8/30/2021 near Mather house

Lecture 10

9/27/2021

Improper Integration 2

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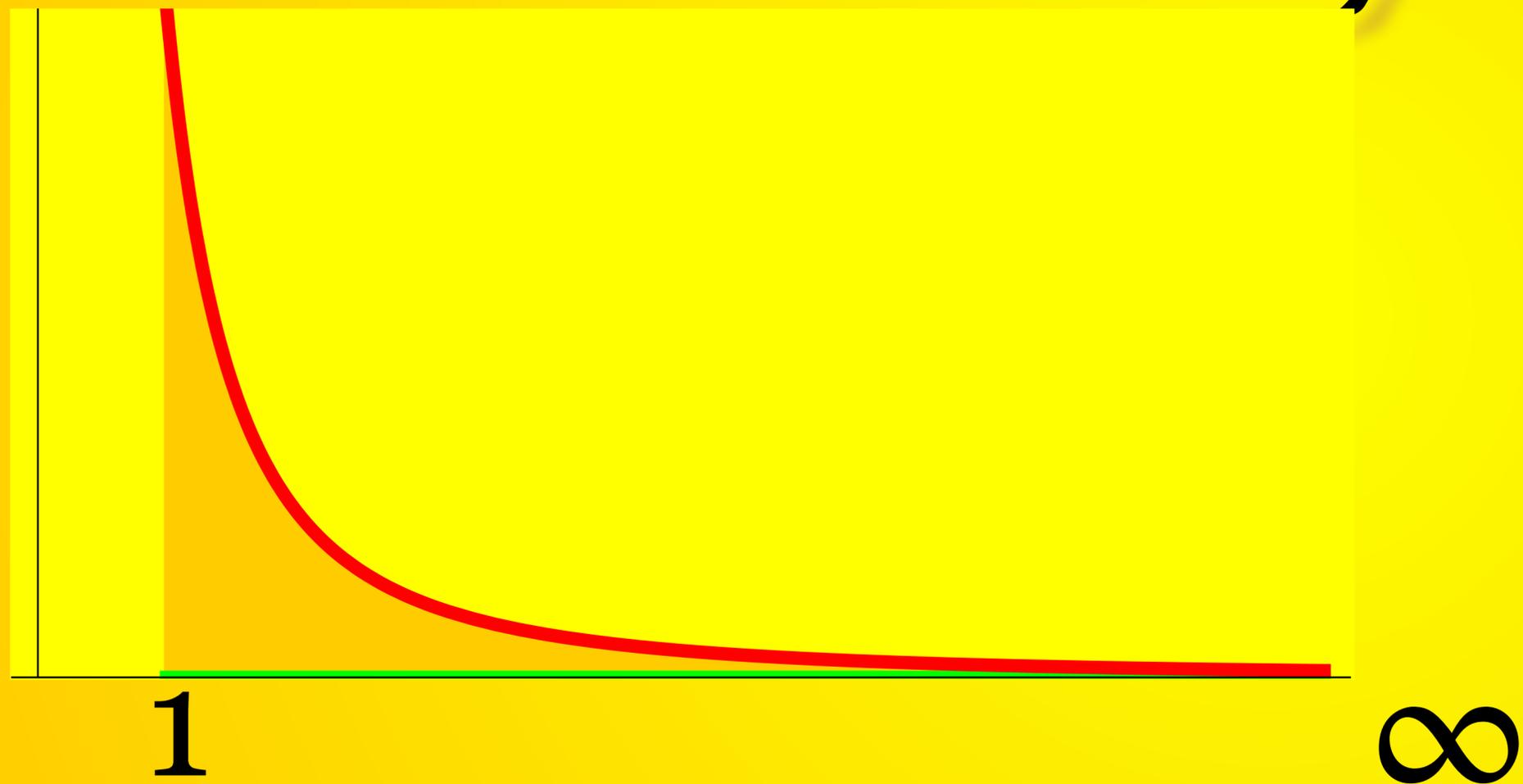
4) Worksheet problems

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Review

Motivation

In Stats, many distributions are defined on unbounded domains or are unbounded

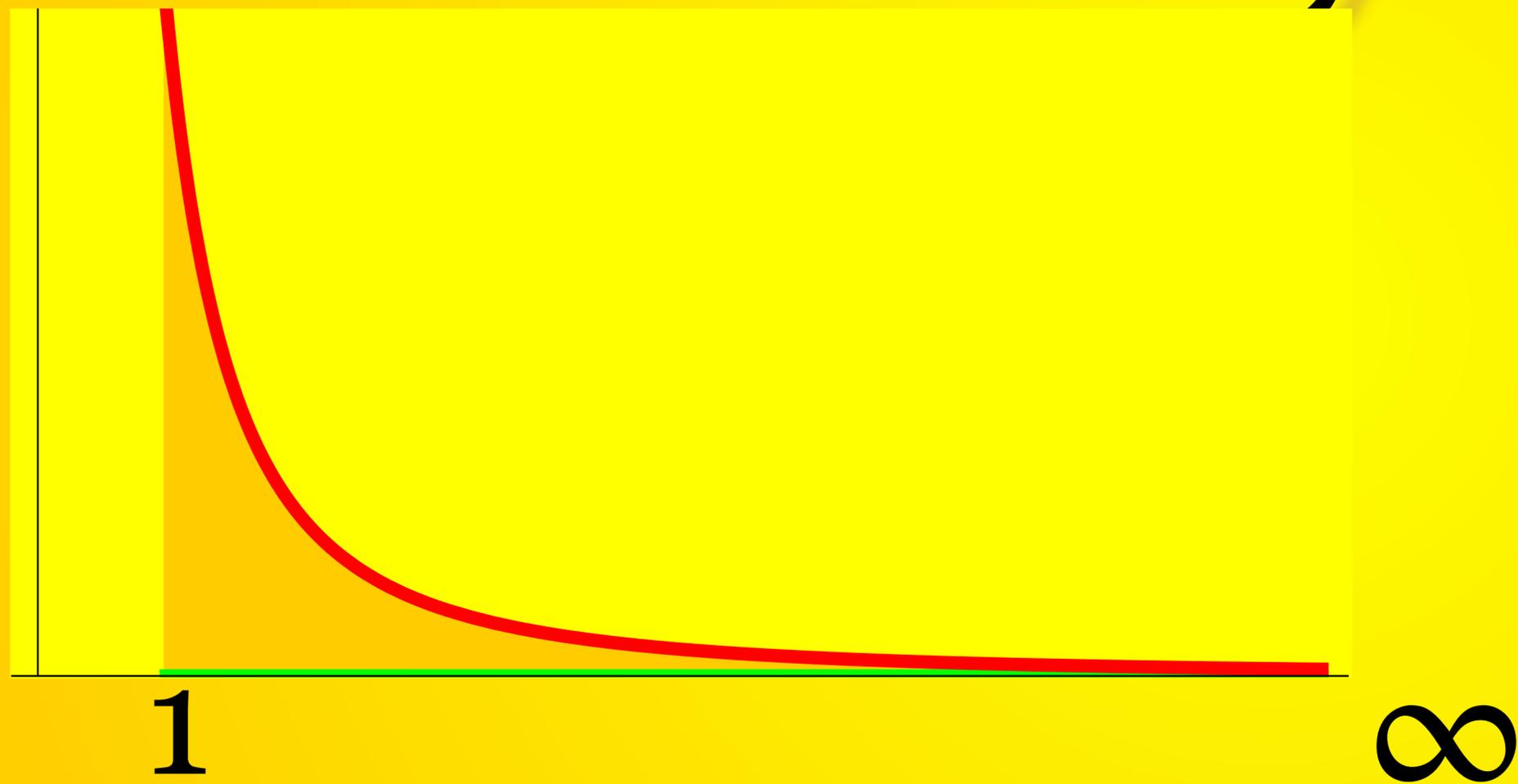$$\frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

$$\frac{1}{\sqrt{1-x^2}}\frac{1}{\pi}$$

Example 1

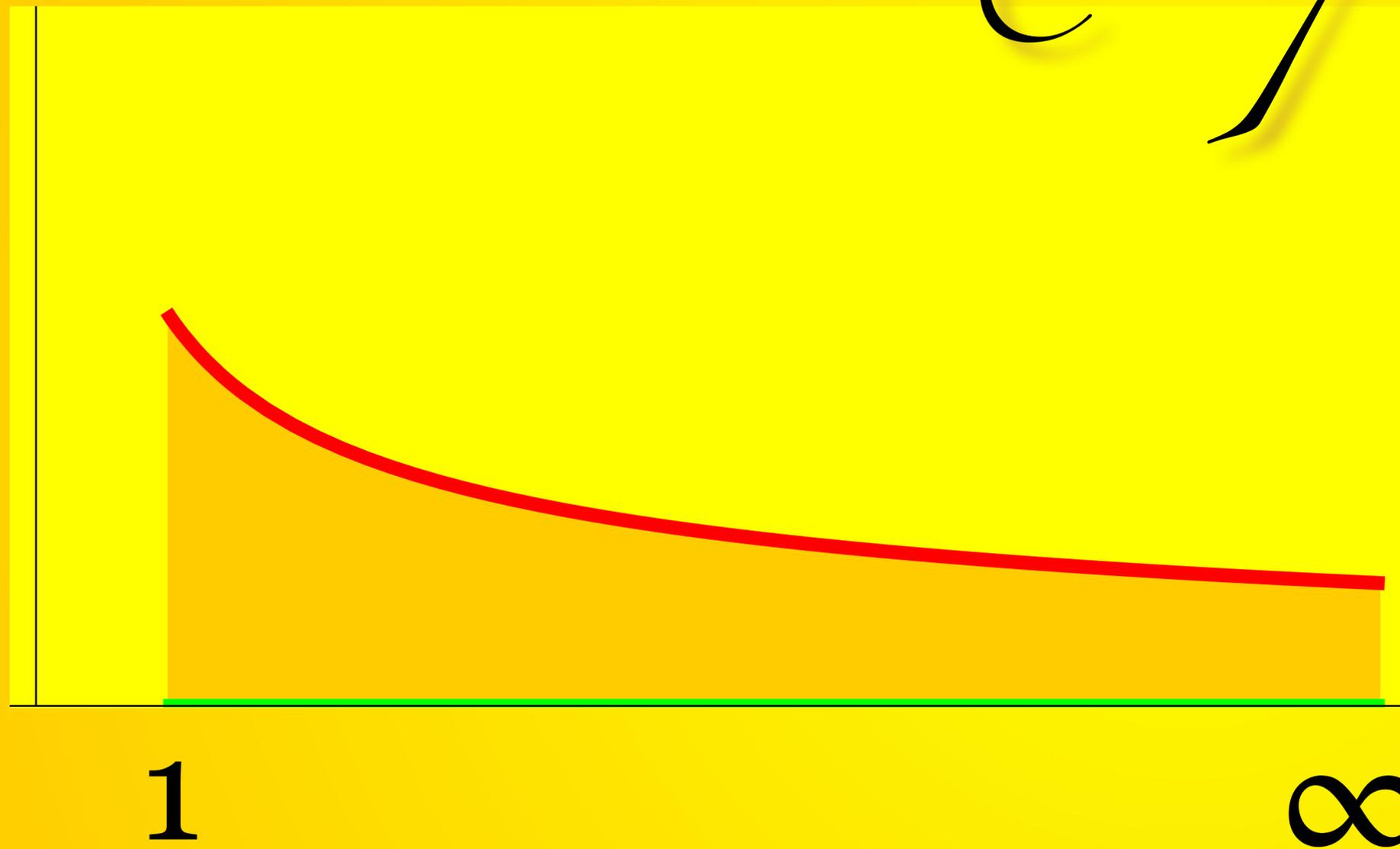

$$\int_1^\infty \frac{1}{x^3} dx$$

Region is unbounded

Example 2

$$\int_1^\infty \frac{1}{e^x}dx$$

1   ∞

Does this integral converge?

Example 3

$$\int_1^\infty \frac{1}{x^{1/3}} dx$$

Does this integral converge?

p-integrals

$$\int_1^\infty \frac{1}{x^p} dx$$

converges for p>1 and diverges else.

comparison

If $0 \le g \le f$ *and*

$$\int_1^\infty f(x)dx \ converges$$

then $\int_1^\infty g(x)\ dx$ *converges.*

Quiz

*converge
or
diverge?*

$$\int_1^\infty \frac{1}{x^2}dx$$

$$\int_1^\infty \frac{1}{\sqrt{x}}dx$$

$$\int_1^\infty \frac{1}{x}dx$$

$$\int_1^\infty \frac{\sin^2(x)}{x^3}dx$$

$$\int_1^\infty \frac{\sin^2(x)}{x^{1/4}}dx$$

$$\int_1^\infty \frac{\sin^2(x)}{x+1}dx$$

A shocking example

Compute this integral!
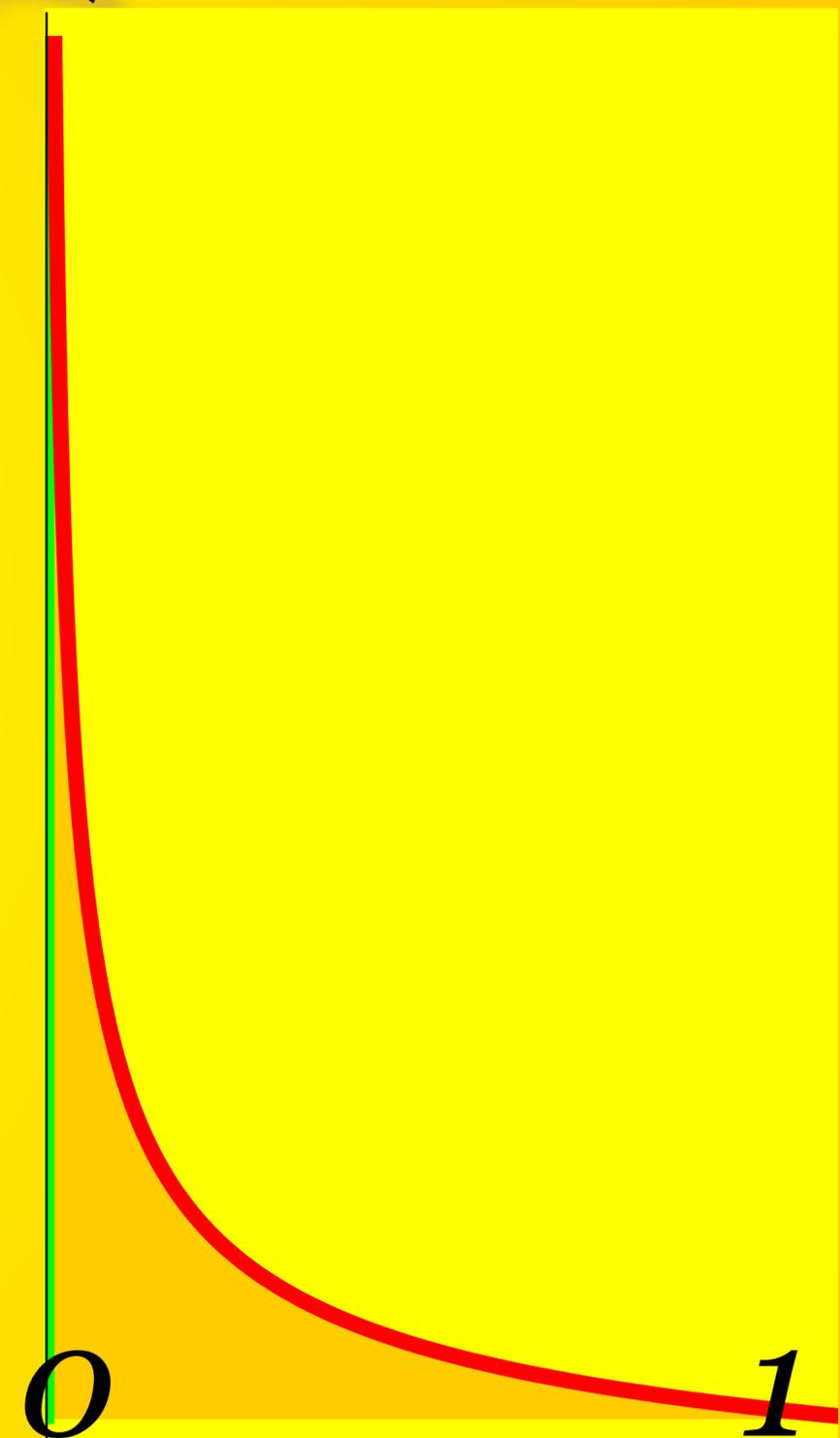
$$\int_{-1}^{1} x^{-2} dx$$

p-integrals part 2
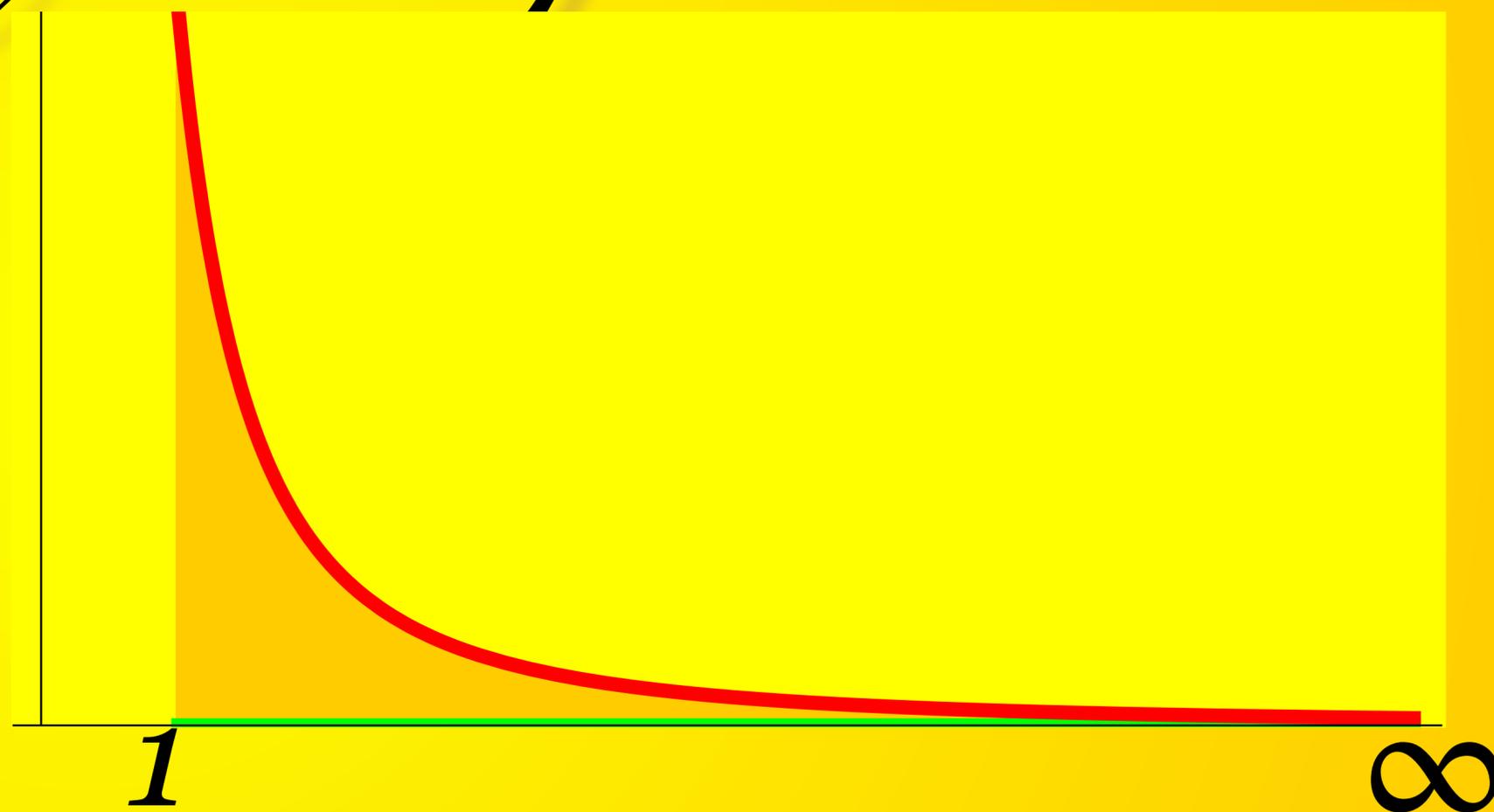
p-integrals part 2

$$\int_0^1 \frac{1}{x^p} dx$$

converges for
p<1 and
diverges else.

0

1

$$\int_1^\infty \frac{1}{x^p} dx$$

converges for p>1 and diverges else.

What is wrong here?

$$\int_{-2}^{2} \frac{1}{x} dx = \lim_{a \to 0} \int_{-2}^{-a} \frac{1}{x} dx + \int_{a}^{2} \frac{1}{x} dx$$

$$= \lim_{a \to 0} (\log|a| - \log|2|) + (\log|2| - \log|a|)$$

$$= \lim_{a \to 0} 0 = 0$$

What is wrong here?

$$\lim_{b \to \infty} \int_{-b}^{b} x^3 \, dx = x^4 \Big|_{-b}^{b} = 0$$

So, the integral $\int_{-\infty}^{\infty} x^3 \, dx$ exists.

Work sheet problems

The End