8/30/2021 near Mather house

Lecture 6

9/20/2021

Numerical Integration I

Table of Contents

Part I

Numerical Methods

Motivation

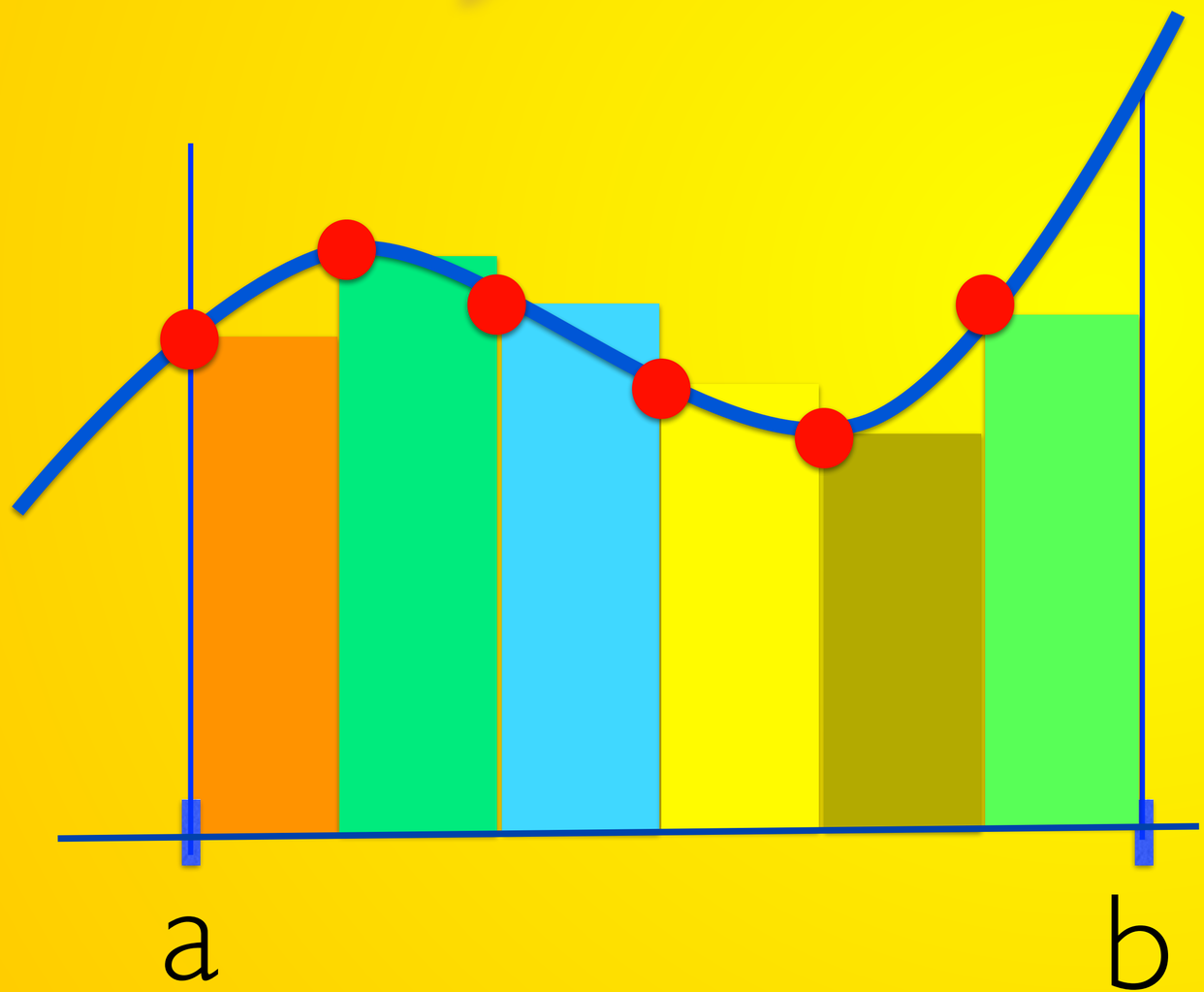
Most integrals can not be evaluted analytically, like $\int \sin(\sin(x))\, dx$

We still want to compute things
and also have an idea how close we are.

Part 2

Numerical Methods

Left Riemann Sum



$$x_k = a + k\frac{b-a}{n}$$

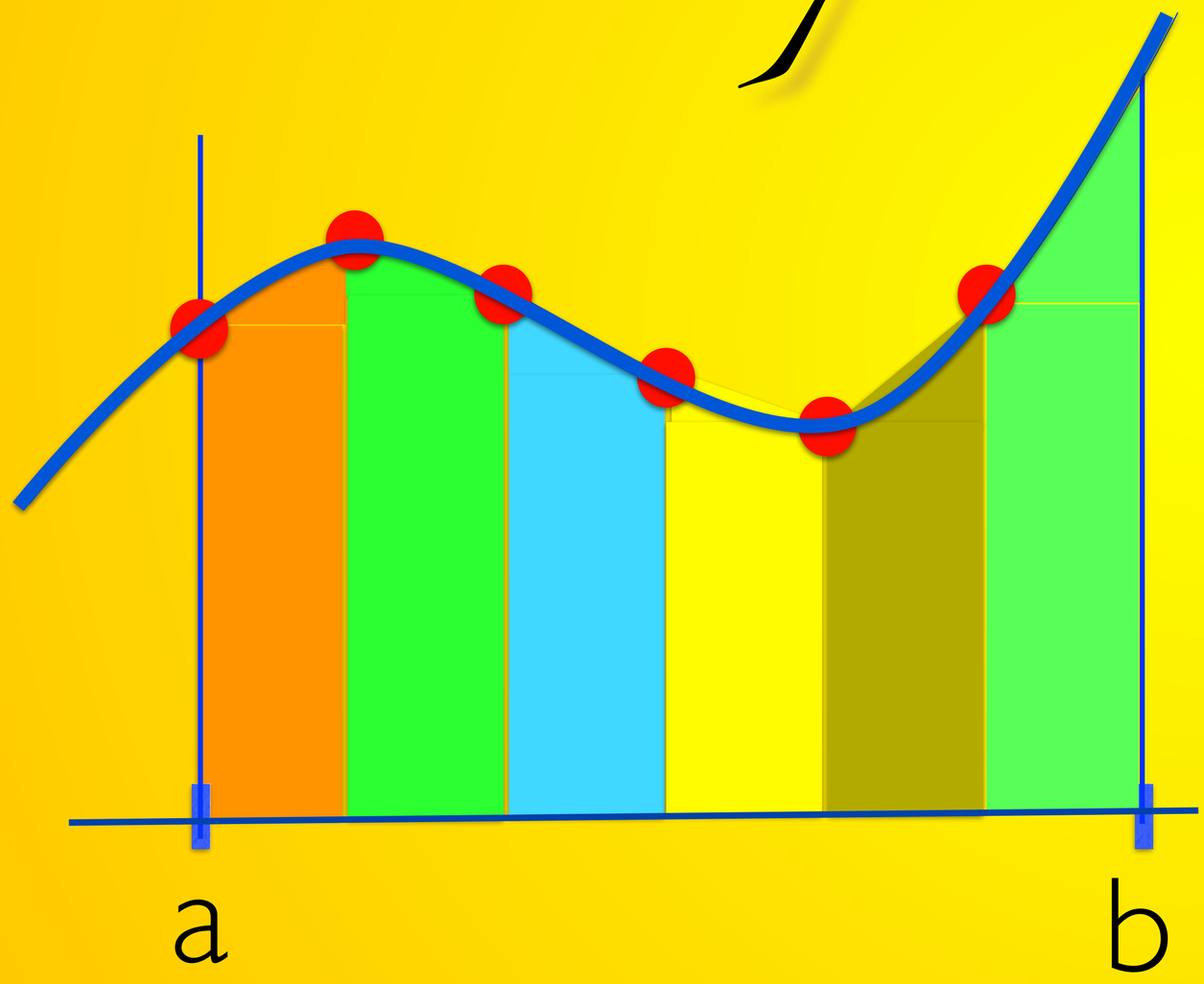
$$L = \sum_{k=0}^{n-1} f(x_k)\Delta(x_k)$$

Right Riemann Sum

$$x_k = a + k\frac{b-a}{n}$$

$$R = \sum_{k=1}^{n} f(x_k)\Delta(x_k)$$

Trapezoid Rule



$$x_k = a + k\frac{b-a}{n}$$

$$\frac{L+R}{2} = \sum_{k=1}^{n} f(x_k)\Delta(x_k)$$

Experiment

Trapezoid

In[1]:=
```
Clear[x]; R = Integrate[ Sin[x], {x, 0, 1}] // N
```
Out[1]=
```
0.459698
```

In[2]:=
```
n = 10; A = Sum[ Sin[ k / n], {k, 0, n - 1}] / n  // N
```
Out[2]=
```
0.417241
```

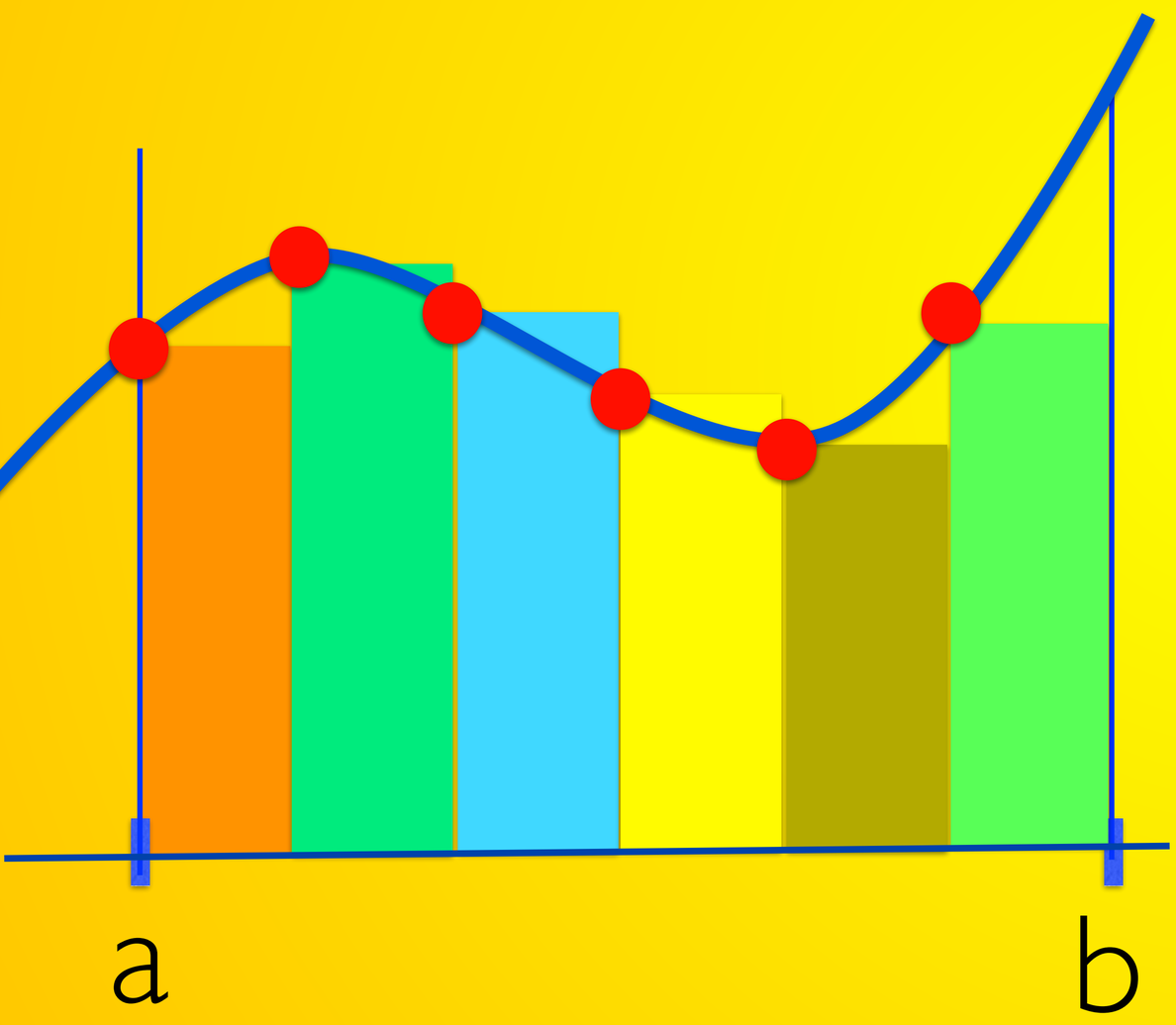
In[3]:=
```
n = 10; B = Sum[ Sin[ (k + 1) / n], {k, 0, n - 1}] / n  // N
```
Out[3]=
```
0.501388
```

In[4]:=
```
(A + B) / 2  // N
```
Out[4]=
```
0.459315
```

Part 3

Error bounds

Error Bound



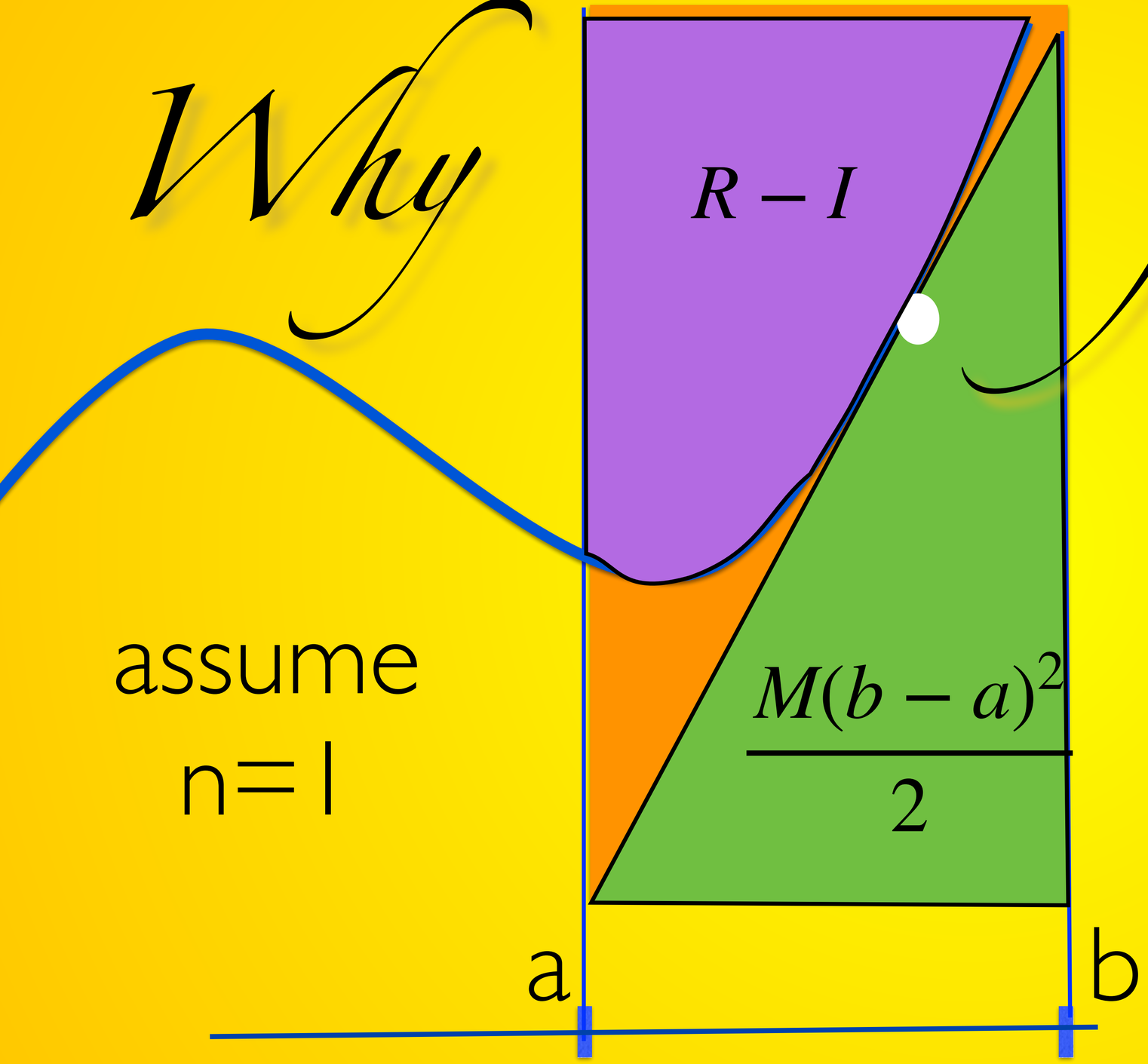
assume: $|f'(x)| \leq M$

Then the error in each slice is bound by $(\Delta x)^2 M/2$

The total error bound

$$\frac{M(b-a)^2}{2n}$$

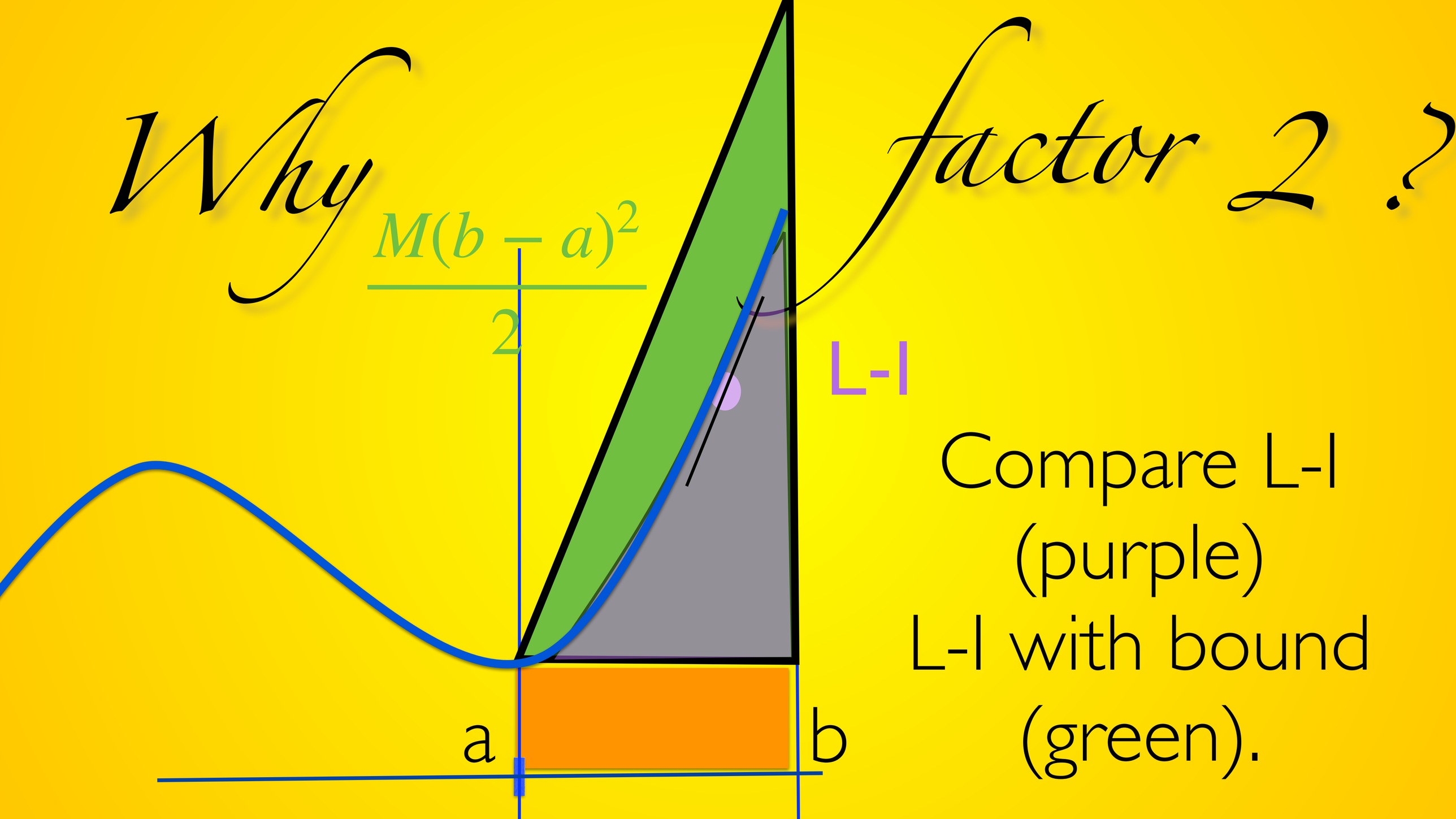
Why $\frac{1}{2}$ factor 2 ?



assume
n=1

In the linear case the factor 2 was clear. In general, we need a picture

Why $\frac{M(b-a)^2}{2}$ factor 2 ?



Compare L-I (purple) L-I with bound (green).

Part 4

Work sheet problems

THE END