8/30/2021 near Mather house

Lecture 7

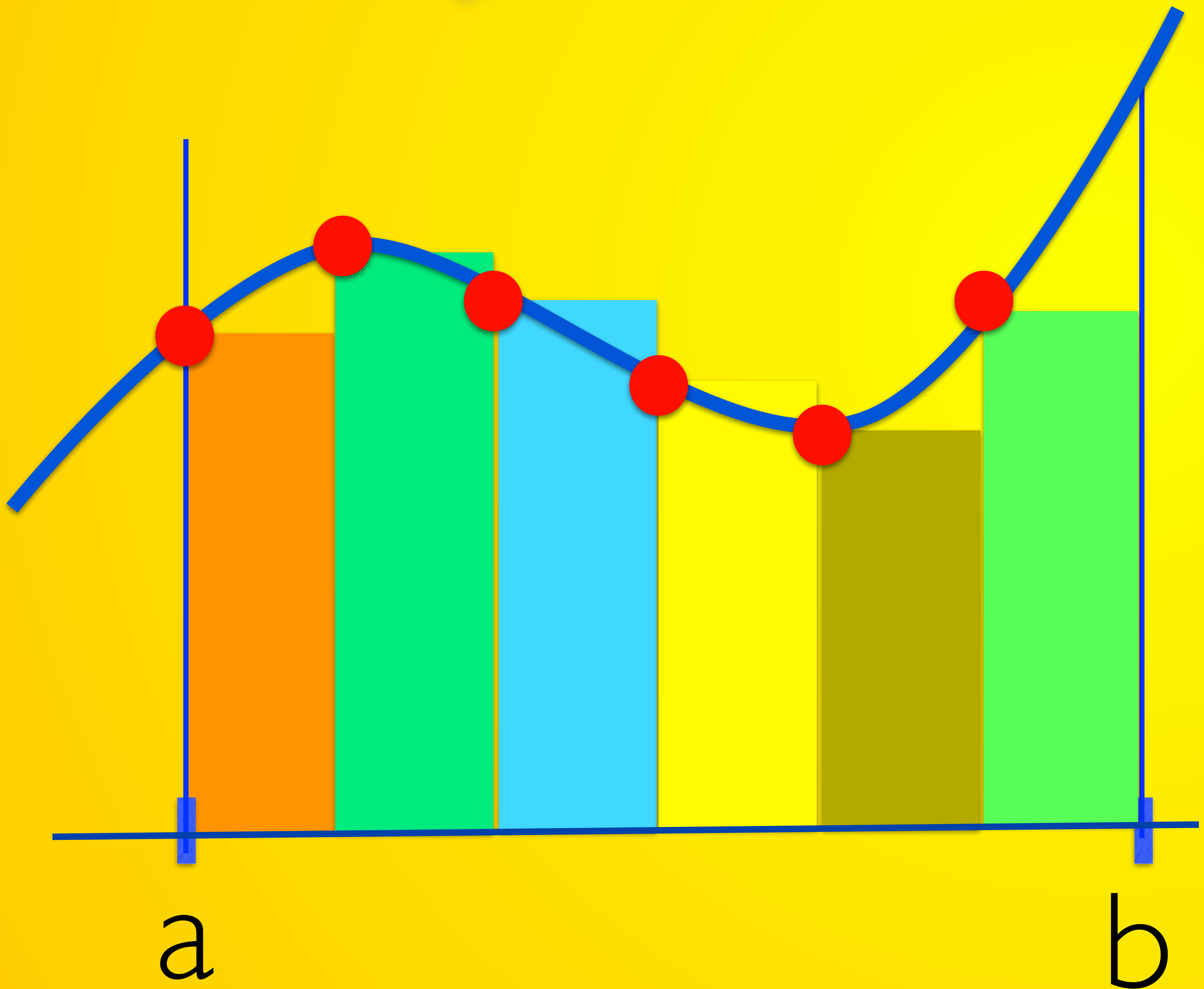
9/22/2021

Numerical Integration 2

Table of Contents

1 Overview

Left Riemann Sum



$$x_k = a + k\frac{b-a}{n}$$

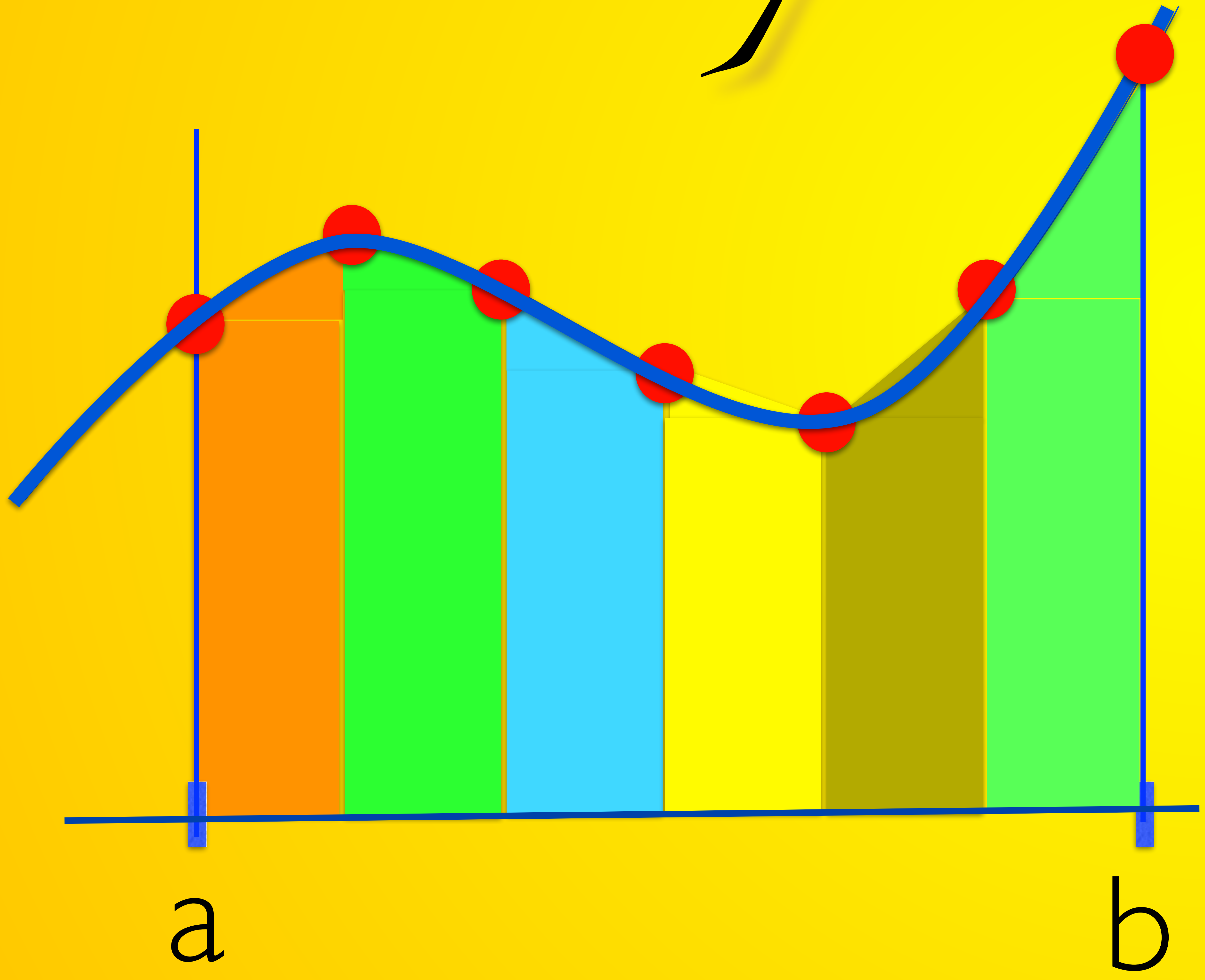
$$L_n = \sum_{k=0}^{n-1} f(x_k)\Delta(x)$$

Right Riemann Sum

$$x_k = a + k\frac{b-a}{n}$$

$$R_n = \sum_{k=1}^{n} f(x_k)\Delta(x)$$

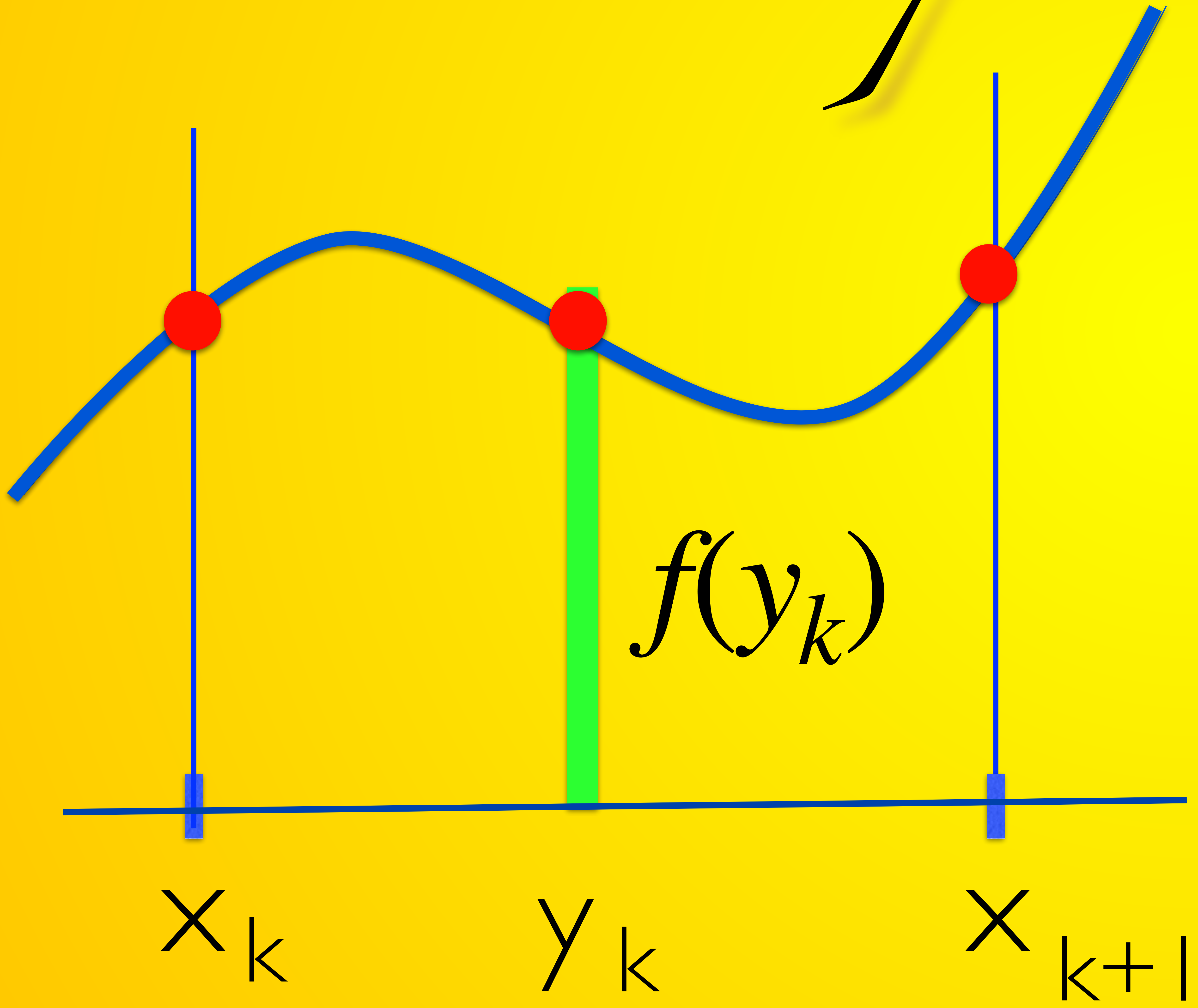
Trapezoid Rule



$$x_k = a + k\frac{b-a}{n}$$

$$T_n = \frac{L_n + R_n}{2}$$

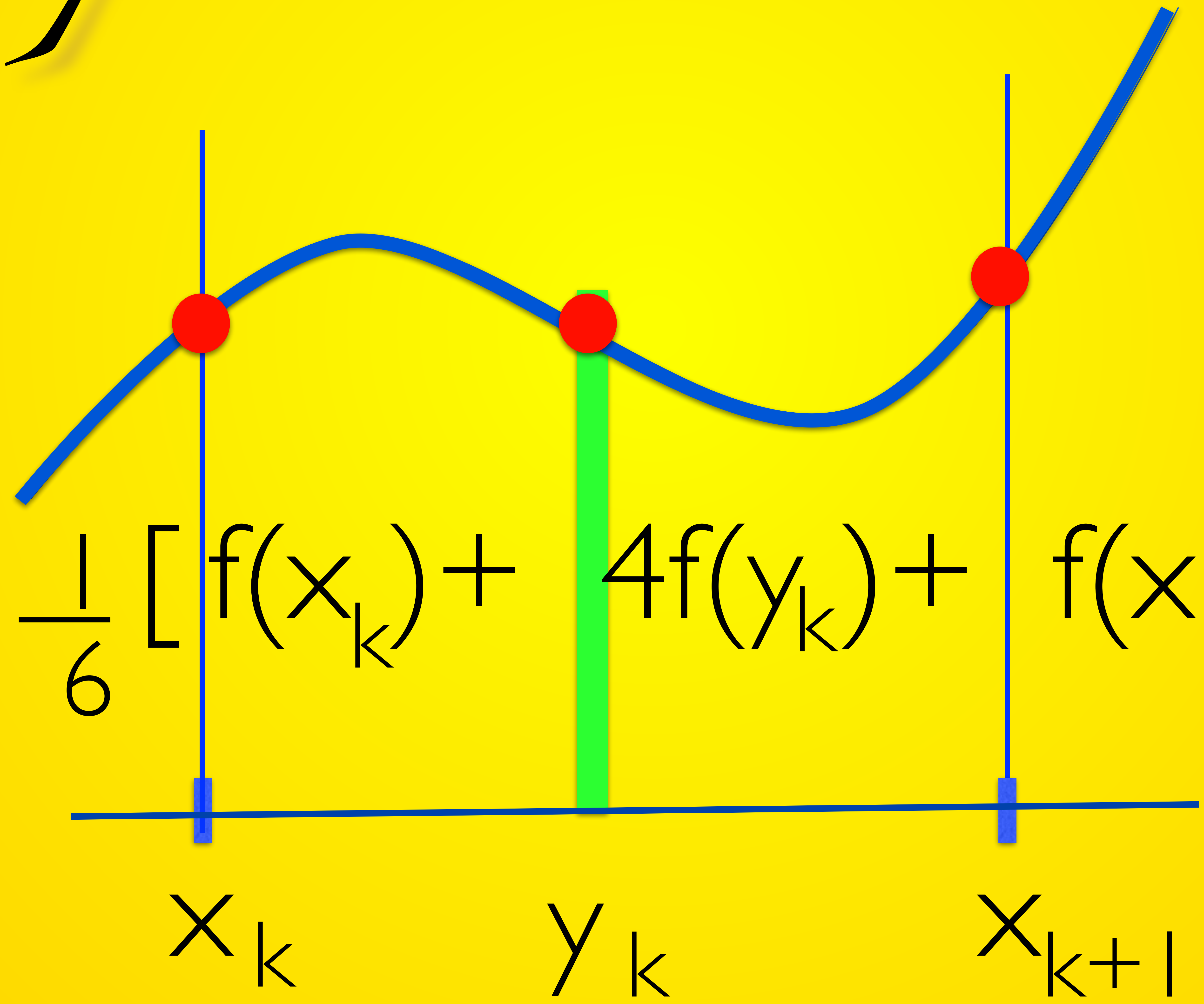
Midpoint Rule



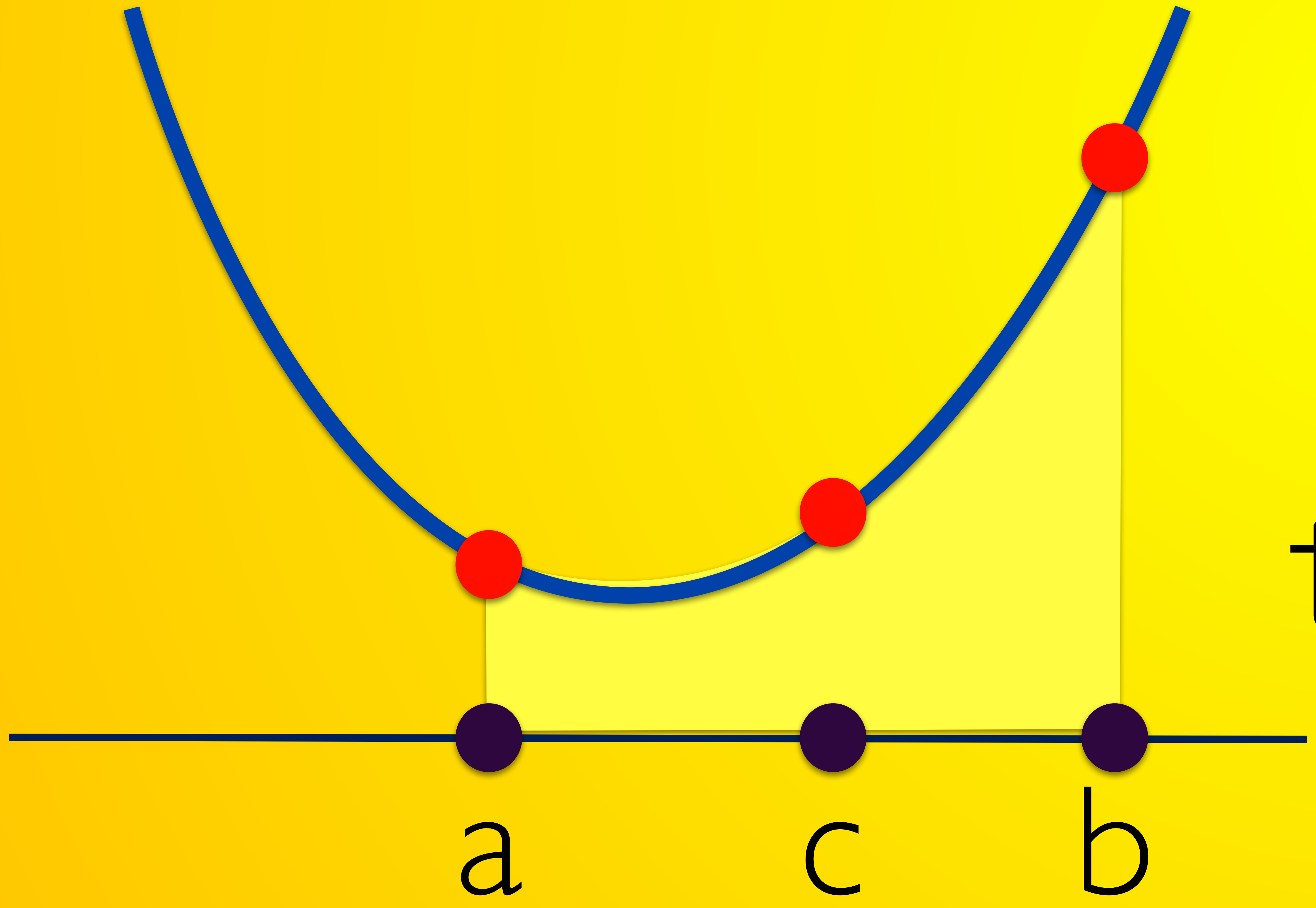
$$M_n = \sum_{k=1}^{n} f(y_k)\Delta(x)$$

Simpson Method

$$\sum_{k=1}^{n} \frac{1}{6}\left[f(x_k) + 4f(y_k) + f(x_{k+1}) \right] \Delta(x)$$



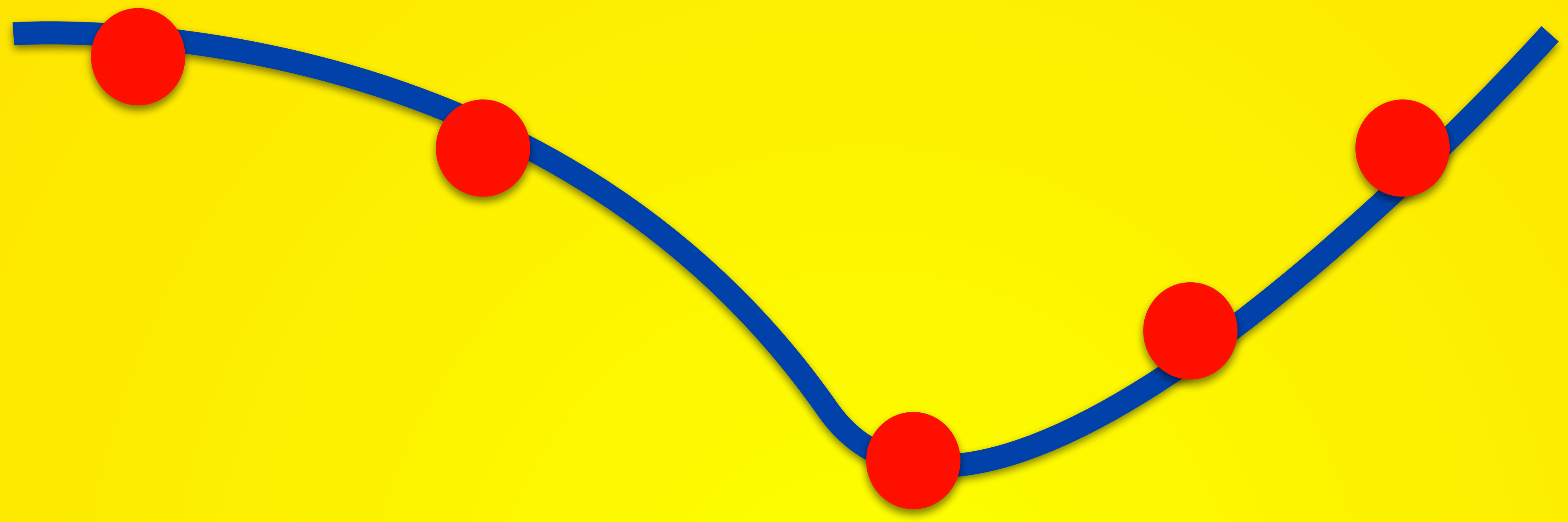
Example



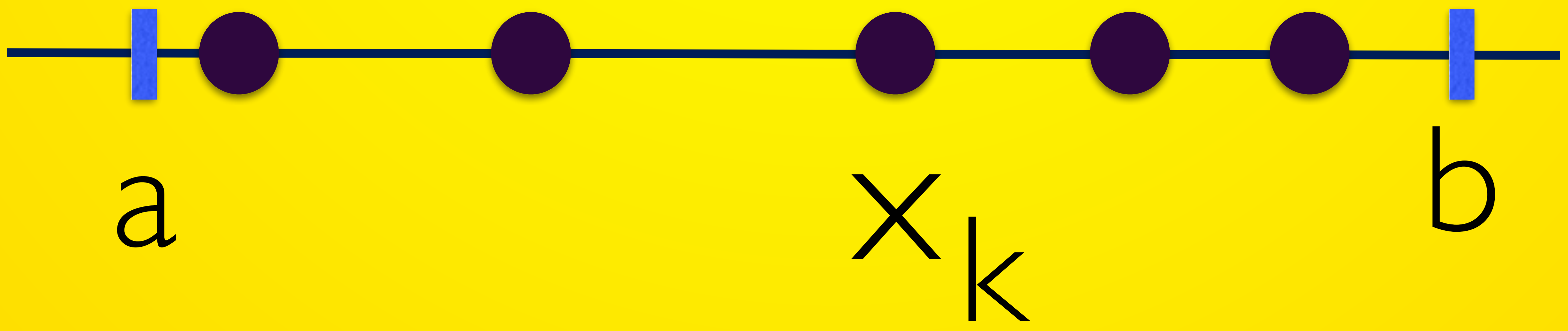
[f(a)+4f(c)+f(b)]/6 = Area

For the Parabola,
the result is correct.

Monte Carlo



$$\sum_{k=1}^{n} f(x_k)\,\Delta(x)$$



x_k uniformly distributed in [a,b]

Numerics Pioneers

Kepler

Riemann

Simpson

Origin

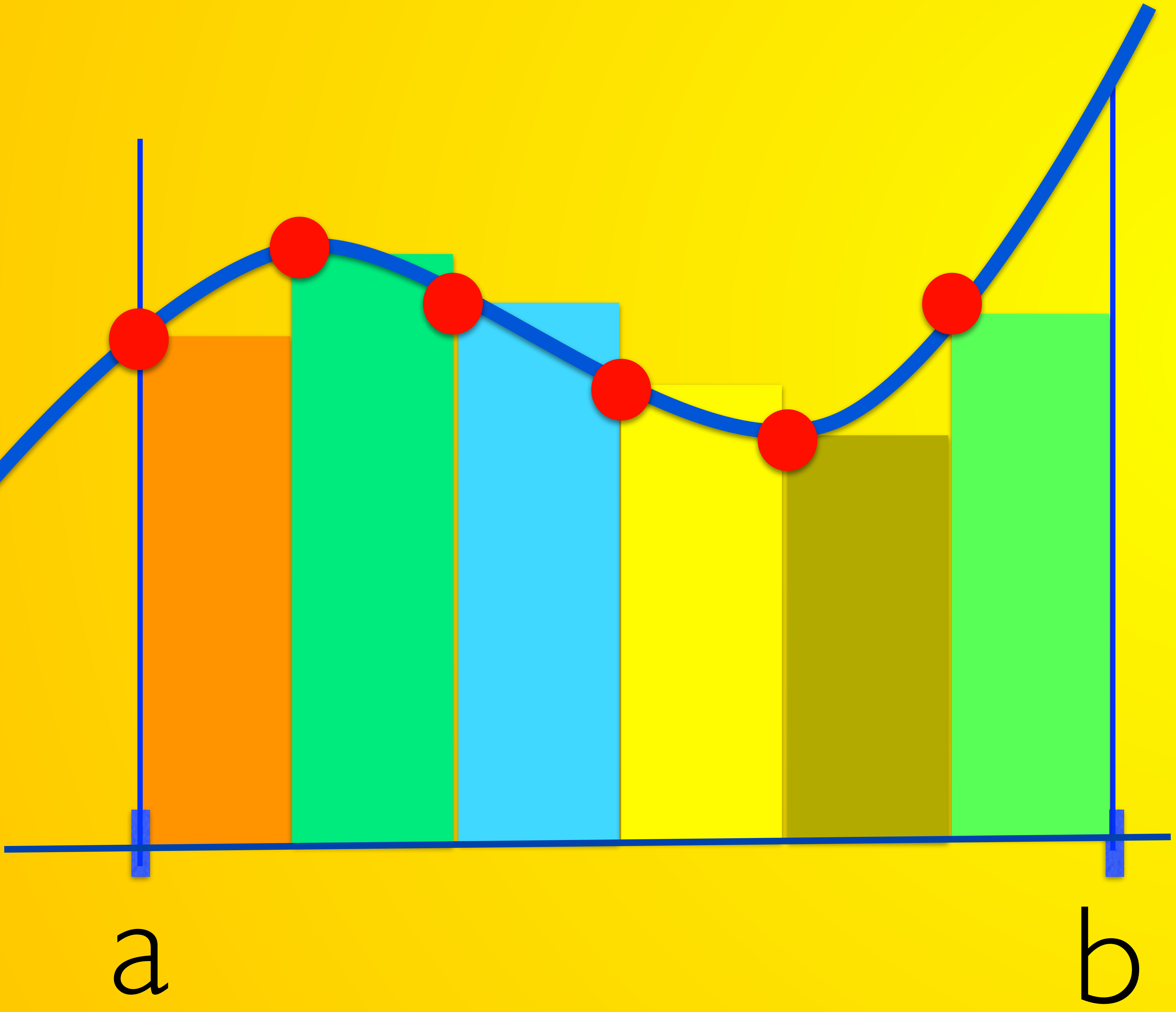
Thomas Simpson
(1710-1761)

Johannes Kepler
(1571-1630)

Part 2

Error bounds

Ln and Rn error bound



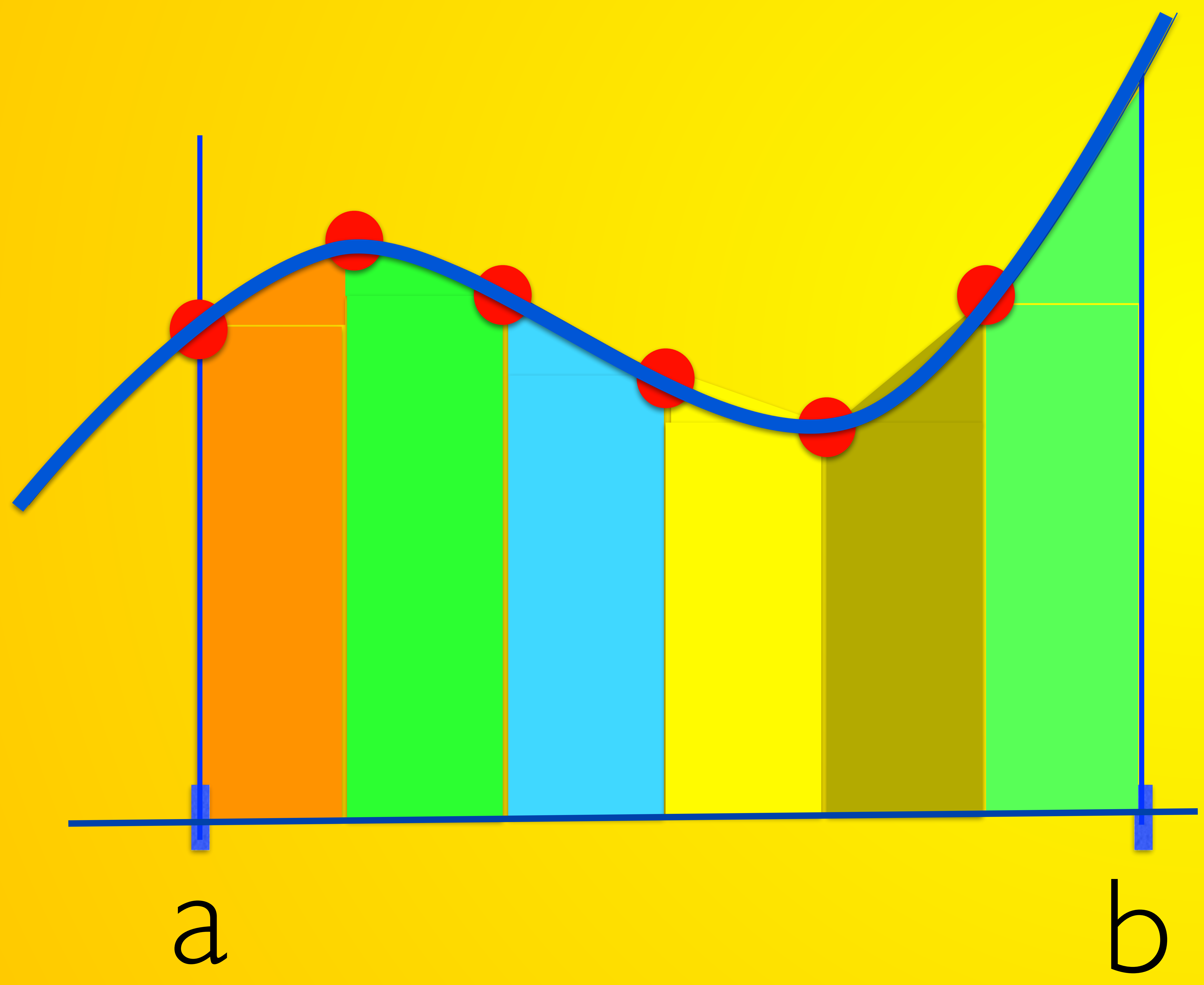
assume: $|f'(x)| \leq M$

Then the error in each slice is bound by $(\Delta x)^2 M/2$

The total error is bound by

$$\frac{M(b-a)^2}{2n}$$

Tn error bound

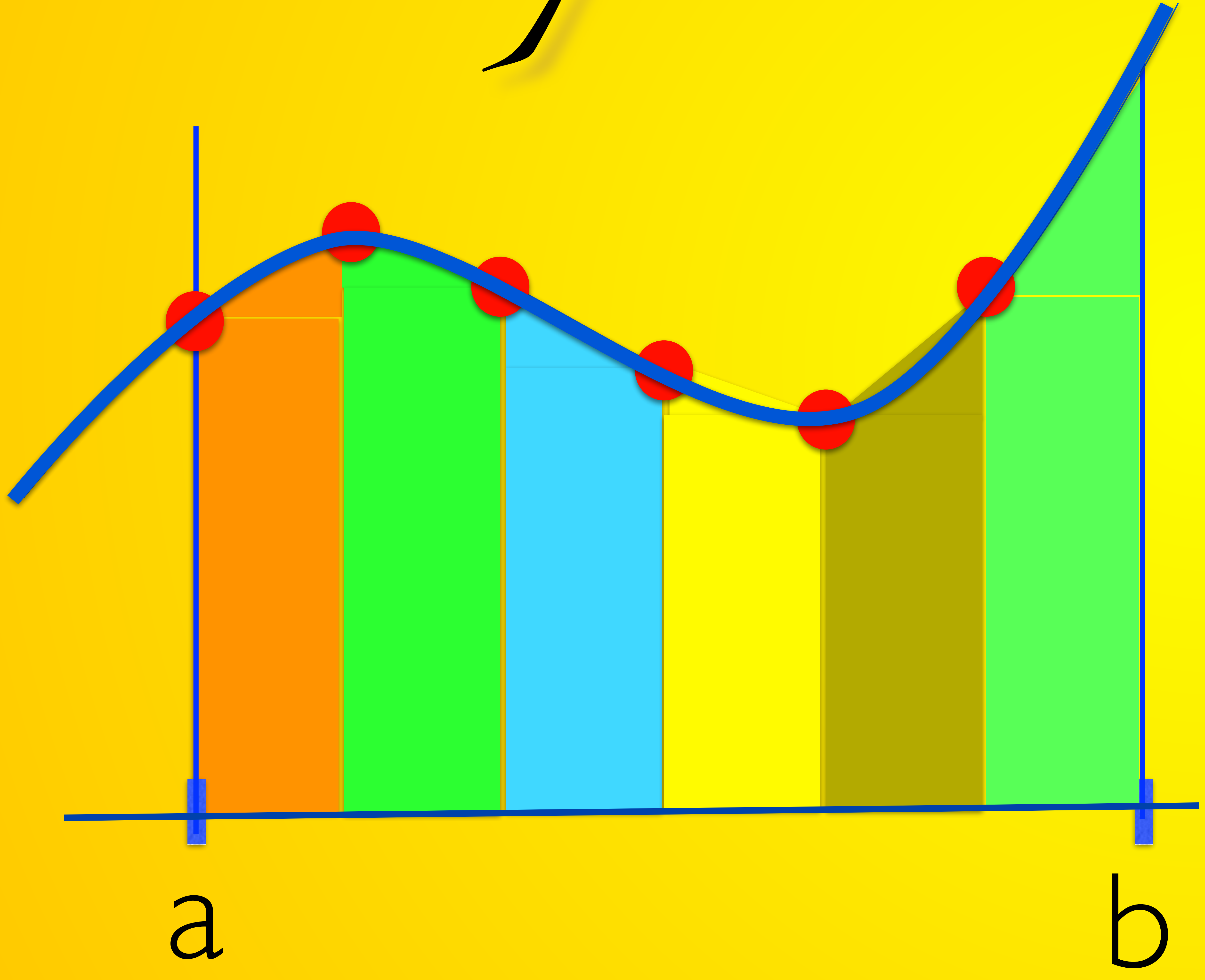


assume: $|f''(x)| \leq M_{(2)}$

The total error is bound by

$$\frac{M_{(2)}(b-a)^3}{12n^2}$$

Simpson error bound



assume:

$$|f^{(4)}(x)| \leq M_{(4)}$$

The total error is bound by

$$\frac{M_{(4)}(b-a)^5}{180(2n)^4}$$

Experiment

Trapezoid

In[1]:=
```
Clear[x]; R = Integrate[ Sin[x], {x, 0, 1}] // N
```
Out[1]= 0.459698

In[2]:=
```
n = 10; A = Sum[ Sin[ k / n], {k, 0, n - 1}] / n // N
```
Out[2]= 0.417241

In[3]:=
```
n = 10; B = Sum[ Sin[ (k + 1) / n], {k, 0, n - 1}] / n // N
```
Out[3]= 0.501388

In[4]:=
```
(A + B) / 2 // N
```
Out[4]= 0.459315

Part 4

Work sheet problems

THE END