8/30/2021 near Mather house

Lecture 11

9/29/2021

Taylor
Series 1

Table of Contents

About the exam

- Monday 6-7:30 PM, up to improper stuff
- Practice exams and review material
- Problems are original ones.
- Do practice problems without solutions

Main Themes: integration

1) Area, Volume, Density, Slicing sums

write down Riemann sums Δx, including density $\rho(x)$

2) Left, Right, Trapezoid, Midpoint rules

perform methods for small n, estimate errors using M_n

3) Improper int, p-integrals, Comparison

diverge or converge? write limits, split up cases!

Today:

How does one compute

$$\sqrt{103}$$

$$\sin(0.4)$$

$$\ln(1.2)$$

The answer is to approximate with a polynomial. For example $a + bx + cx^2$

Motivation

√101?

What does linear approximation tell?

$$\sqrt{101}$$

$$\sqrt{100 + x} = 10 + ax$$

Can you see what a is?

Lets try with a quadratic function:

$$\sqrt{100 + x} = 10 + ax + bx^2$$

Can you see what a,b are?

Degree 2 polynomial

Lets pretend f(x) is a polynomial:

$$f(x) = a + bx + cx^2$$

$$f'(x) = b + 2cx$$
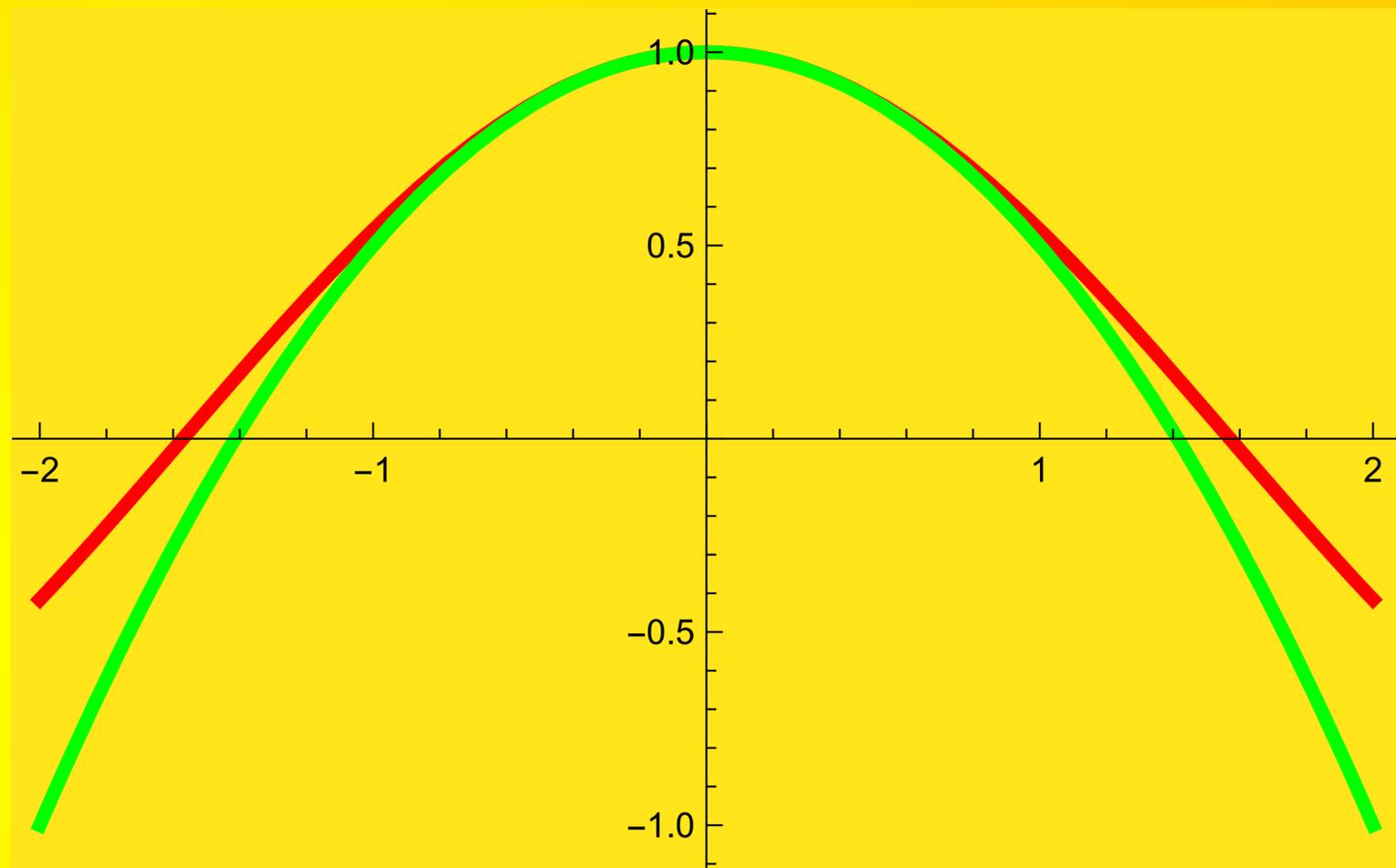
$$f''(x) = 2c$$

At x=0, we see

f(0)=0, f'(0)=b, f''(0)/2=c

$$f(x) = \cos(x)$$

$$f(x) \sim 1 - x^2/2$$

Quadratic Taylor

$$f(x) \sim f(0) + f'(0)x + f''(0)\frac{x^2}{2}$$

$$\cos(x) \sim 1 + 0 \cdot x + (-1)\frac{x^2}{2}$$

Example 2

What does the quadratic approximation
give?

$$\sqrt{104}$$

How would you go about a cubic
approximation?

Cubic Taylor

$$f(x) \sim f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{6}$$

Example

$$f(x) \sim f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{6}$$

$$\log(1 + x) \sim x - x^2/2 + x^3/3$$

Preview

On Friday, we continue with this and especially also look at approximations at other points.

The End