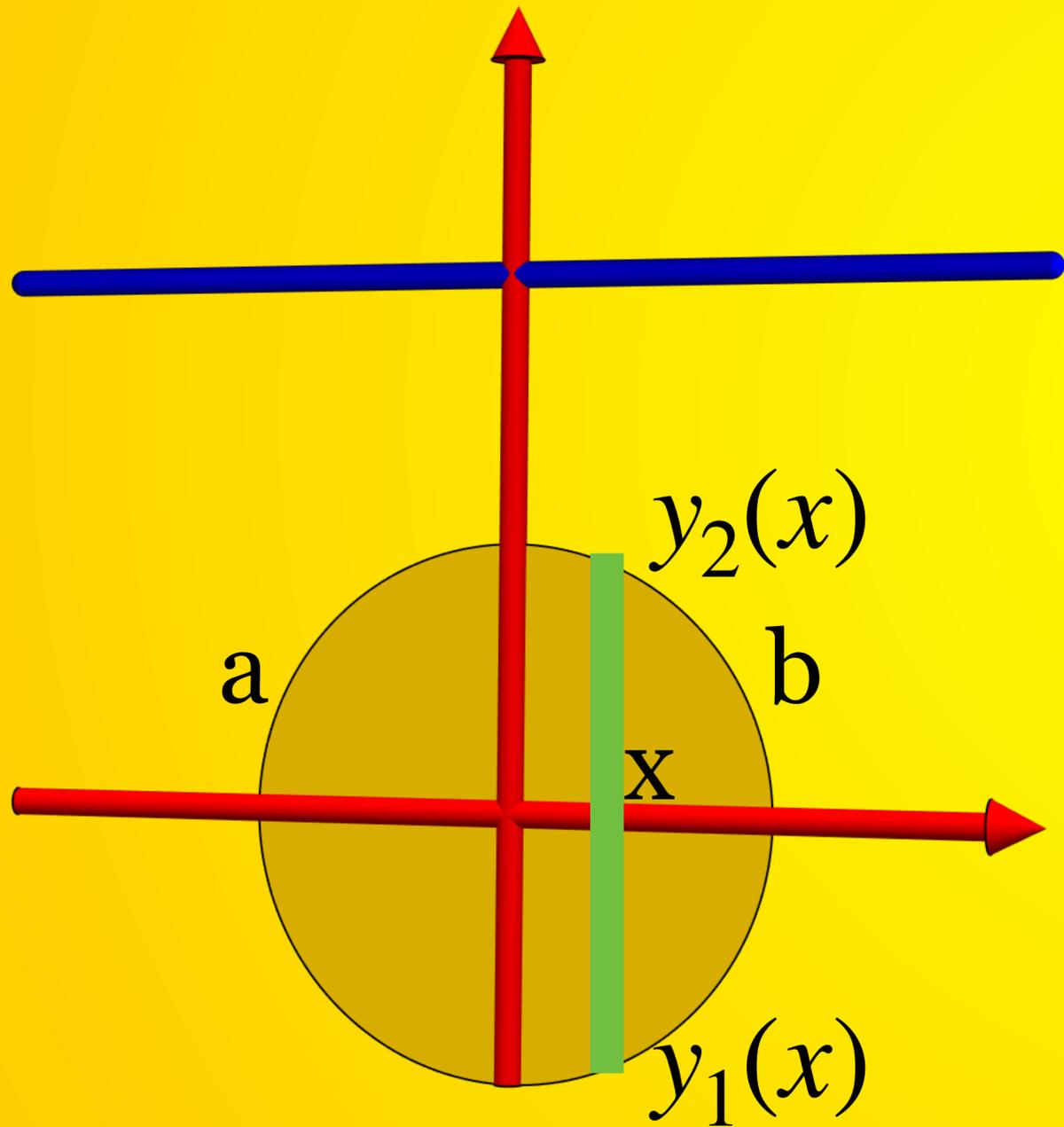
- Wednesday, 9/15 from 6-7:30.
- Test will be remote. Download from Gradescope in a 15 minute window starting 6pm and uploading within 90 minutes of your download time. We will give more instructions closer to the test date.
- People with AEO letters should contact Cliff about getting extra time
- People with conflicts should contact Cliff about starting earlier or later.
- There will be make-up Techniques test in mid-October where people can recoup any and all lost points.
- No aids of any kind (electronic, notes, books, people, ...) can be used during the test. Just you and your brain.
- Study material is on the Exam Information (Techniques test) page of the Math 1b website. This includes practice problems with solutions, practice exams with solutions and videos.
- The sections on Monday, 9/13 will be reviewing for the test.

Part 2

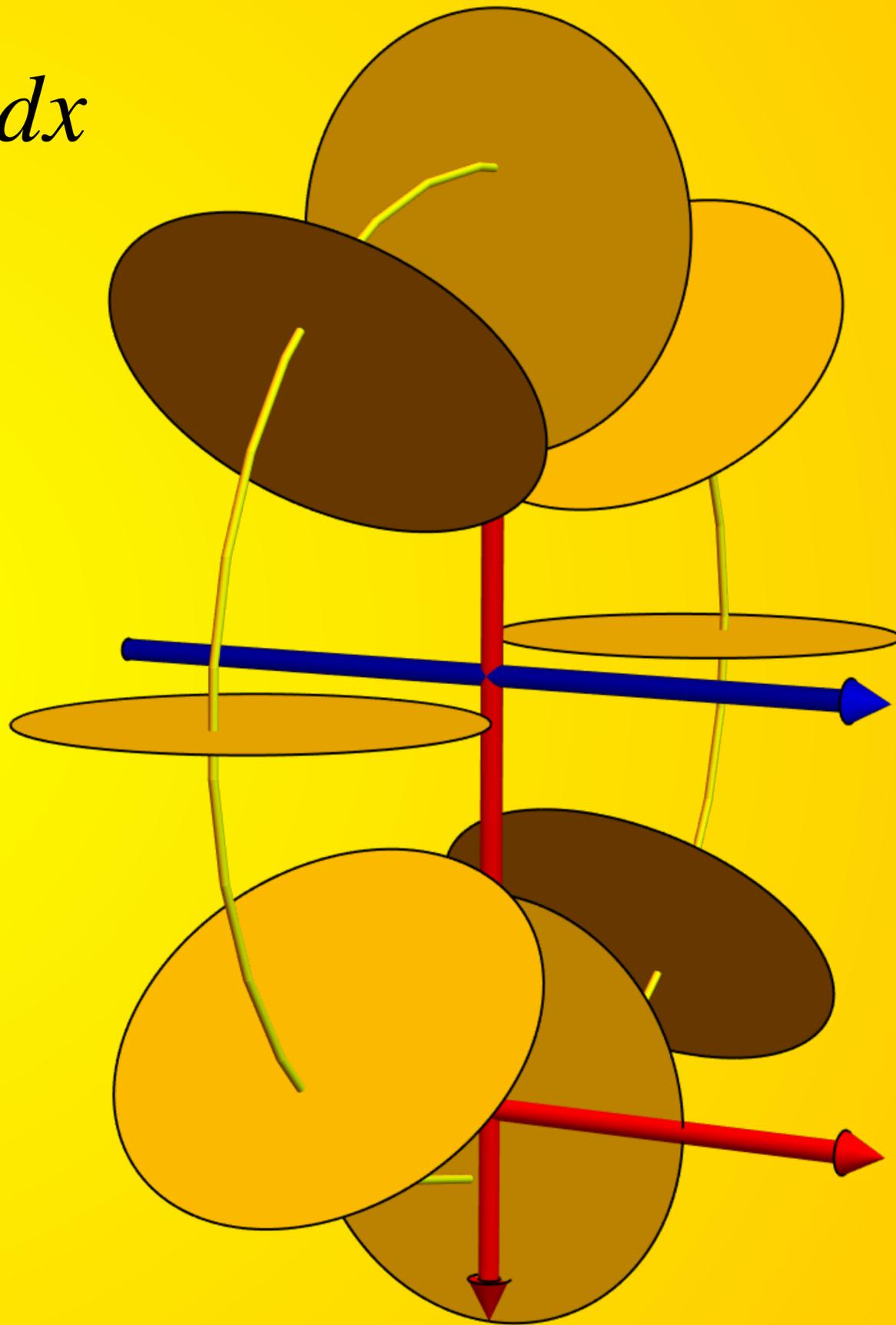
Revolution Integrals

See Handout

$$\int_a^b \pi(y_1(x)^2 - y_2(x)^2) dx$$

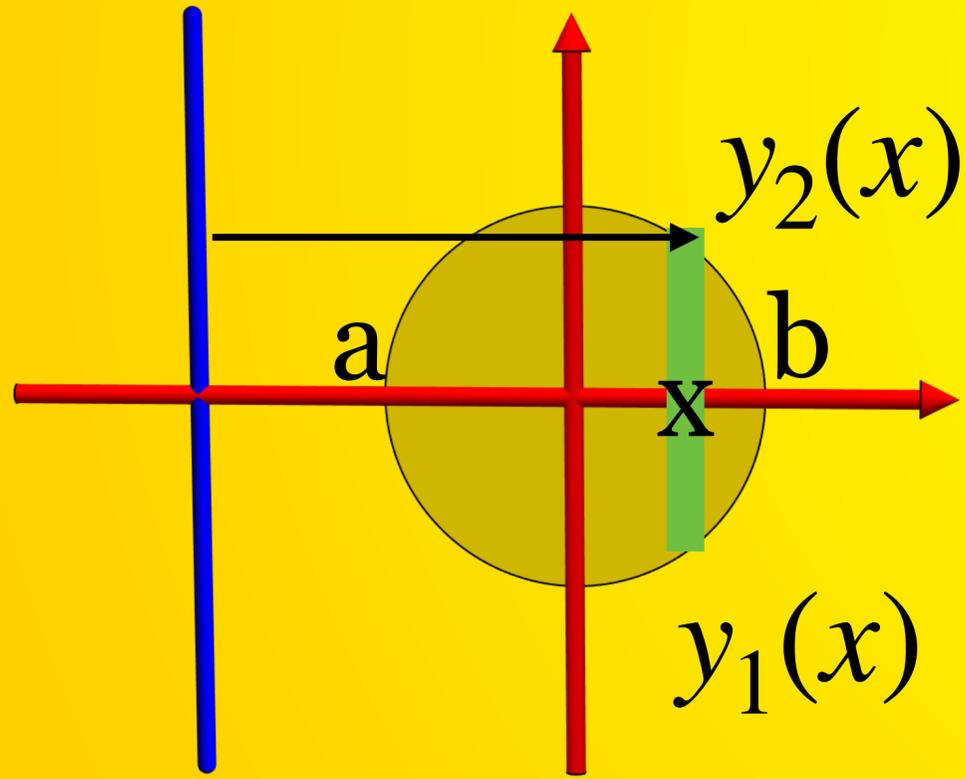


Spin it
around the
blue line.

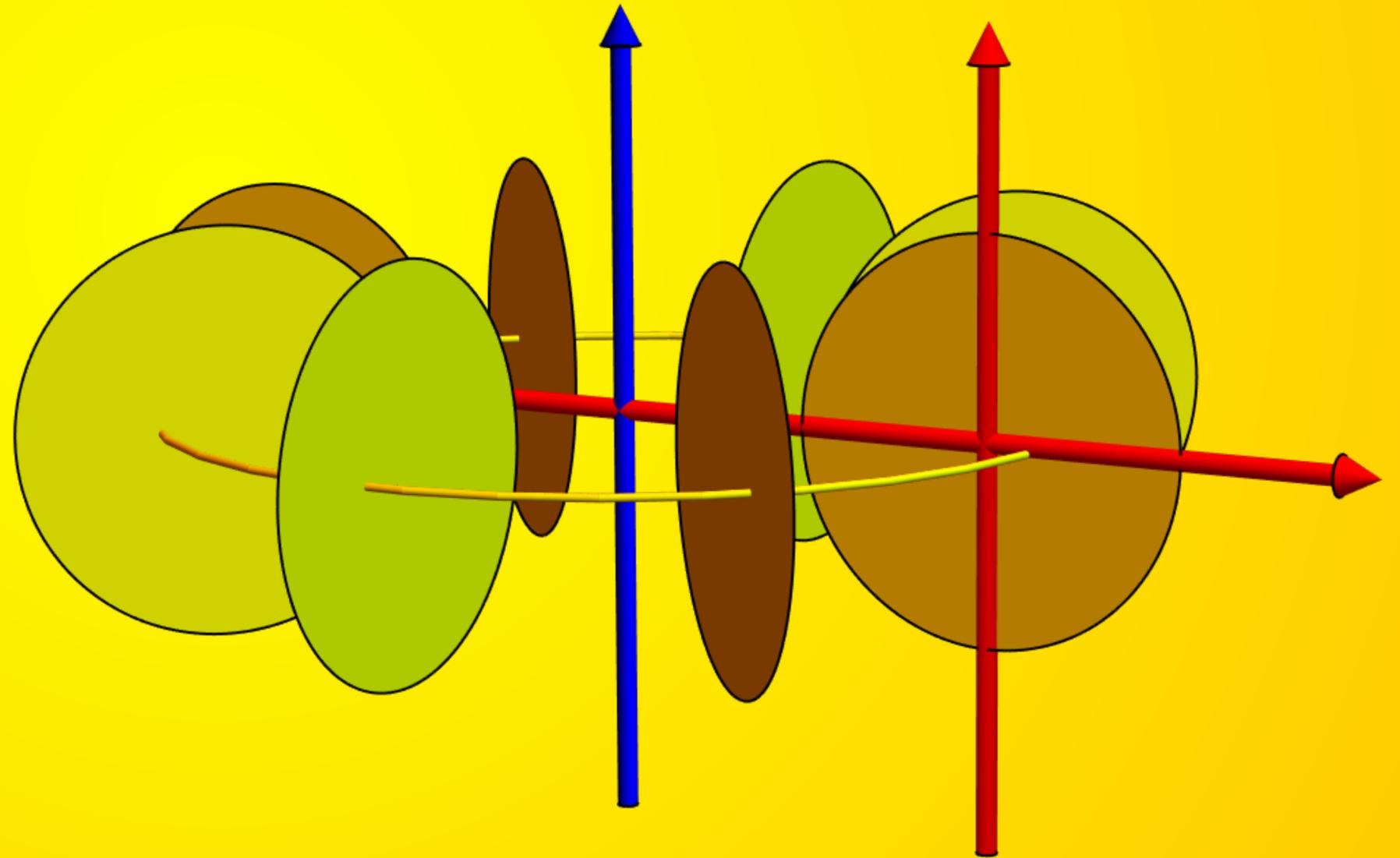


$$\int_a^b 2\pi r(x)(y_2(x) - y_1(x)) dx$$

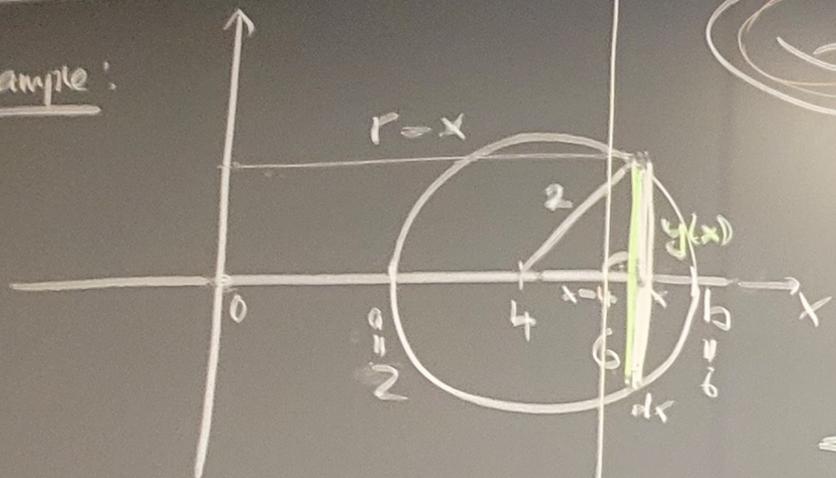
$$r(x) = c + x$$



Spin it
around the
blue line.



Example:



Pappus: Area (b) · length of the circle through center of mass

Symmetry: could do with substitution simpler.

$$2\pi \int_2^6 x \cdot 2\sqrt{2^2 - (x-4)^2} dx$$

$$= 4\pi \int_2^6 x \sqrt{4 - (x-4)^2} dx$$

$$16\pi \int_{-2}^2 u \sqrt{4-u^2} du$$

length of circle C
Area Circle

$$= 4\pi \int_{-2}^2 (u+4) \sqrt{4-u^2} du$$

$$\int \sqrt{4-x^2} dx \rightarrow \text{trig subst.}$$

$$x = 2 \sin u$$

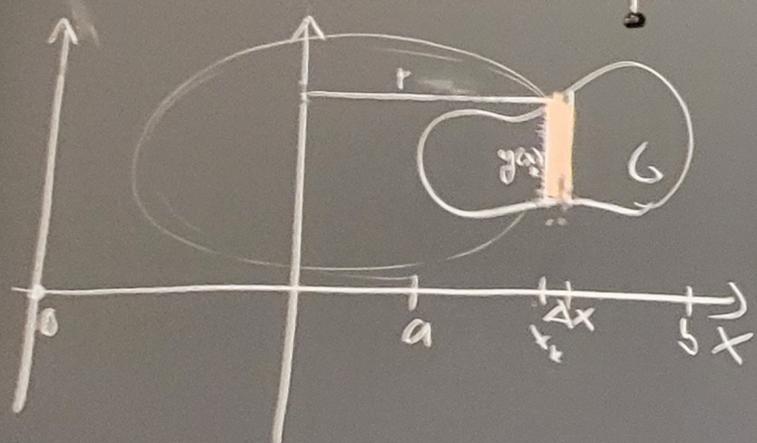
$$\sqrt{4-x^2} = 2 \cos u$$

$$dx = 2 \cos u$$

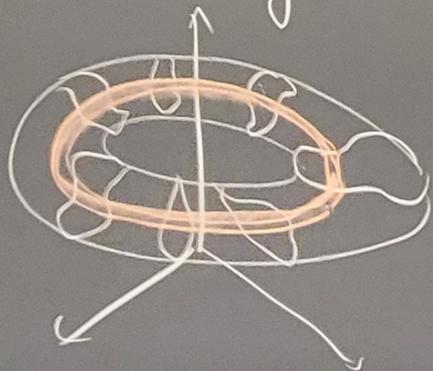
$$\int x \sqrt{4-x^2} dx = \int \left(\frac{1}{2}\right) \sqrt{u} du$$

$$\left(\frac{1}{2}\right) \frac{2}{3} u^{3/2}$$

Lecture 5 Revolution integrals



Volume of a solid of revolution obtained by rotating G around an axis.



$$2\pi \sum_{k=1}^n r(x_k) y(x_k) \Delta x$$

$$\downarrow n \rightarrow \infty$$

$$2\pi \int_a^b r(x) y(x) dx$$

Tonight: Technique test 6-7:30 PM

For last minute questions: I can be available after class today or at 12 Noon outside. (432)

$$\lim_{x \rightarrow 0} \frac{x^3}{1+e^x} = \lim_{x \rightarrow 0} \frac{x}{1+e^x} = 0 \checkmark$$

$$\lim_{x \rightarrow 0} \frac{x}{1+x} \cdot \frac{1}{x} = 1$$

Part 3

Work sheet

Good luck in the technique test!

THE END