8/30/2021 near Mather house

Lecture 6

9/17/2021

3D Density Problems

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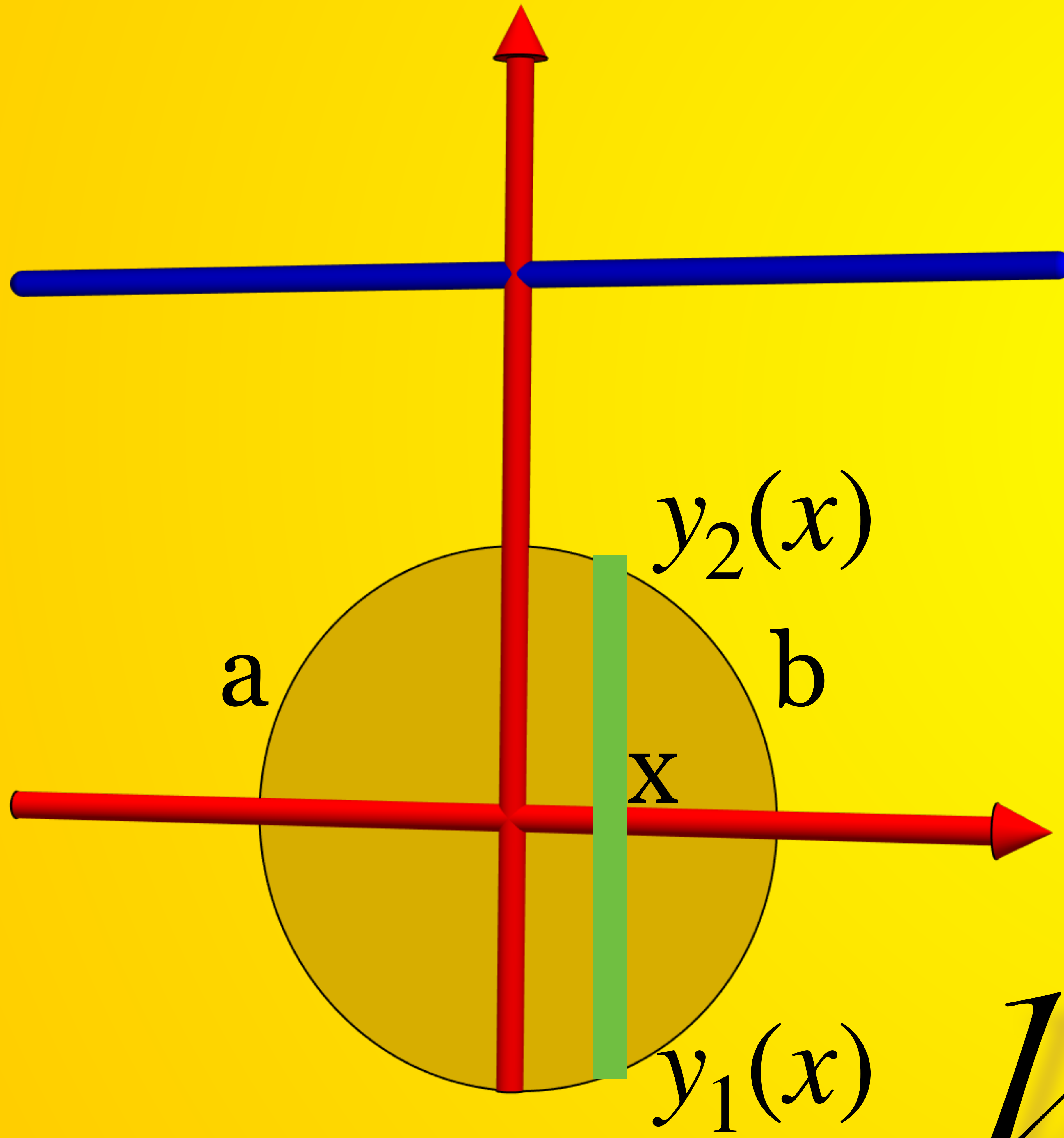
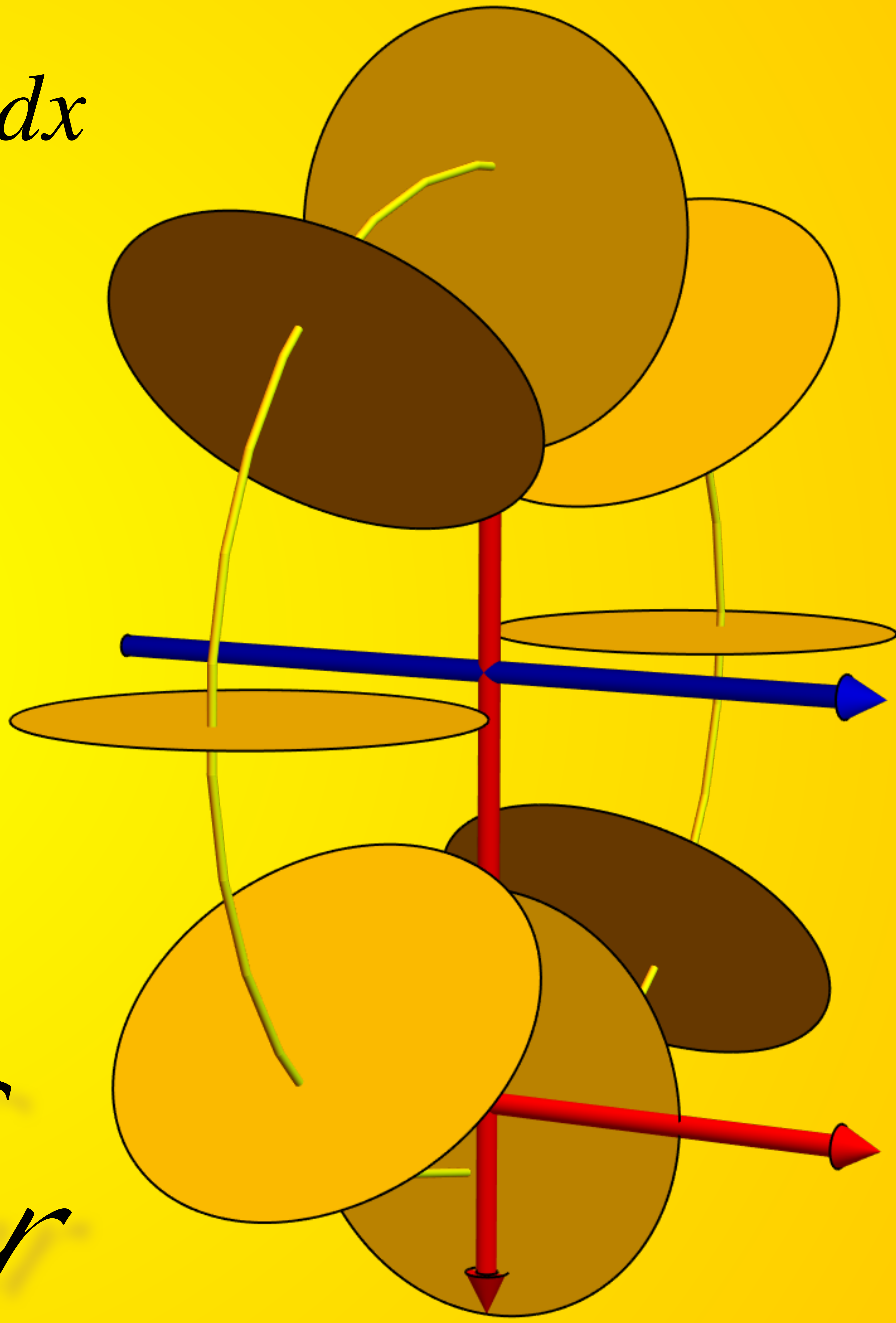
Part I

Reminders and Review

$$\int_a^b \pi(y_1(x)^2 - y_2(x)^2)\, dx$$



Spin it around the blue line.

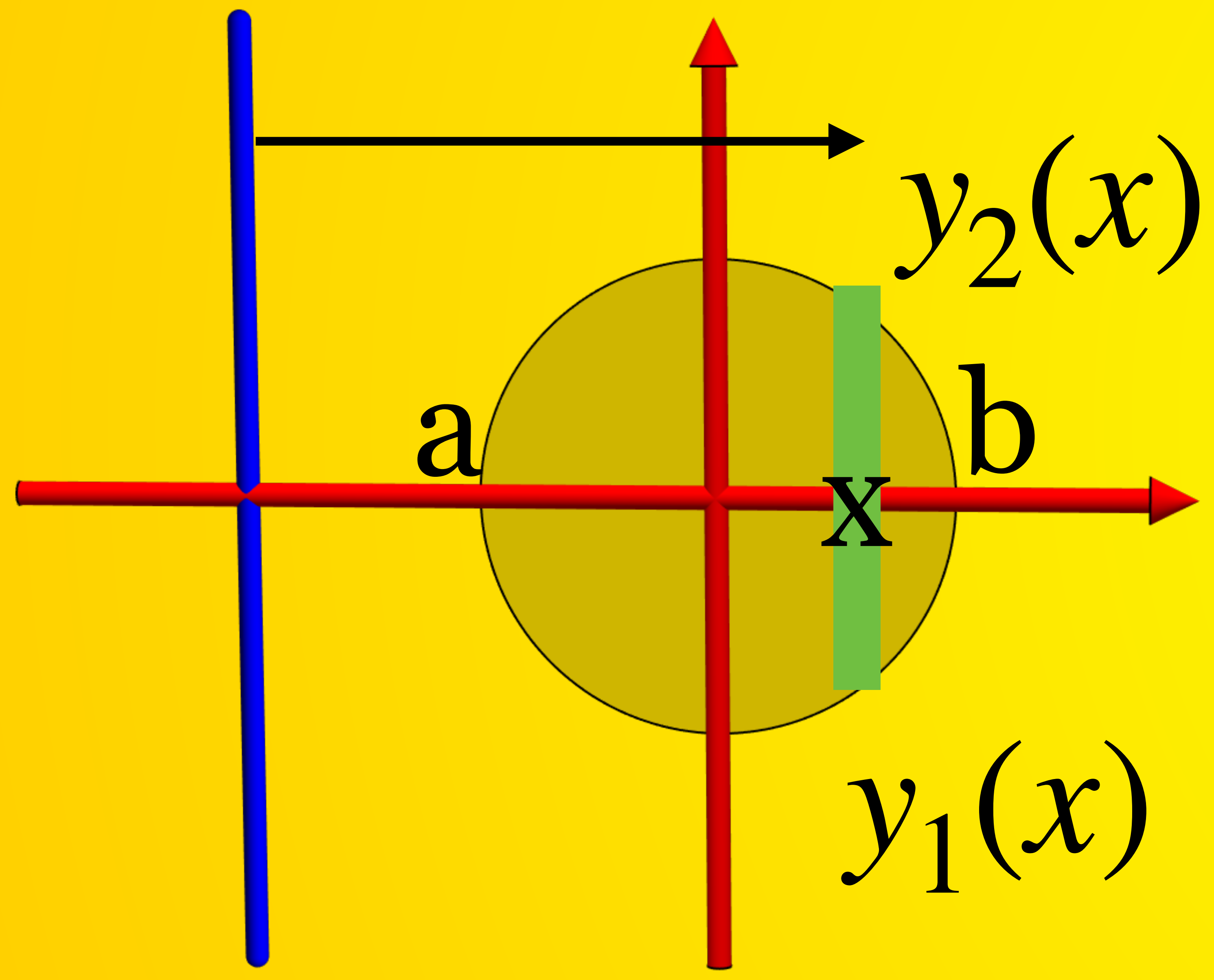


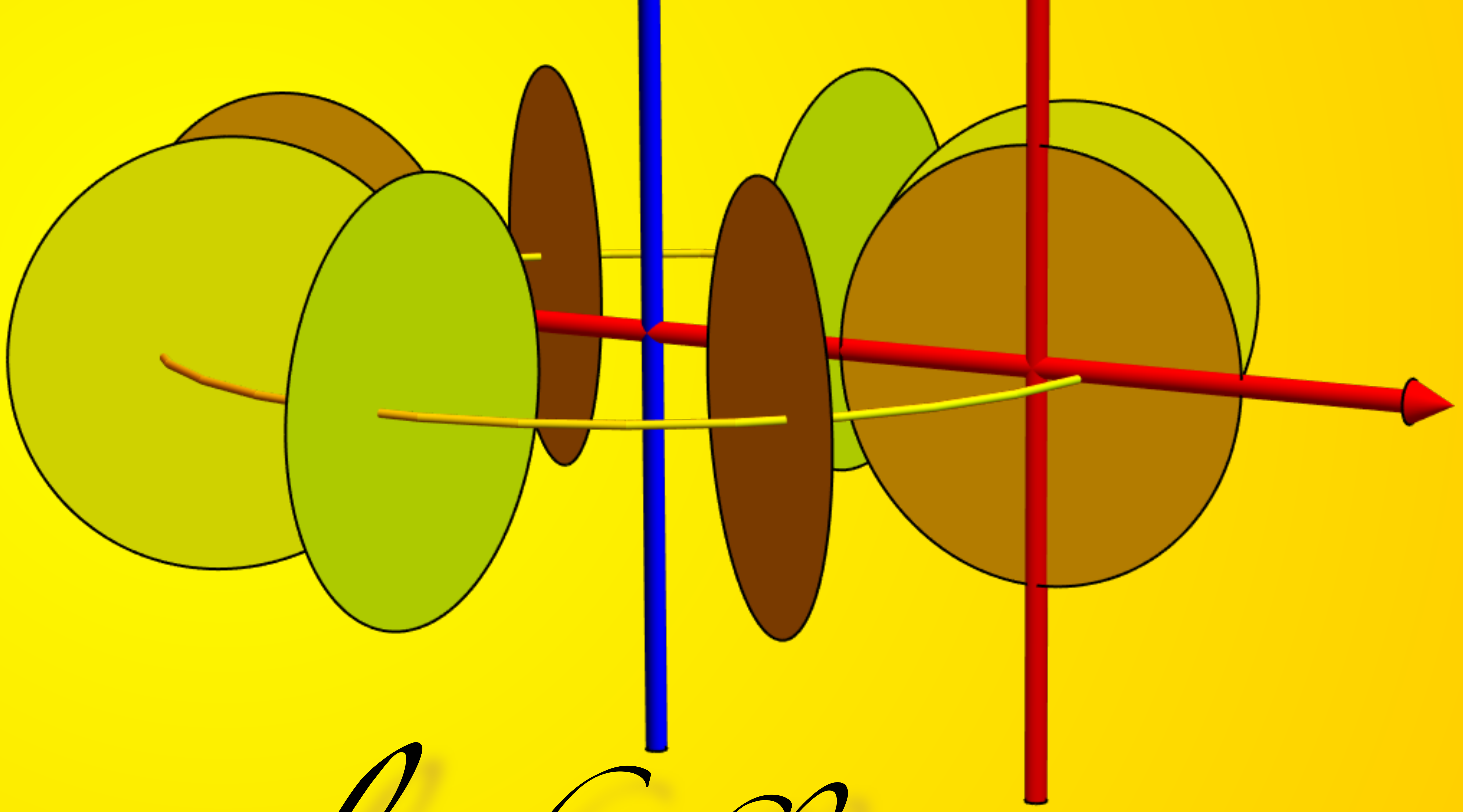
Washer

$$\int_a^b 2\pi r(x)(y_2(x) - y_1(x))\,dx$$

Spin it around the blue line.

$$r(x) = c + x$$





Cylindrical shell

Homework 5
Due Monday

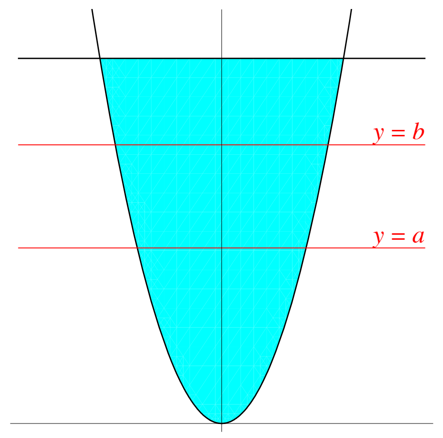
(a) Which chef puts more garlic on a pizza? (You should be able to figure this out without doing any calculations; think about which parts of each chef's pizza are most densely covered with garlic and which parts are least densely covered.) Explain.

(b) Find the amount of garlic on each of Chef Paul's pizzas.

(c) Find the amount of garlic on each of Chef Tim's pizzas.

5. Weekly problem #4 is now due. This problem is optional, and worth 5 points extra credit on this assignment. Here it is:

Let $\mathcal{R}$ be the region bounded by $y = x^2$ and $y = 9$. Find a and b so that the horizontal lines $y = a$ and $y = b$ split the region $\mathcal{R}$ into 3 pieces of equal area.



W. You can now also begin working on Weekly Problems #8 – #11.

Problem Set 05 – 3-dimensional Density Problems

As usual, we expect you to give thorough explanations when writing up these problems; see Problem Set 02 for a reminder of the things you should include when writing up integration application problems.

1. An enormous mountain of height 7500 meters is shaped like a cone with a base radius of 5000 m. Due to various geological factors such as pressure and heat, the density of the mountain varies with the height h and is given by the function $\rho(h)$ kg/m^3.

   (a) Write a general Riemann sum that approximates the mass of the mountain.

   (b) Write an integral giving the exact mass of the mountain.

   Use the following fact to check your answer if you like: If $\rho(h) = \frac{1}{h+2500}$, then the mass of the mountain is approximately 4.957×10^7 kg.

2. The density of a ball of ice is greatest at the center and decreases with distance from the center of the ball. The ball is 12 centimeters in radius and the density is given by $\rho(x)$ grams per cubic centimeter, where x represents the distance from the center of the ball in centimeters.

   (a) Write a general Riemann sum that approximates the mass of the ball. Please define all notation you use; that is, if you use things like Δx or x_k (and you should!), give formulas for them. You may want to look back at Problem Set 01, especially at #2 and #4.

   (b) By taking an appropriate limit of the Riemann sum you found in (a), write a definite integral giving the mass of the ball. Please state exactly what limit you are using.

3. In the town of Lybonrehc, there has been a nuclear reactor meltdown. Fortunately, the reactor has a containment building which keeps radioactive iodine 131 from being released into the air. The containment building is hemispherical with a radius of 100 feet (the flat side of the hemisphere lies on the ground). The density of iodine in the building is $\rho(h)$ g/cubic feet, where h is the height from the floor (in feet). Write an integral that gives the amount of iodine in the building.

4. Weekly Problem #3 is now due. Here it is:

   The Three Aces pizzeria has two chefs, Paul and Tim. Both toss lots of garlic on their pizzas.

   - Chef Paul tosses garlic so that the density of garlic on his pizza is given by
   $$g(x) = \frac{1}{1+x^2} \text{ ounces per square inch of pizza,}$$
   where x is distance from the center of the pizza.
   - Chef Tim tosses garlic so that the density of garlic on his pizza is given by
   $$g(x) = \frac{1}{1+x^2} \text{ ounces per square inch of pizza,}$$
   where x is distance from the edge of the pizza.

   Each pizza is 14 inches in diameter.

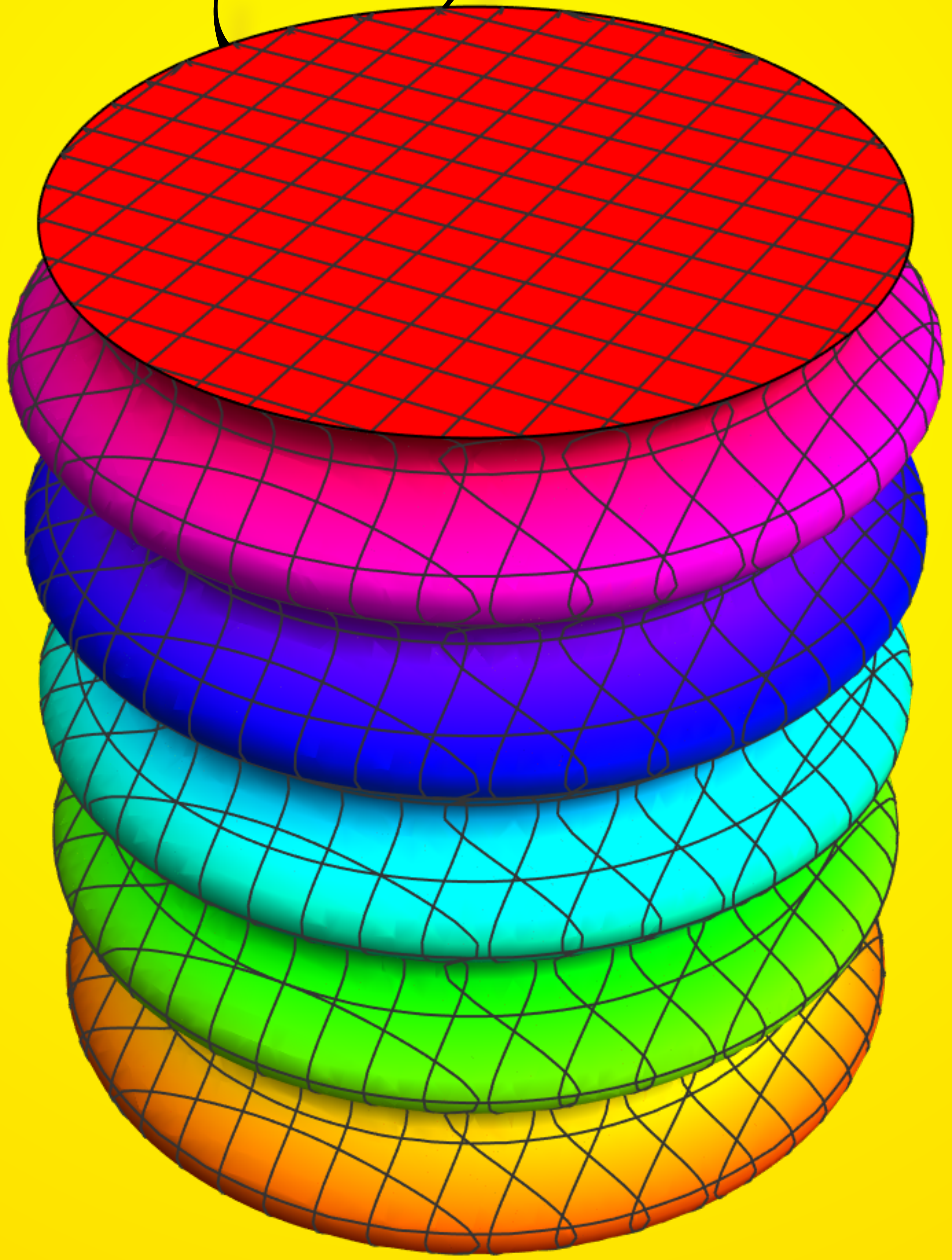
Dates:

October 4th. First midterm.

Part 2

3D Density Problems

Wedding Cake

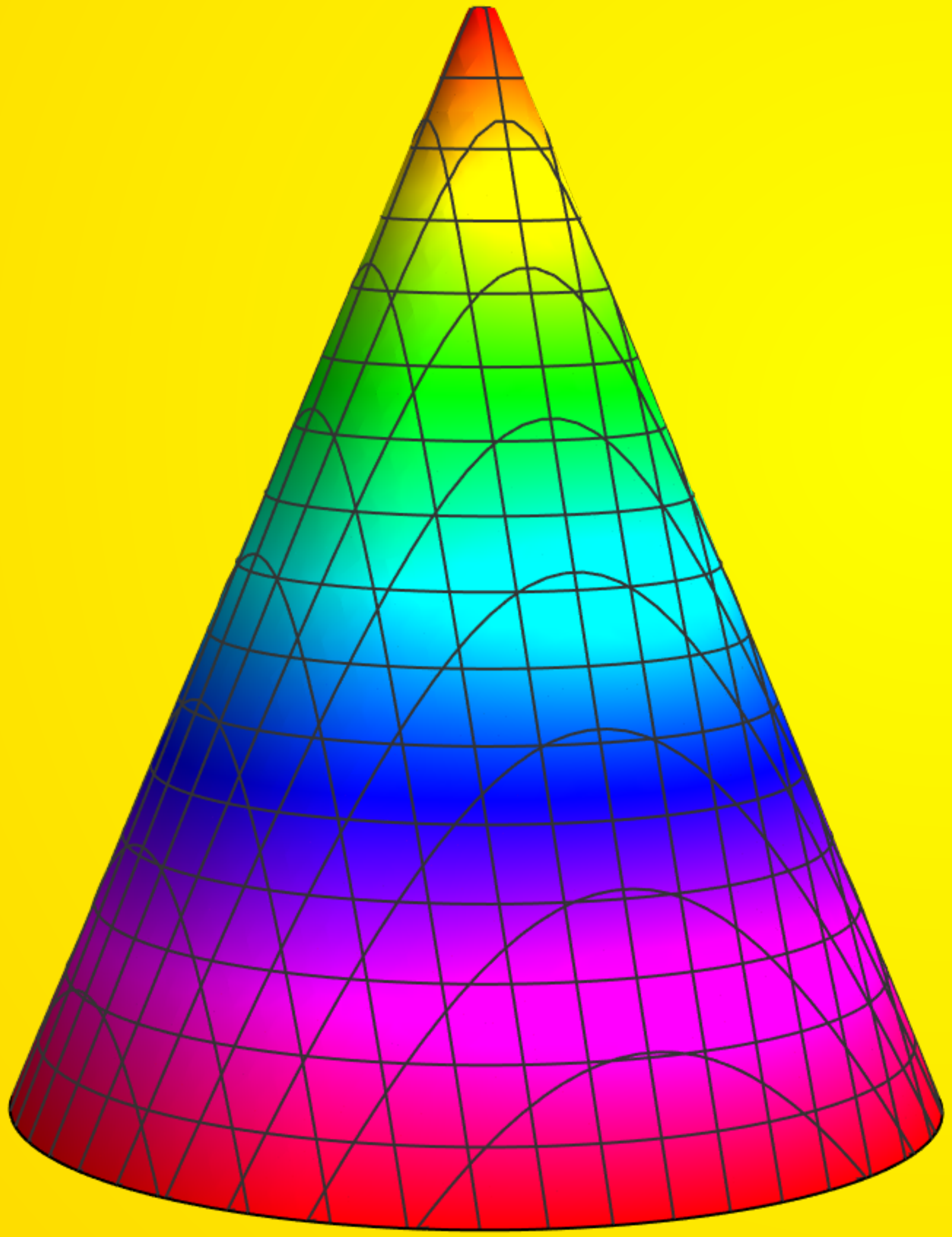


see lecture

Part 3

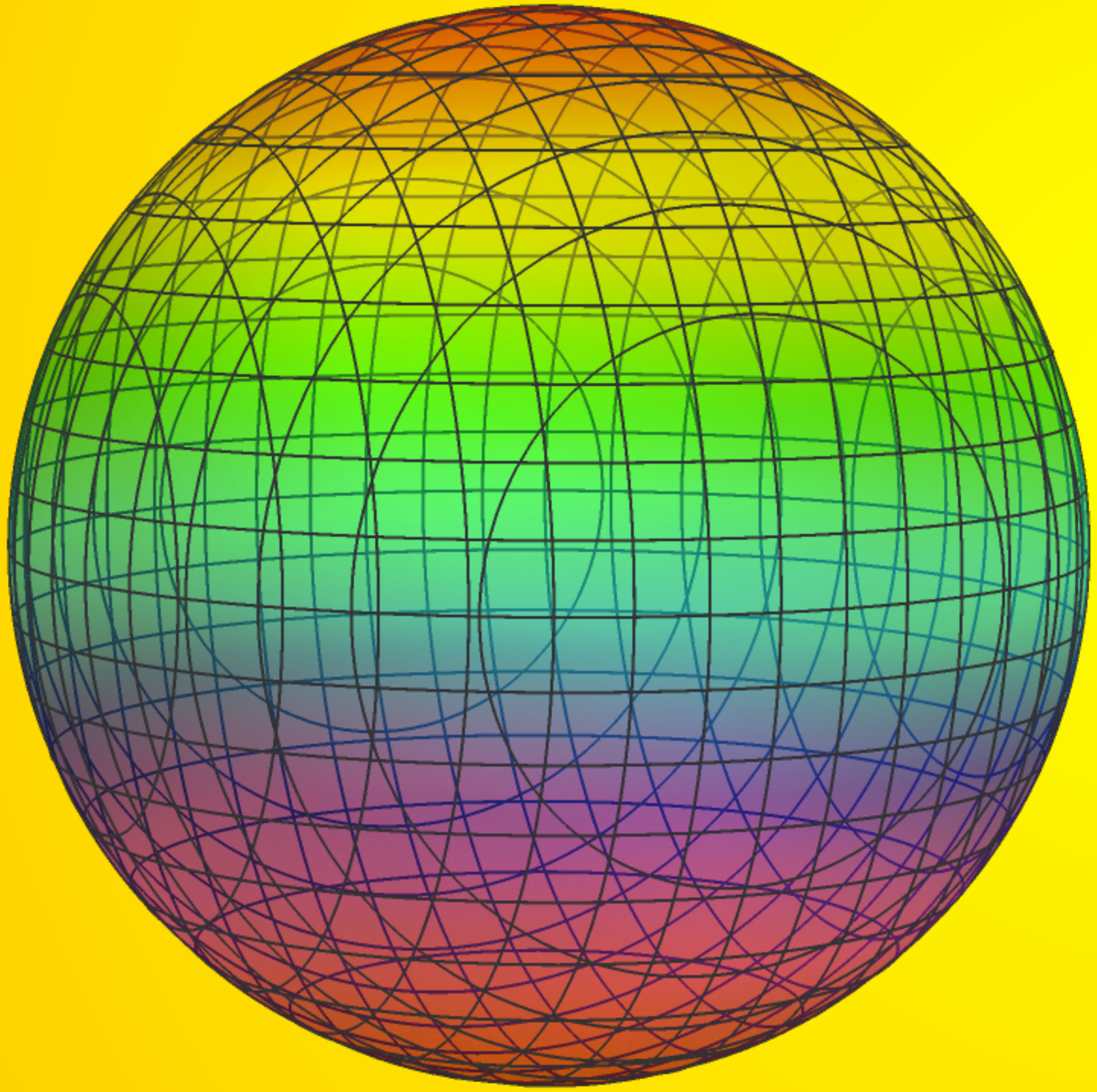
Work sheet problems

Cone



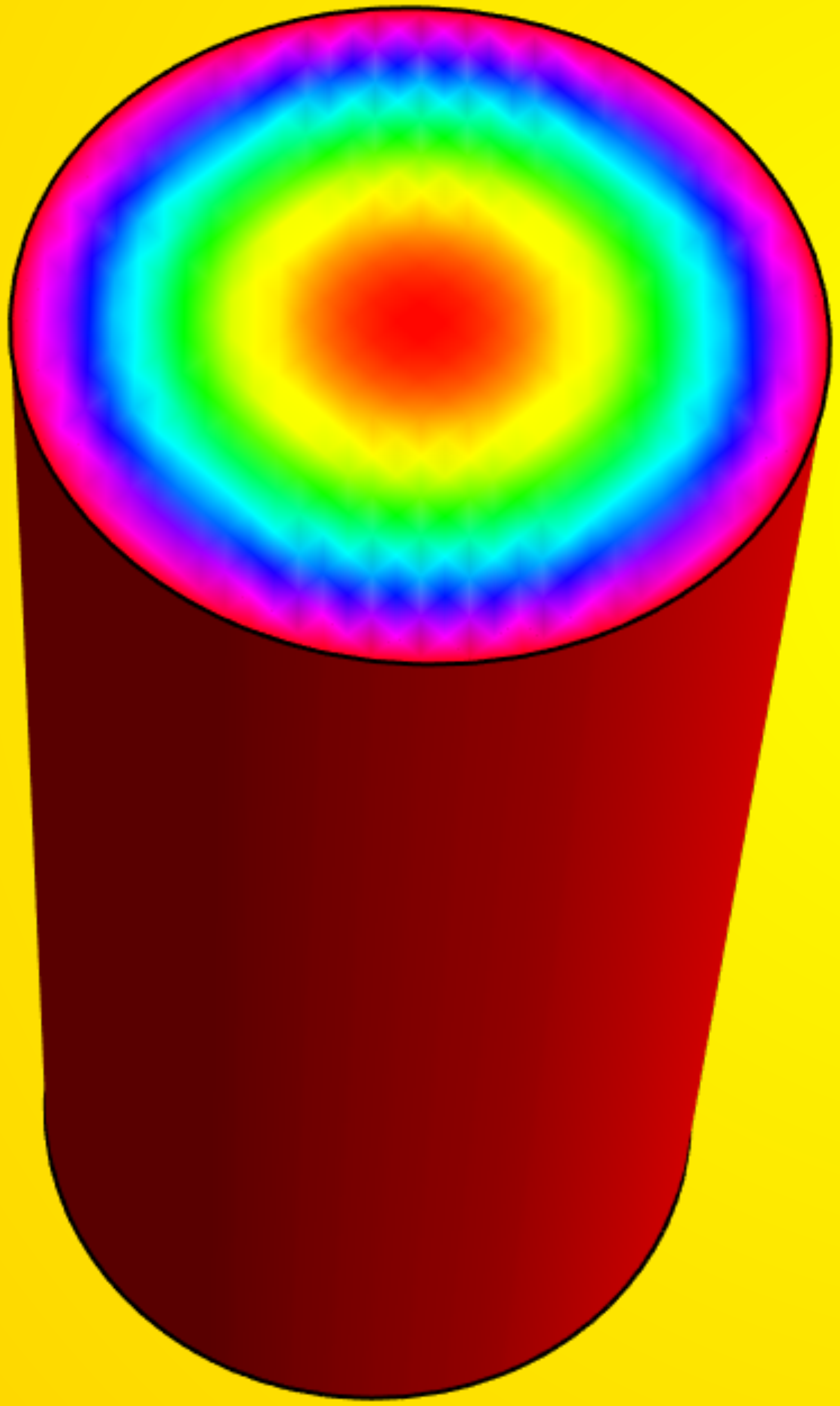
Use Disk shells

Planet



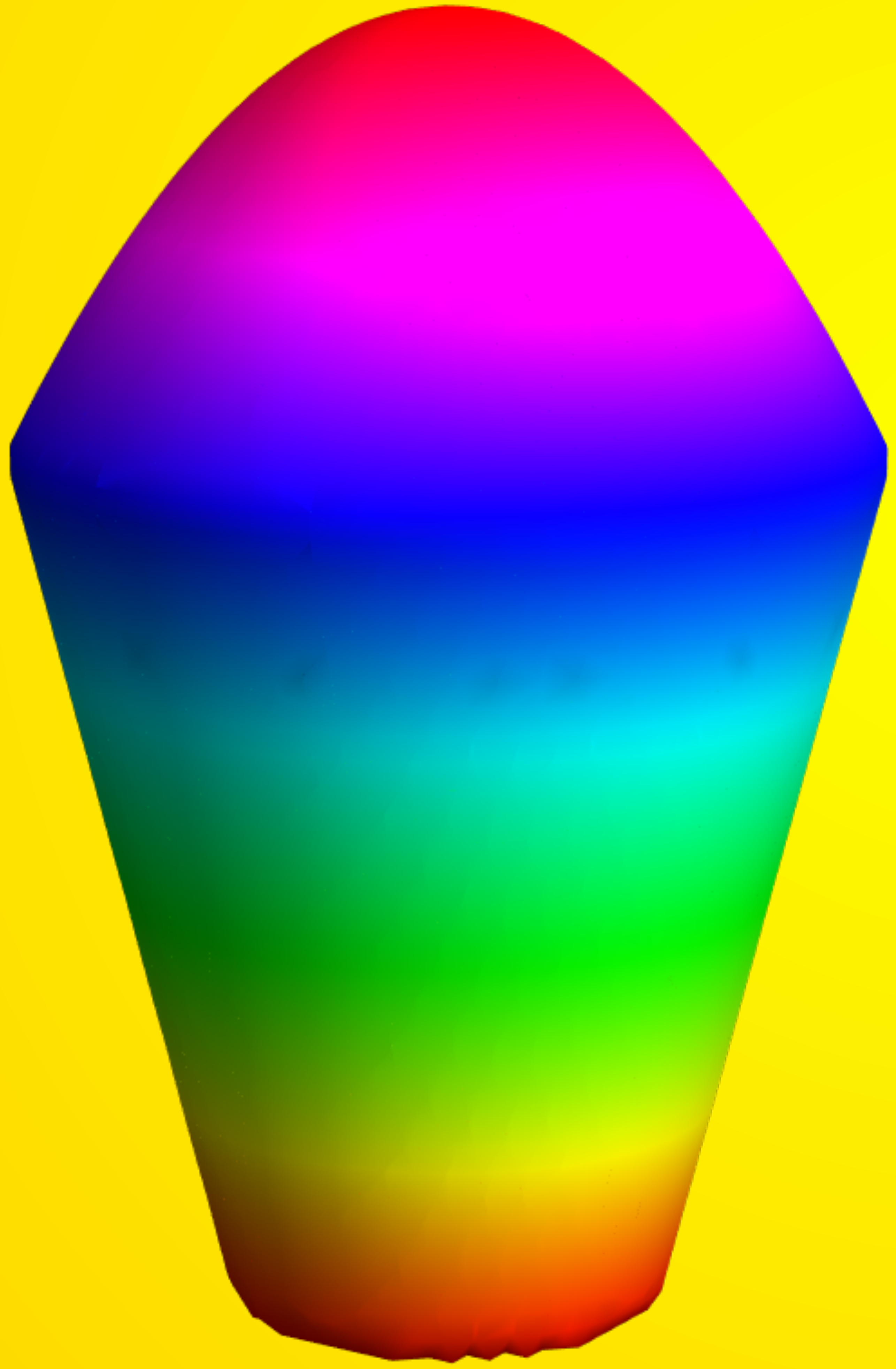
Use spherical shells

Candle



Use cylindrical shells

Muffin



Next week

Numerical integration

Error Estimates

Improper integrals

THE END