8/30/2021 near Mather house

Lecture 12

10/1/2021

Taylor Series 2

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Taylor Approximation

Key Expression

$$P_n(x) = f(c) + f'(c)(x - c) + f''(c)\frac{(x - c)^2}{2} + \ldots + f^{(n)}(c)\frac{(x - c)^n}{n!}$$

$$\log(1 + x) \sim x - x^2/2 + x^3/3$$

Example 1

$$f(x) = \exp(x)$$

Find the 6'th degree approximation

Example 1

$$\log(1 + x) \sim x + x^2/2 + x^3/6 + x^4/24 + x^5/120$$

Example 2

$f(x) = \exp(x)$

Find the 6'th degree approximation at c=0.

Example 1

$$\log(1 + x) \sim x - x^2/2 + x^3/3$$

Example 2

$$f(x) = \log(1 + x)$$

Find the third degree approximation

Example 2

$$\log(1 + x) \sim x - x^2/2 + x^3/3$$

Worksheet

Mathematica

https://downloads.fas.harvard.edu/download

Mathematica (Student) ⬀   MacOS X

CREATING A WOLFRAM ACCOUNT: Before downloading and installing Mathematica you must create a Wolfram account. If you have already done so, please proceed to the instructions for "DOWNLOADING AND INSTALLING" below. To create a Wolfram account, please click here. You must use your **Harvard email address**. An email with a link to validate your Wolfram ID will be sent to the address you provide.

DOWNLOADING AND INSTALLING: After creating and validating your Wolfram account, sign in to the Wolfram User Portal and complete the Wolfram Activation Key Request Form. When selecting a product on the form, choose a version of "Mathematica for sites (single machine)." Click Submit. An activation key will be generated and emailed to you. Click the link for "Product Summary page" [if necessary, please refer to the System requirements link to determine compatibility]. Click "Get Downloads" and select "Download" next to the appropriate platform. Follow the installation instructions and enter the activation key when prompted.

EXPIRING AND EXPIRED INSTALLATIONS: Annual re-activation is required. Prior to expiration, you will receive an email from Wolfram with instructions for re-activation. For installations of Mathematica that have already expired, you will be prompted to request and enter an activation key when you launch the application.

Mathematica

```
In[1]:= Taylor[f_, c_, n_] := Function[x, Sum[D[f[y], {y, k}] (x - c)^k/k! /. y → c, {k, 0, n}]]
        g = Taylor[Sin, 0, 7]; g[x]
        Manipulate[Plot[{Sin[x], Taylor[Sin, 0, k][x]}, {x, 0, 2 Pi}, PlotRange → {-2, 2}], {k, 1, 20, 1}]
```
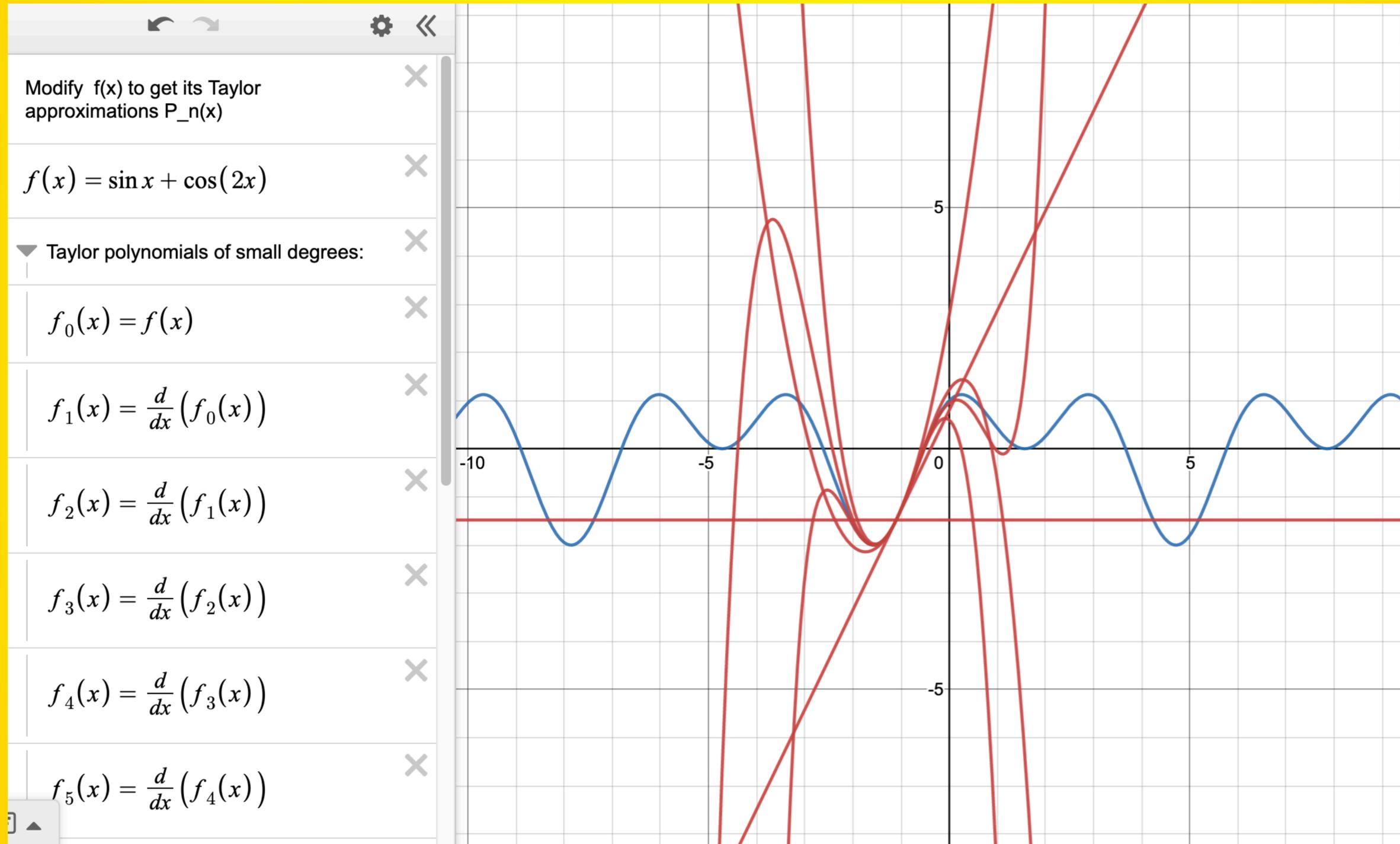
$$\text{Out[2]= } x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$$

Out[3]=
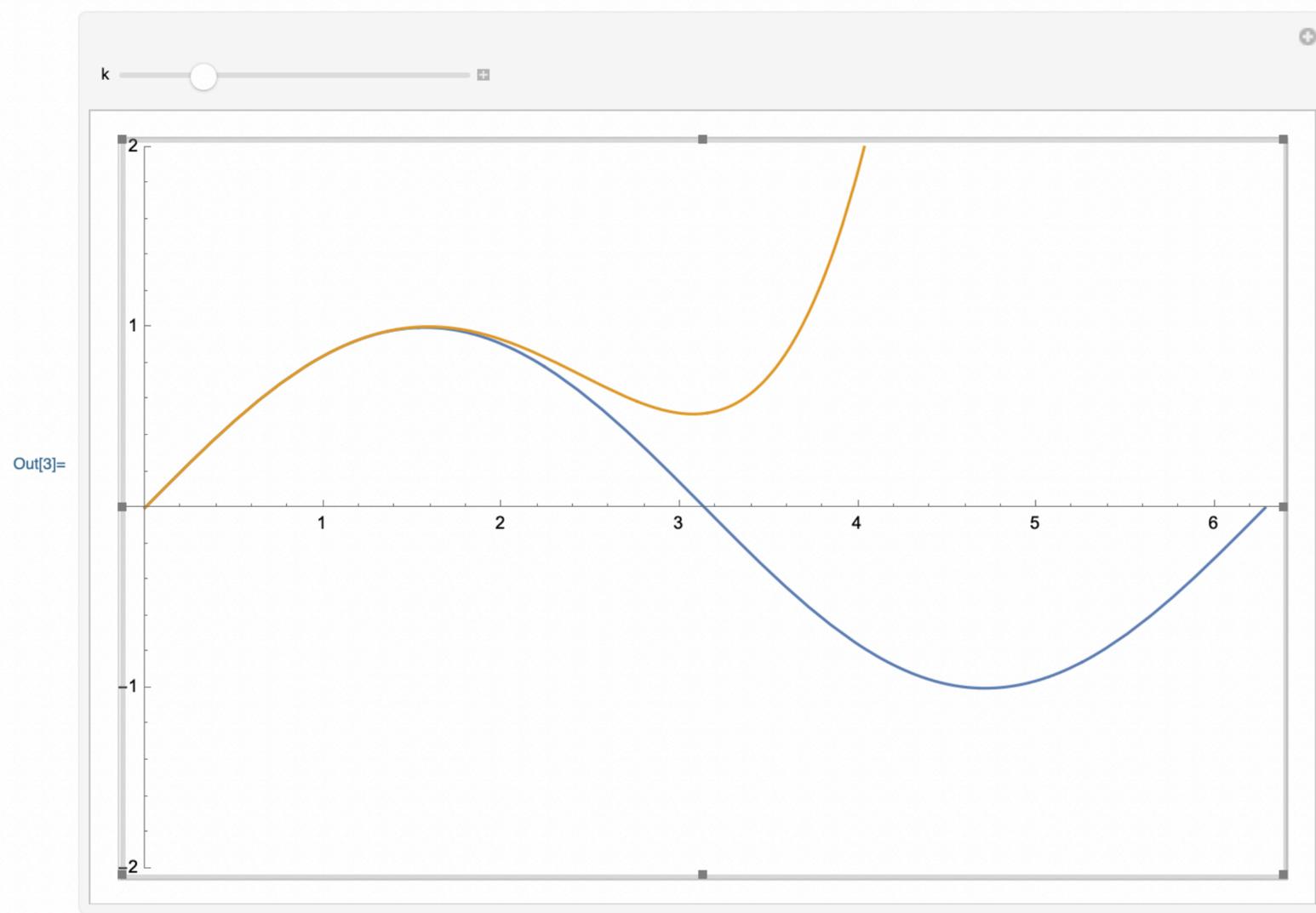
Taylor[f_, c_, n_] := Function[x,
Sum[D[f[y], {y, k}] (x - c)^k/k! /. y -> c,
{k, 0, n}]];

g = Taylor[Sin, 0, 7]; g[x]

Desmos

Modify $f(x)$ to get its Taylor approximations $P\_n(x)$

$f(x) = \sin x + \cos(2x)$

▼ Taylor polynomials of small degrees:

$f_0(x) = f(x)$

$f_1(x) = \dfrac{d}{dx}\left(f_0(x)\right)$

$f_2(x) = \dfrac{d}{dx}\left(f_1(x)\right)$

$f_3(x) = \dfrac{d}{dx}\left(f_2(x)\right)$

$f_4(x) = \dfrac{d}{dx}\left(f_3(x)\right)$

$f_5(x) = \dfrac{d}{dx}\left(f_4(x)\right)$

The End