8/30/2021 near Mather house

Lecture 15

10/8/2021

Convergence of Series

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Series

$$S = a_1 + a_2 + a_3 + \dots$$

is called a series written in ... notation.

$$S = \sum_{k=1}^{\infty} a_k$$

is the sum notation

Convergece

The series converges if the partial sums

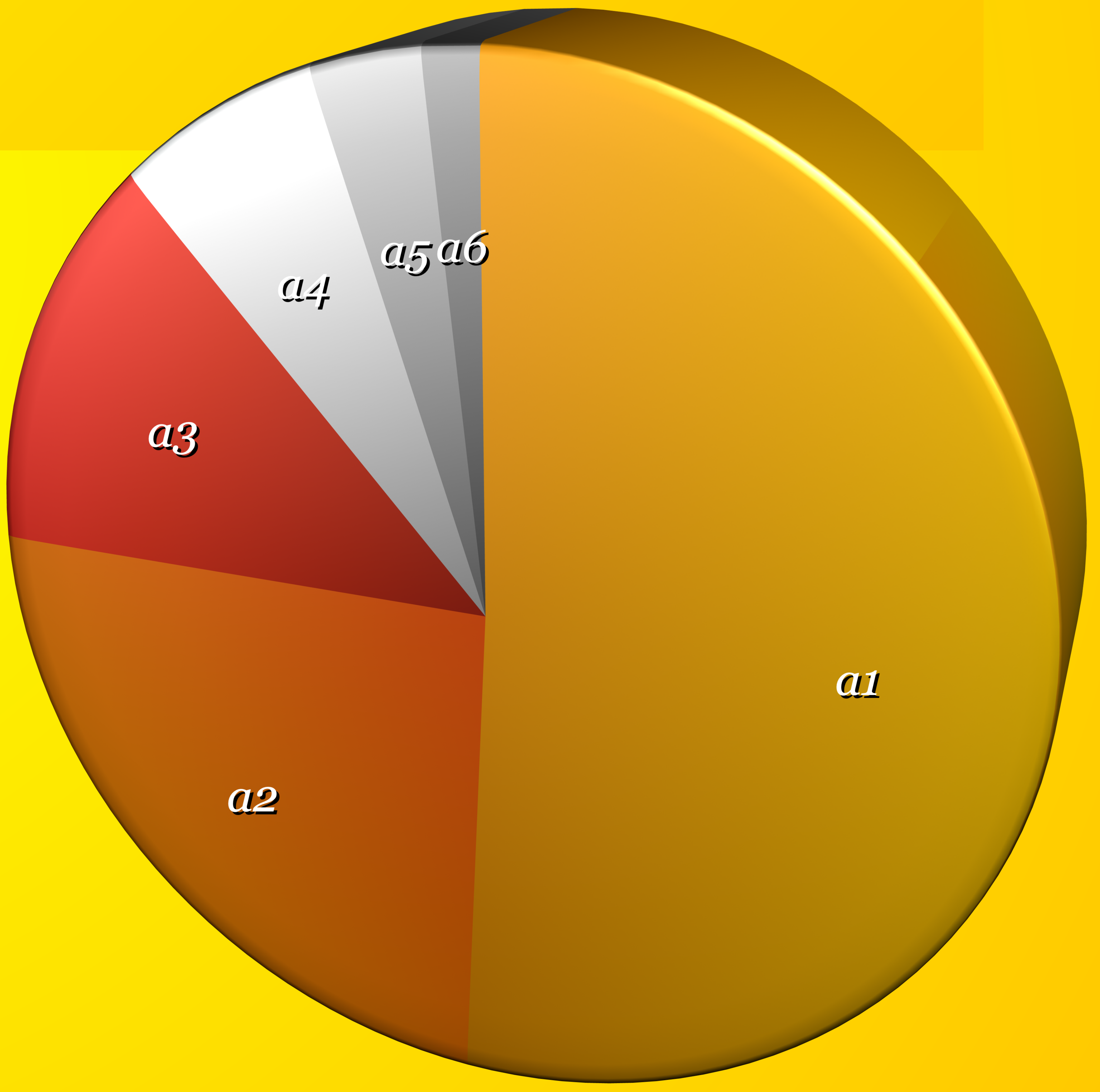
$$S_n = a_1 + a_2 + \ldots + a_n$$

converge to a number S.
It converges absolutely if the sum of the absolute values converges.

Example

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

converges to 1.
Take half a pie, add a quarter (half of the rest) an eigth (half of the rest) etc



Example

$$S = 1 + 1 + 1 + \dots$$

in general, $a_n \to 0$ is necessary for a series to converge.

Example

$$S = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots.$$

converges. Do you know to what?

Example

$$S = \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} \cdots$$

converges. Do you know to what?

write it out 0.1+0.01+0.001+....

Example

$$S = 1 + 1 + 1 + 1 + 1 + \ldots$$

diverges to infinity because $S_n = n$.

Grandi's Series

$$S = 1 - 1 + 1 - 1 + 1 - 1 \ldots$$

is called Grandi's series.

Luigi Grandi 1671 – 1742

Taylor series

$$S = \sum_{k=0}^{\infty} f^{(k)}(0)\frac{x^k}{k!}$$

is an infinite sum for every fixed x.

Example:

$$S = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \ldots$$

Example

$$S = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots.$$

converges to ......

Example

$$S = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

converges to ……

Example

$$S = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots$$

converges to ......

Example

$$S = x + x^2/2 + x^3/3 + x^4/4 + x^5/5 + \ldots$$

converges to ln(1/(1-x)). Why?

Worksheet

Reminders

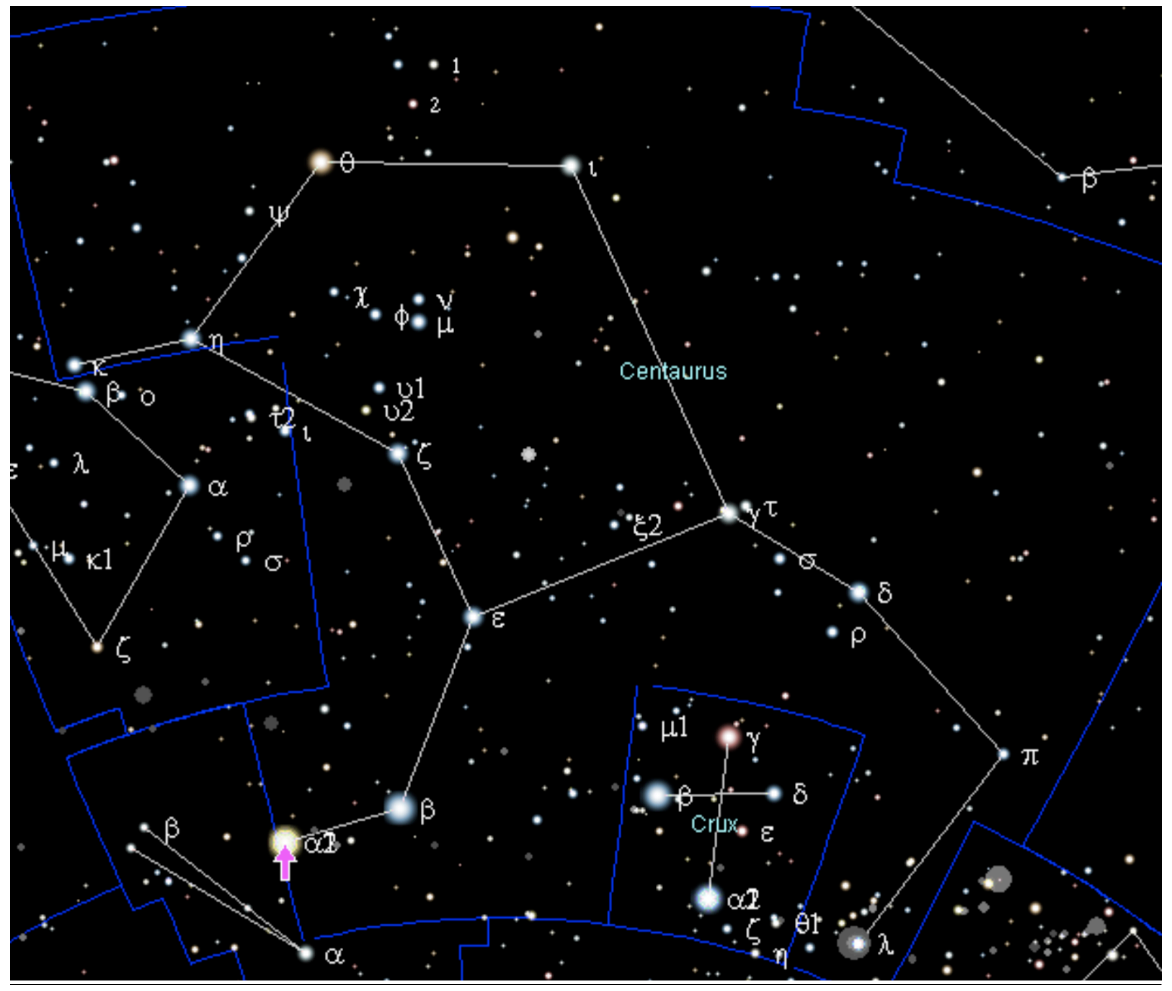
QRD 3

The distance to the stars

Then nearest star to Earth is the sun. How far away is the next nearest star? And, how far away are the other stars? How can you possibly measure that? Probably you know that the next nearest star is named Proxima Centauri. It is too dim to be seen without a telescope. However, Proxima Centauri is part of a triple star system with the other two stars being so close to each other that we see them as the single bright star Alpha Centauri; a very bright star visible from near the equator or farther south (not from Boston anyway). What follows is an image from the Wikipedia article on Alpha Centauri. It is the brightest star, about center height on the left; Proxima Centauri is the reddish star at the center of the small red circle which is down and to the right of Alpha Centauri.

The two bright stars are (left) Alpha Centauri and (right) Beta Centauri, both binaries. The faint red star in the center of the red circle, at right angles to both and south-east of Alpha is Proxima Centauri, intensely red, smaller in size, weaker in brightness and a distant third element in a triple star system with the main close pair forming Alpha Centauri. Taken with Canon 85mm f/1.8 lens with 11 frames stacked, each frame exposed 30 seconds.

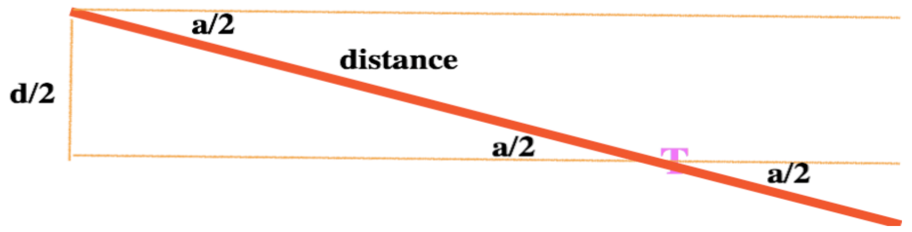
If you are near to or south of the equator, you can easily see Alpha Centauri. The small pink arrow points to Alpha Centauri on the following star chart



But how far away is Proxima Centauri? The current most accurate distance measurement comes from the European Space Agency's Gaia satellite (more about this momentarily) which (according to Wikipedia) finds Proxima Centauri to be 4.2465 ± 0.0003 light years.

The unofficial definition of a 'light year' is the distance that light travels in a year. To determine that, you have to decide what precisely you mean by a 'year' which is a subtle issue if you are looking for precision. In any event, the International Astronomical Union (IAU) has an official definition of a light year which is 9,460,730,472,580.8 km, exactly. This is roughly 9.5 *trillion* kilometers.

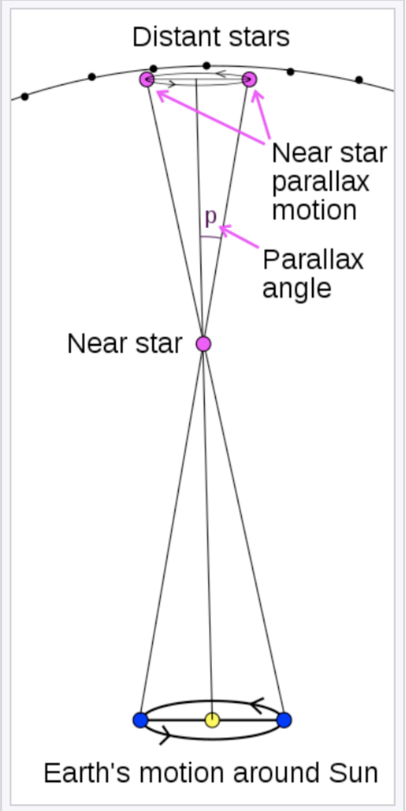
The Gaia space satellite has measured the distance to not just Proxima Centauri but to a *lot* of other stars. The ultimate goal of Gaia is to measure the distances to 20 million stars with accuracy within 1% and distances to 200 million stars with accuracy within 10%. When finished, it will have measured the distances of over 1 billion stars in our galaxy (and lots of other things besides). Take a look at the Gaia mission website if you want to know more https://sci.esa.int/web/gaia/-/47354-fact-sheet. (Gaia measures more than just distance. See these websites https://www.cosmos.esa.int/web/gaia/early-



Because of this, the angle **a**/2 can be determined by the viewer, which is to say you or me or Gaia depending on the context.

PROBLEM 2: *Measure the length of your arm and the distance between your eyes. Use these numbers to determine the angle* **a**/2 *in the case where you hold your finger at arms length and alternately view your finger using just your left eye and then just your right eye. (Give the arm and eye distance measurements to get full credit for this problem.)*

In the astronomical context, the parallax picture is this one (also from Wikipedia):



Stellar parallax motion

What is denoted by **p** in this diagram is what was called **a**/2 in the previous diagrams. The analog of **d**/2 in this setting is the radius of the earth's orbit around the sun (thus, ½ of the diameter which is the analog of **d**). With this geometry understood, then the task of measuring the distance to a star has the following components:

due next week


Proxima Centauri
space.com


Contact

The End