8/30/2021 near Mather house

Lecture 17

10/14/2021

Geometric Series


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due today
space.com


Contact

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Geometric Series

$$S = 1 + x + x^2 + x^3 + x^4 + \ldots.$$

is called the geometric series

$$S = \sum_{k=0}^{\infty} x^k$$

is the sum notation

Explicit formula

We have seen that sometimes

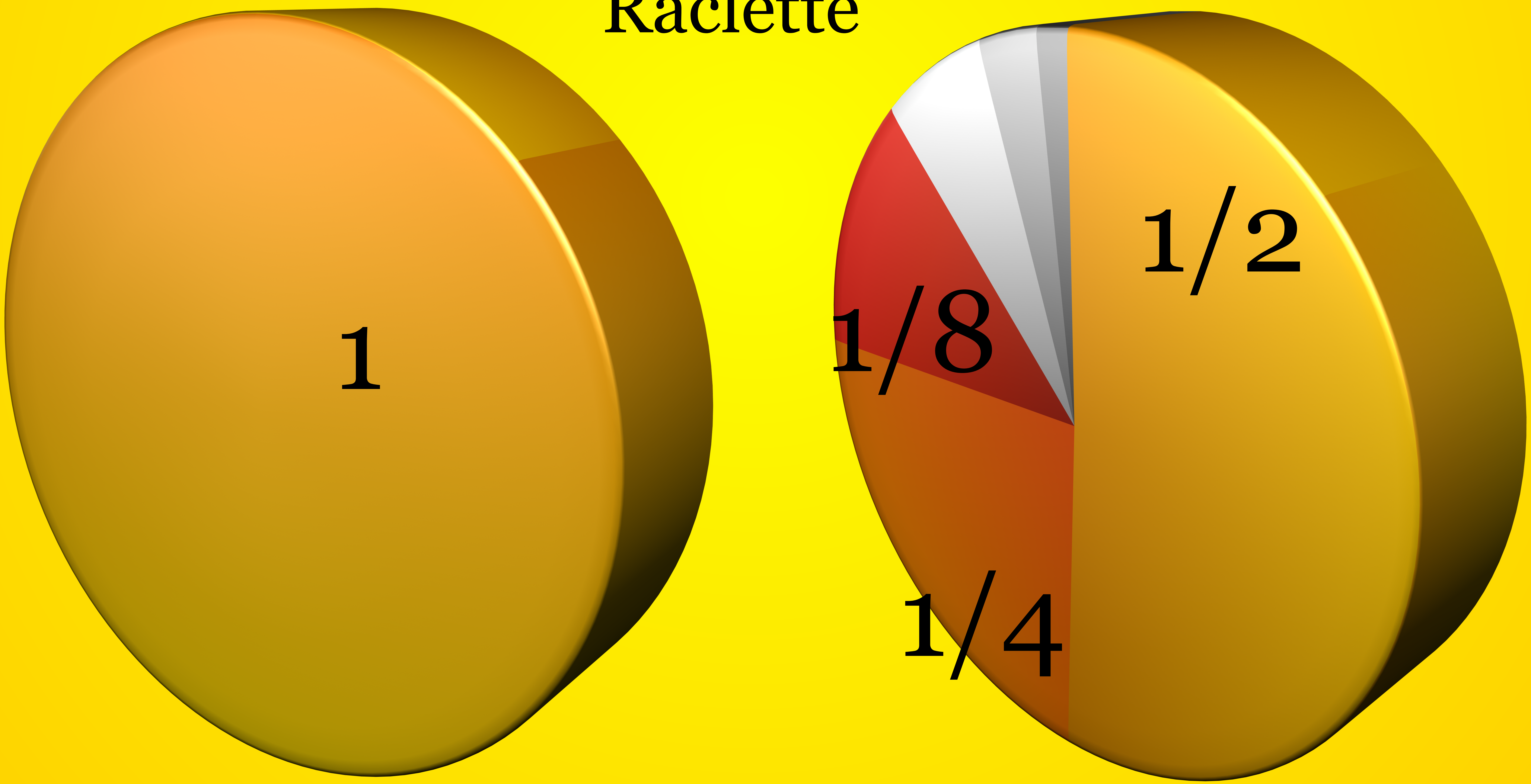
$$1 + x + x^2 + x^3 + \ldots\ldots = \frac{1}{1-x}$$

When do we have equality?

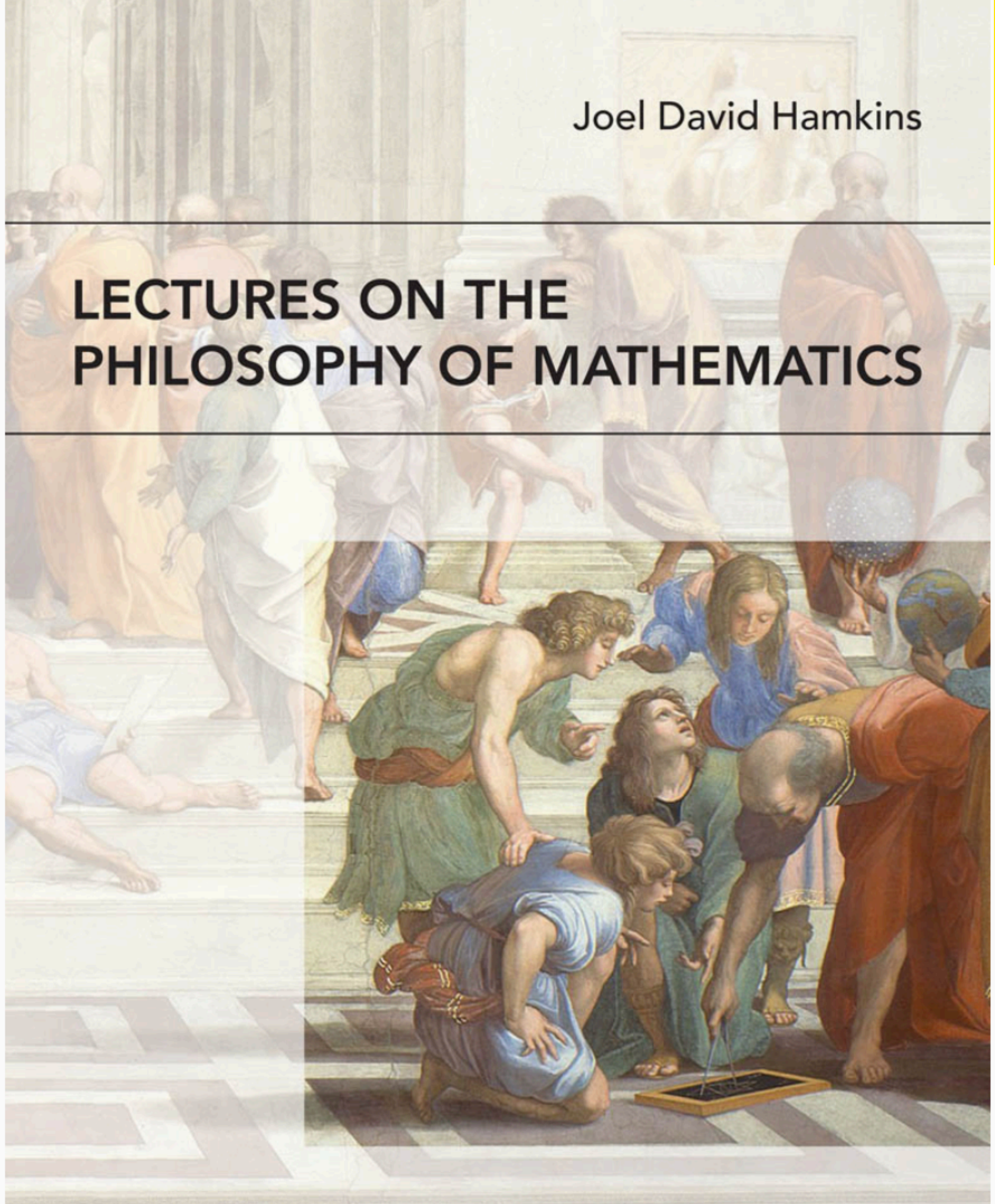
Example

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 2$$

Raclette



Unit 29
Trig Substitution
Plan:
1) Square roots
2) Area of circle
3) General recipe
4) Examples
5) Lunes
5. Raclette
x²+(y−1)²=4
x²+(y+1)²=4
y = √(4−x²) −1
B = 4π/3
A = √3
(2B−A)
2 ∫ √(4−x²) −1 dx
= 2 ∫ 4cos²u du − 4√3
x = 2 sin u
dx = 2 cos u du
√(4−x²) = 2 cos u
2 sin u



From Hemkins

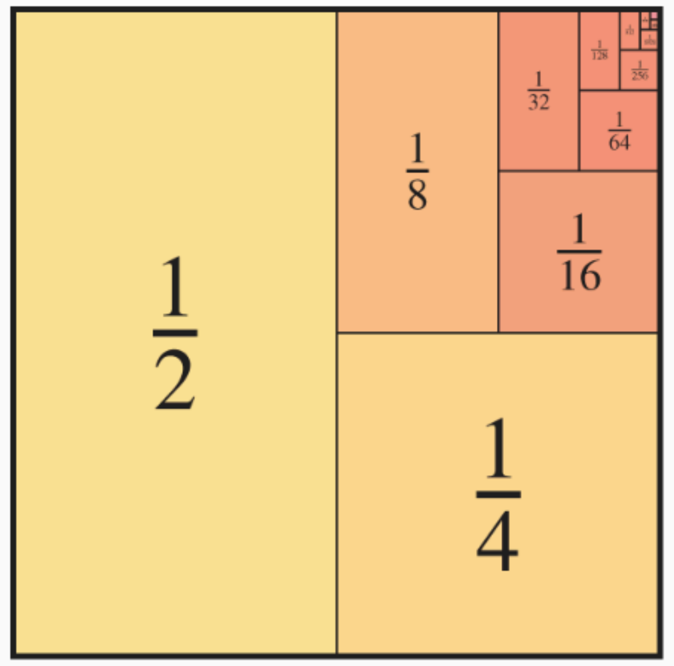
viewing infinite sets as an ordinary part of the mathematical landscape. But let us briefly mention some other older perspectives on infinity.

Zeno of Elea (c. 490–430 BC) argued that all motion is impossible because before you move from here to there, you must get halfway, but before you do this, you must get halfway to the halfway point, and before you do *this*, you must get halfway to that point, and so on, ad infinitum. Thus, he concluded, you can never begin. Similarly, Achilles can never overtake the tortoise in their footrace, because before doing so, he must get to where the tortoise was, but during this time, the tortoise has moved on, and when Achilles reaches *that* point, the tortoise has moved on yet again, ad infinitum. So Achilles can never catch the tortoise. What do you think of these paradoxical arguments?

Many mathematicians take them to be satisfactorily resolved by the ideas of calculus. The integral calculus, in particular, is founded on the idea that one may calculate a finite area by dividing it into successively tiny rectangles and taking a limit of those resulting areas. Similarly, the standard limit analysis of infinite series says, directly in opposition to Zeno, that one can add up an infinite series of numbers, such as

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

and nevertheless achieve a finite sum. In this case, the series sums to 1, a fact that



can be visualized with the help of the diagram here. The whole unit square has area one. This area is partitioned into smaller regions of area $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and so on. Since those regions exhaust the square, it follows that the sum of the series is 1.

Actual versus potential infinity

Another historical idea concerning infinity is Aristotle's distinction between actual and merely potential infinity. In ancient times, potentialists predominated, but now most mathematicians are actualists. The *potentialist* regards infinite collections as infinite only in a sense of unending potentiality. One can have more and more members of an infinite collection, as many (finitely many) as one likes, but it is not sensible to have them as a completed infinite totality. There are infinitely many natural numbers, in that one will never run out of them; we can never have all of them. The *actualist*, in contrast,

Grandi's Series

$$S = 1 - 1 + 1 - 1 + 1 - 1\ldots$$

for x=-1

$$S = 1 + 1 + 1 + 1 + \ldots.$$

for x=1

Cases which do not converge!

Luigi Grandi 1671 – 1742

Finite Formula

From

$$S_n = 1 + x + x^2 + x^3 + \ldots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

we see that the partial sums converge if $|x|<1$.

Other notation

$$S = a + ar + ar^2 + ar^3 + \ldots. = \frac{a}{1 - r}$$

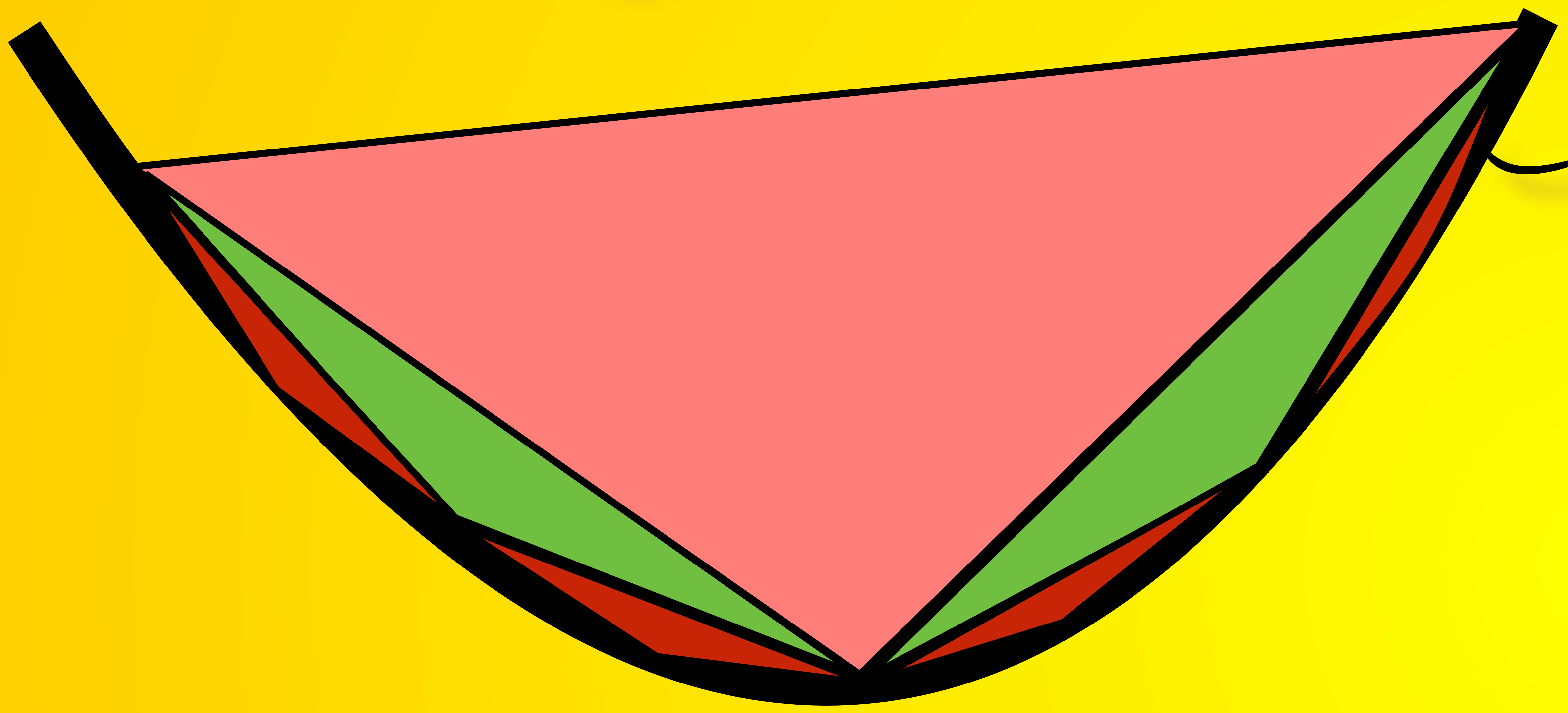
this is the same thing just using r instead of x and by having a constant a. We only need to know r and a to get the result.

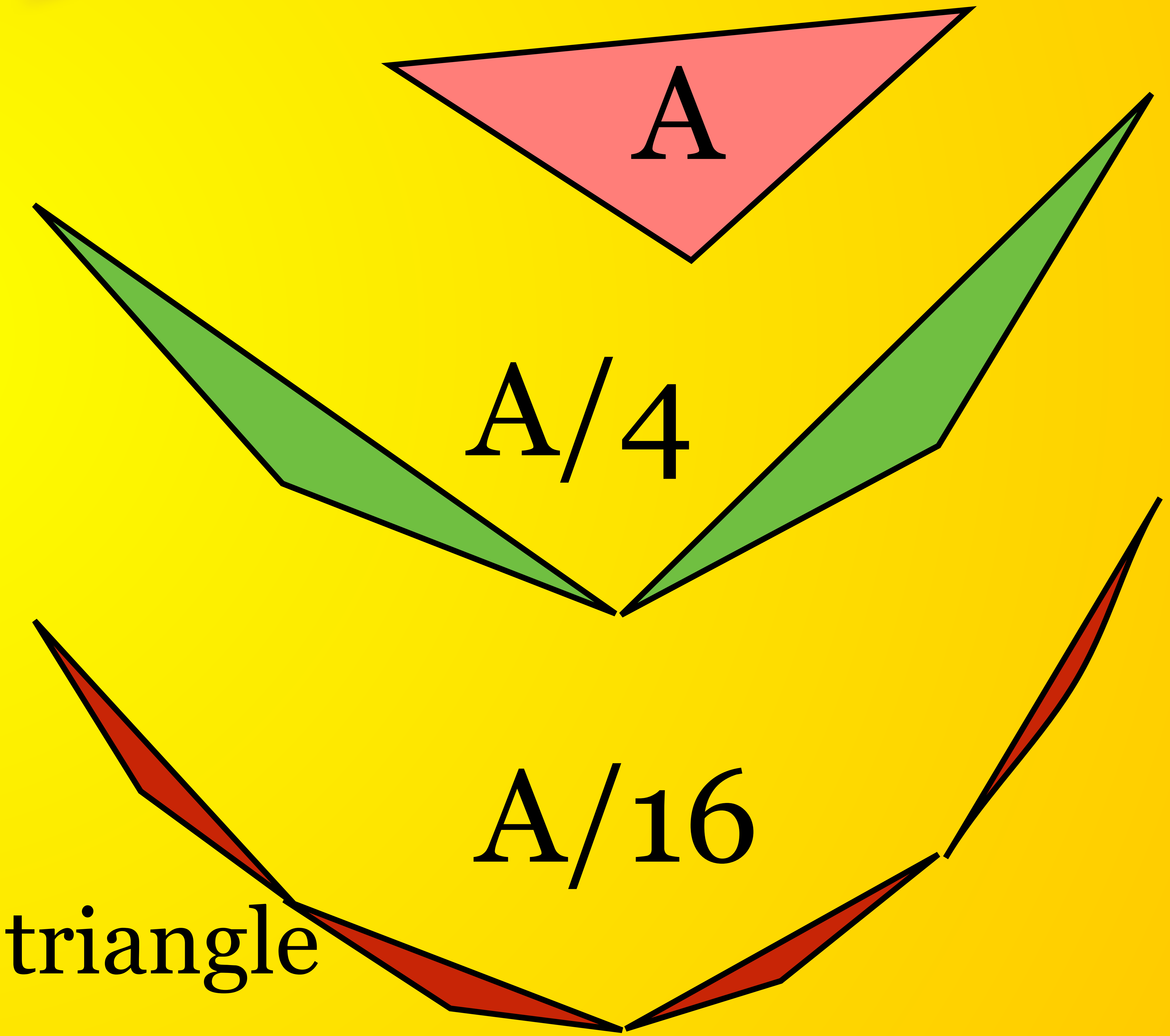
Applications and History

Zeno Paradox

the tortoise runs x times slower than Archilles

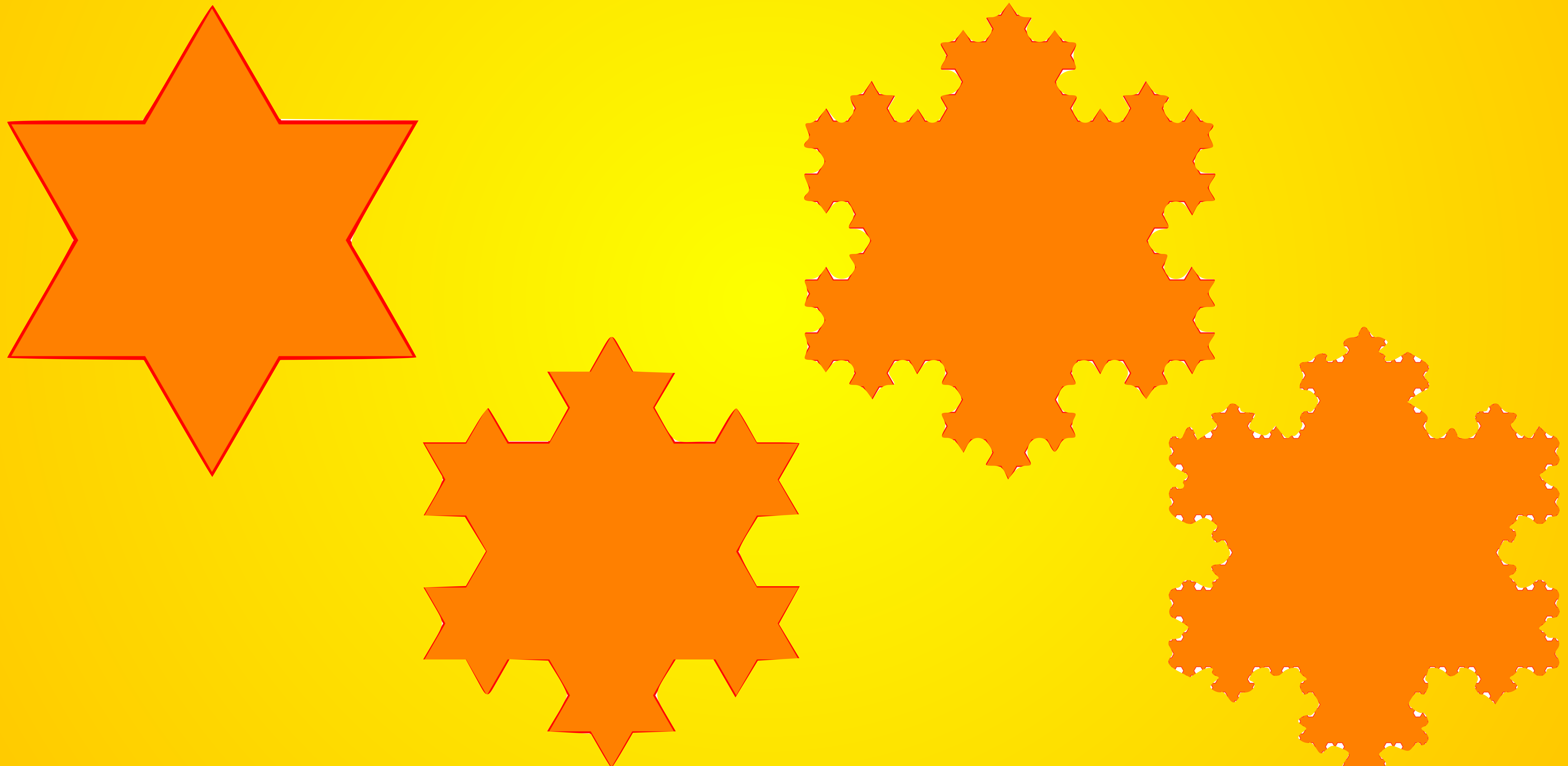
Quadrature of parabola





The area bound by the parabola and the segment is 1/(1-x)=4/3 times the area of the triangle

Fractals



Worksheet

Reminders

The End