8/30/2021 near Mather house

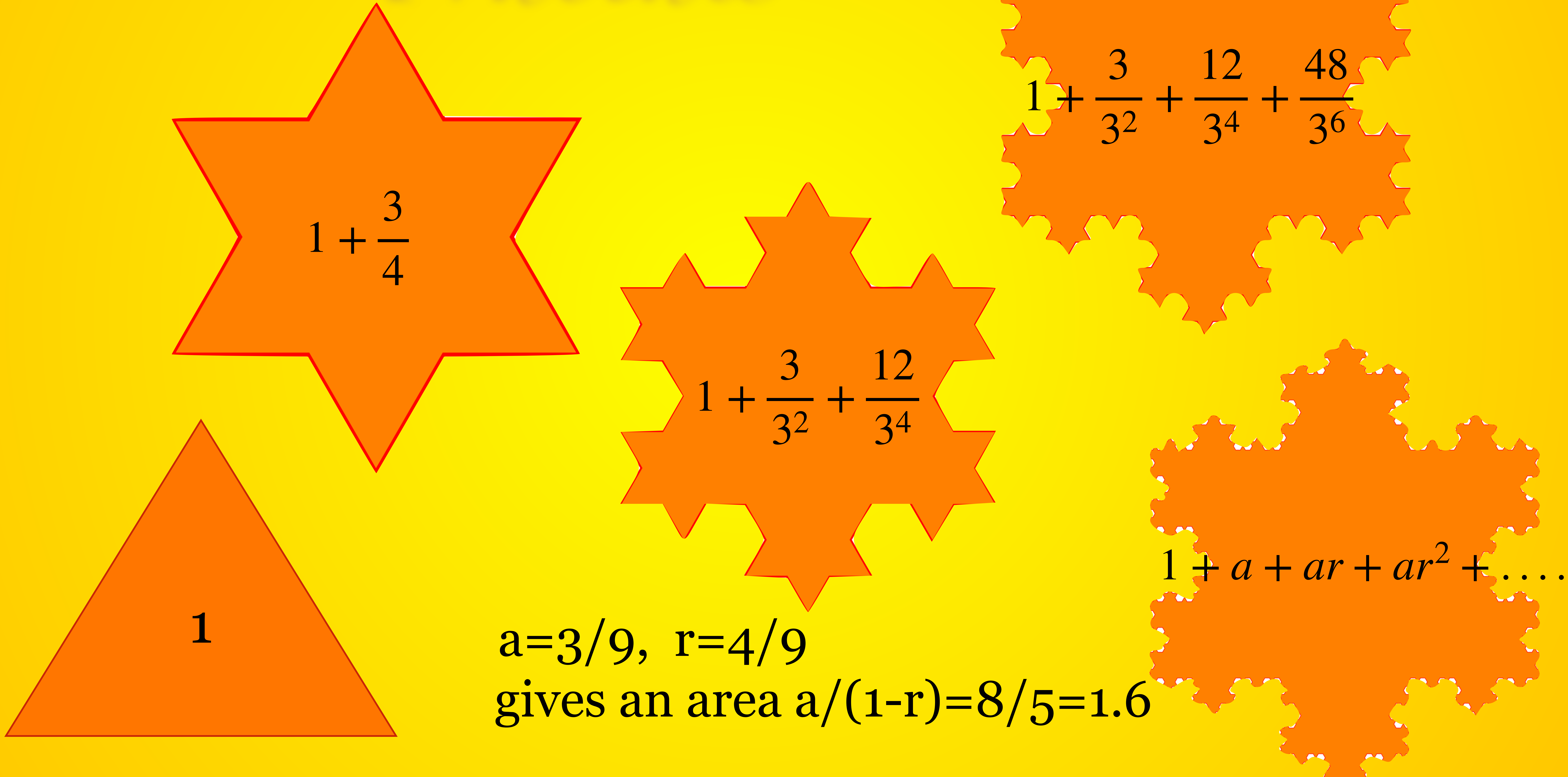
Lecture 18

10/17/2021

p-series and integrals

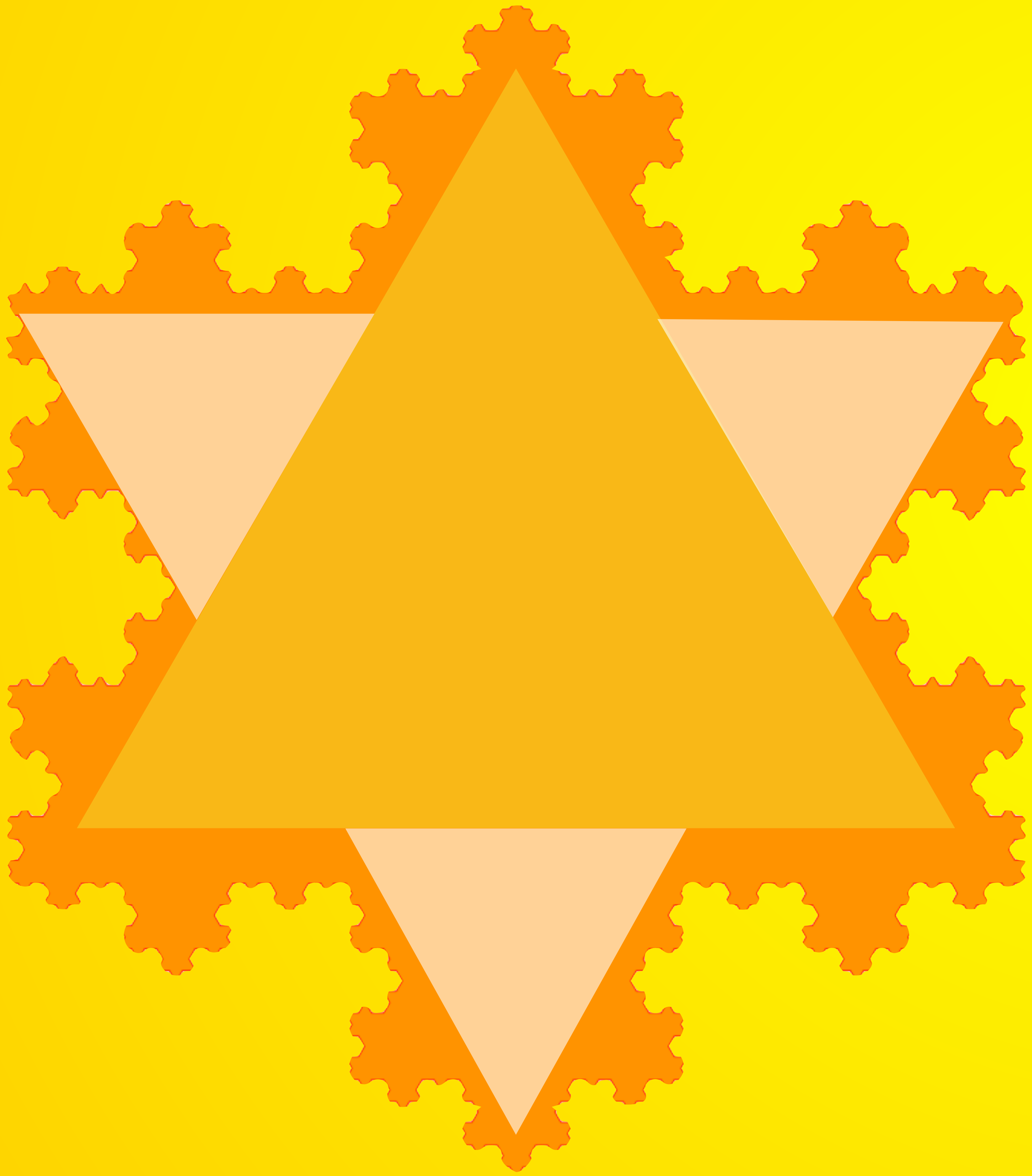
Table of Contents

Fractals



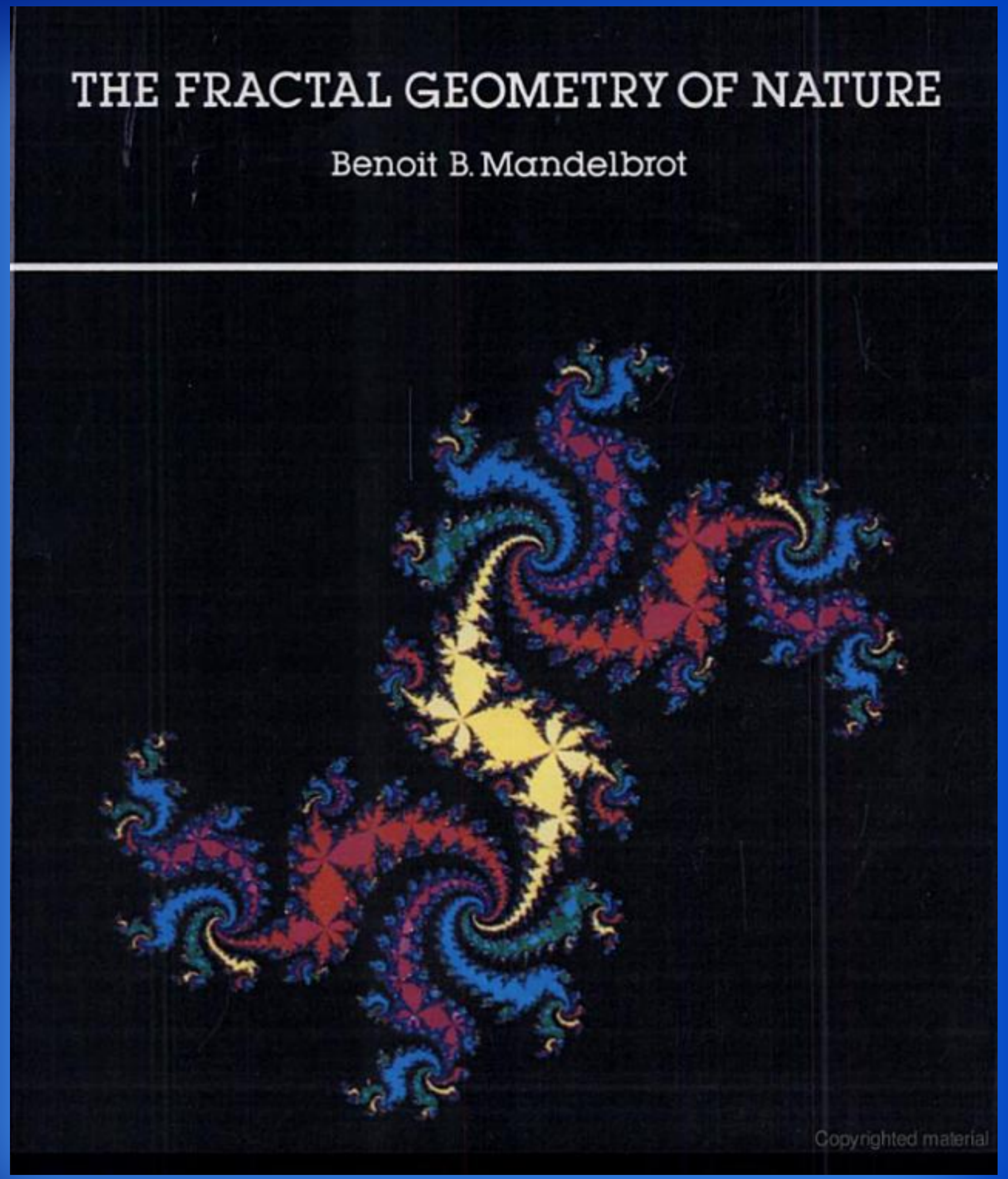
a=3/9, r=4/9
gives an area a/(1-r)=8/5=1.6

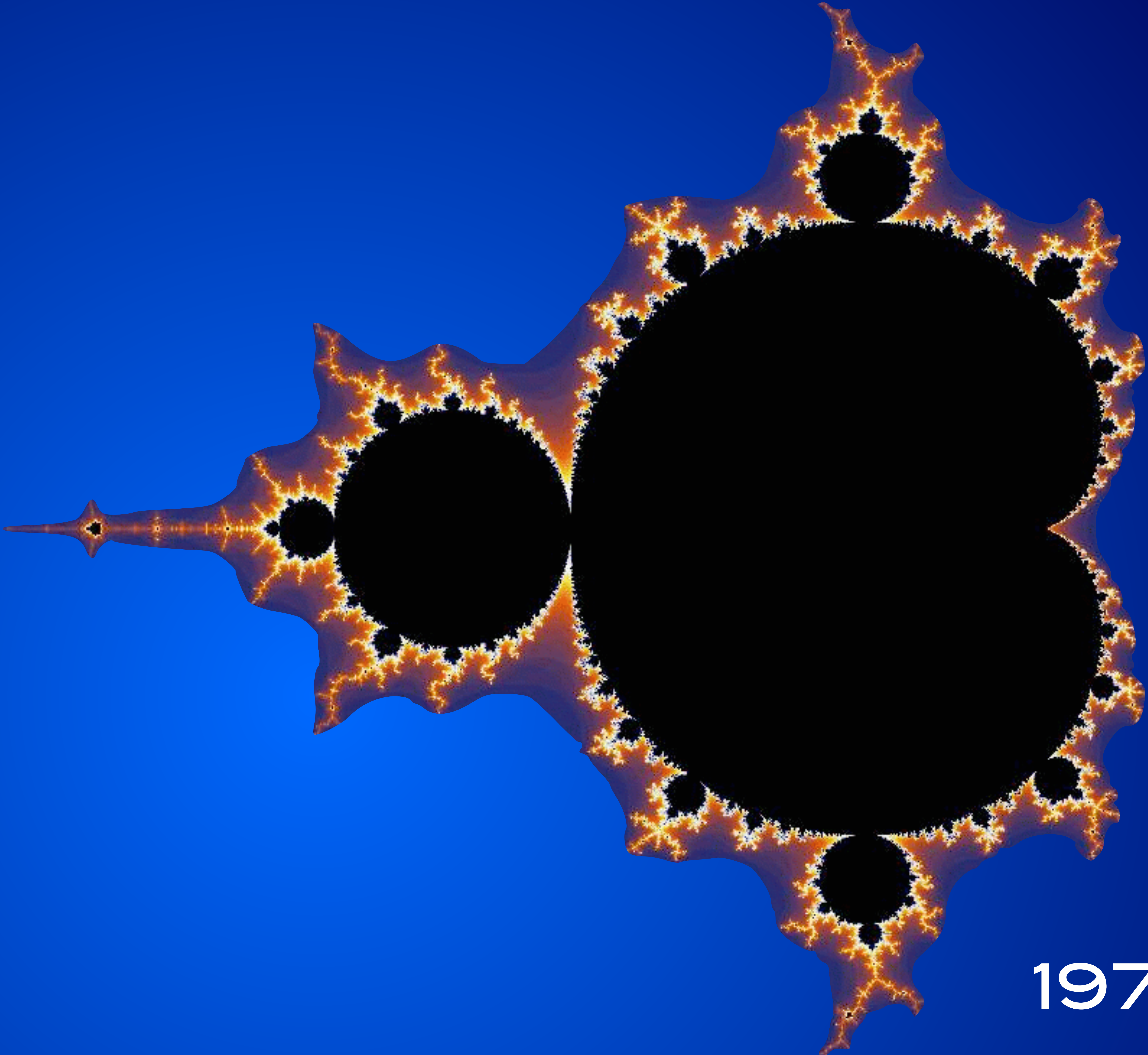
Koch Snowflake



1904

Helge von Koch,
1870-1924


THE FRACTAL GEOMETRY OF NATURE
Benoit B. Mandelbrot
Copyrighted material



1975

Mandelbulb 3D

$$z \rightarrow z^n + c$$

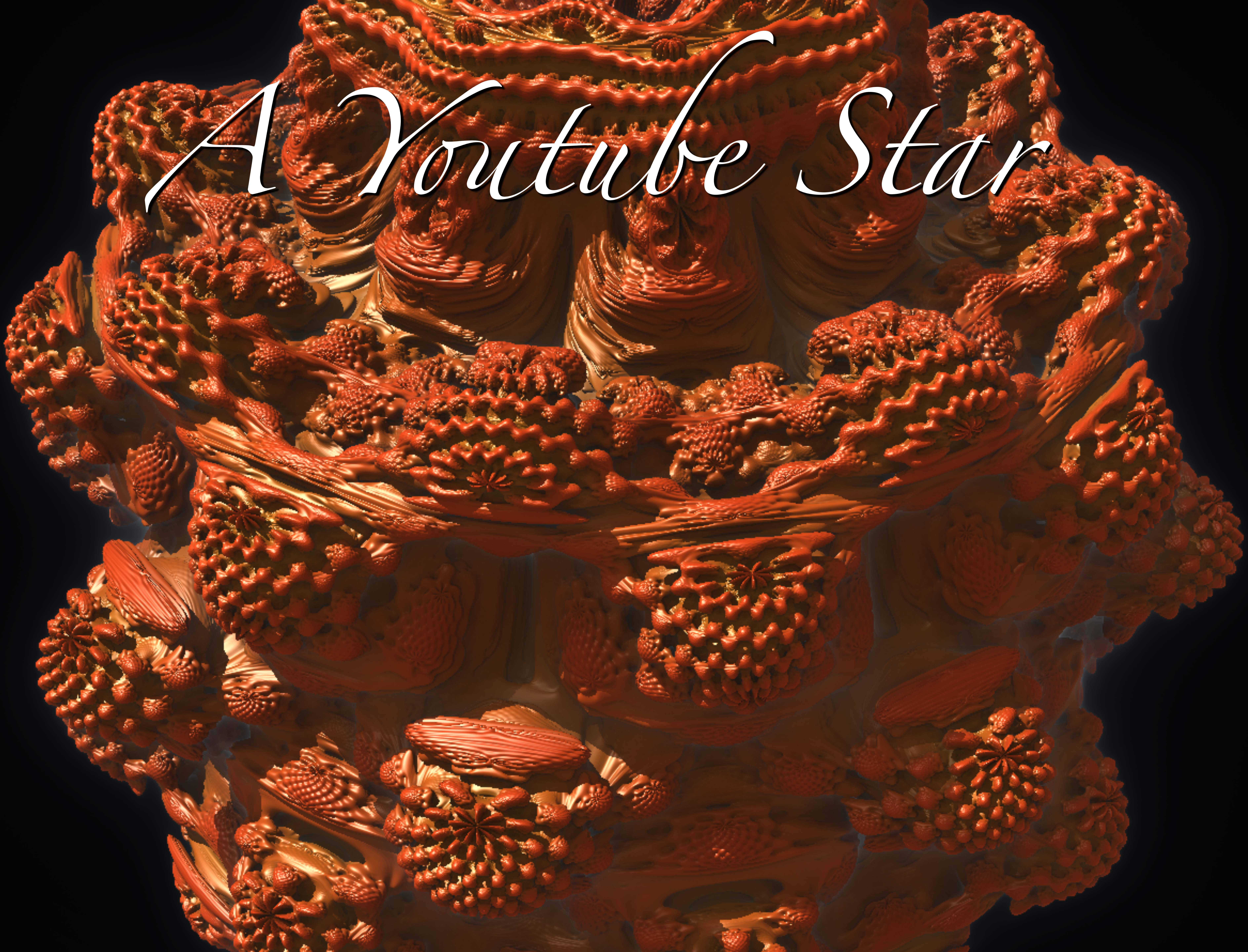
IN SPHERICAL COORDINATES.

DANIEL WHITE 2007
PAUL NYLANDER 2009
INSPIRED BY
RUDY RUCKER

Paul Nylander

Rudy Rucker, 1946-

Daniel White


A Youtube Star


Russ McClay, Youtube 2015

p- Series

$$S = 1/1^p + 1/2^p + 1/3^p + \ldots$$

is called the p- series

$$S = \sum_{k=1}^{\infty} k^{-p}$$

is the sum notation

Example

The p=1 series is the Harmonic series:

$$S = 1 + 1/2 + 1/3 + 1/4 + \ldots$$

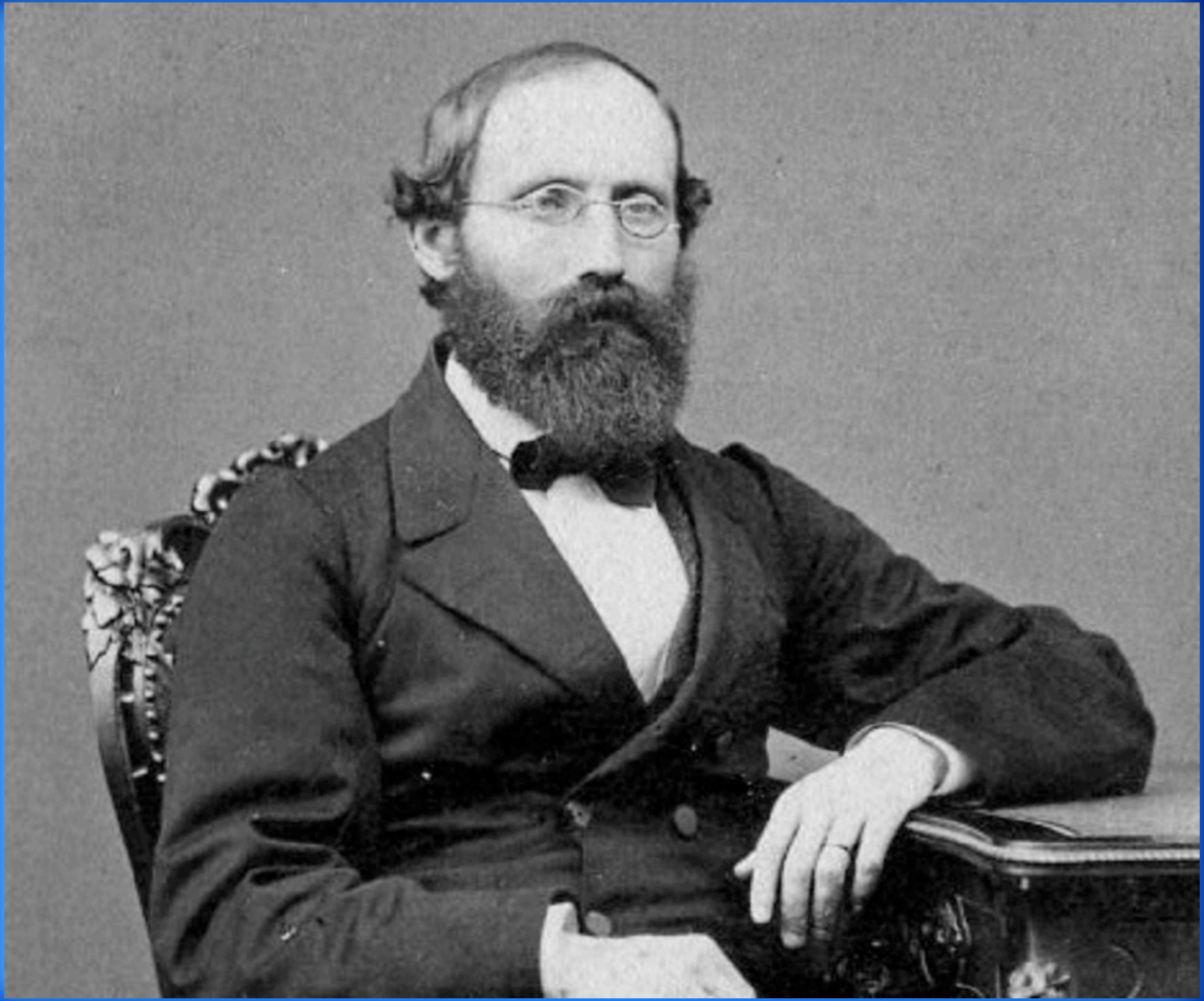
It diverges.

Riemann Hypothesis

$$\zeta(s)=\sum_{n=1}^{\infty} n^{-s}$$ has all roots on $Re(s)=1/2$

The roots

100
98
96
94
92
90
88
86
84
82
80
78
76
74
72
70
68
66
64
62
60
58
56
54
52
50
48
46
44
42
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2



Bernhard Riemann, 1826-1866

Example

$$S = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} = \frac{\pi^2}{6}$$

is the p=2 series. It is called the Basel problem.

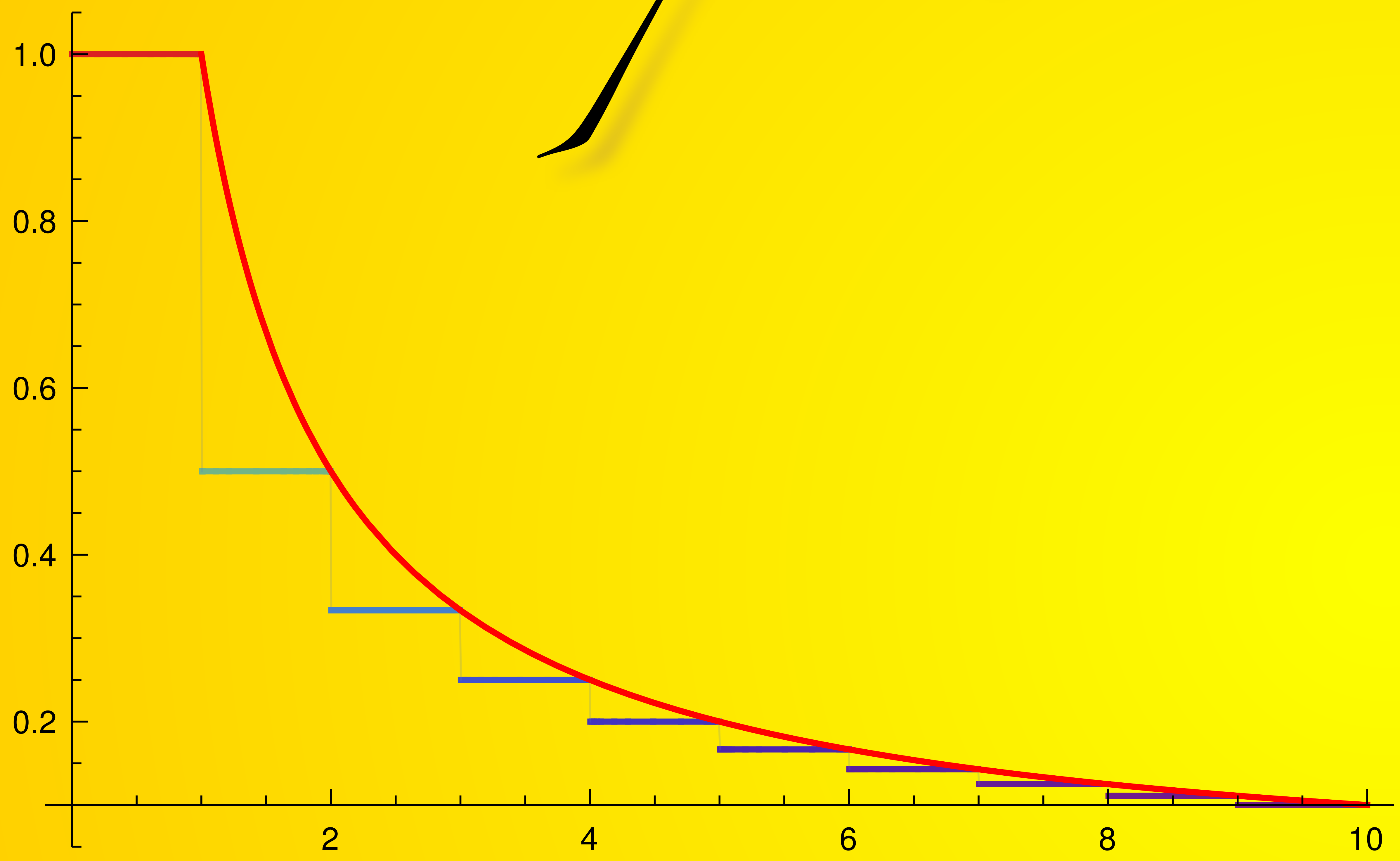
Compare with integral

$$S = 1/1^2 + 1/2^2 + 1/3^2 + \ldots$$

$$\leq 1 + \int_1^\infty \frac{1}{x^2}\, dx = 2$$

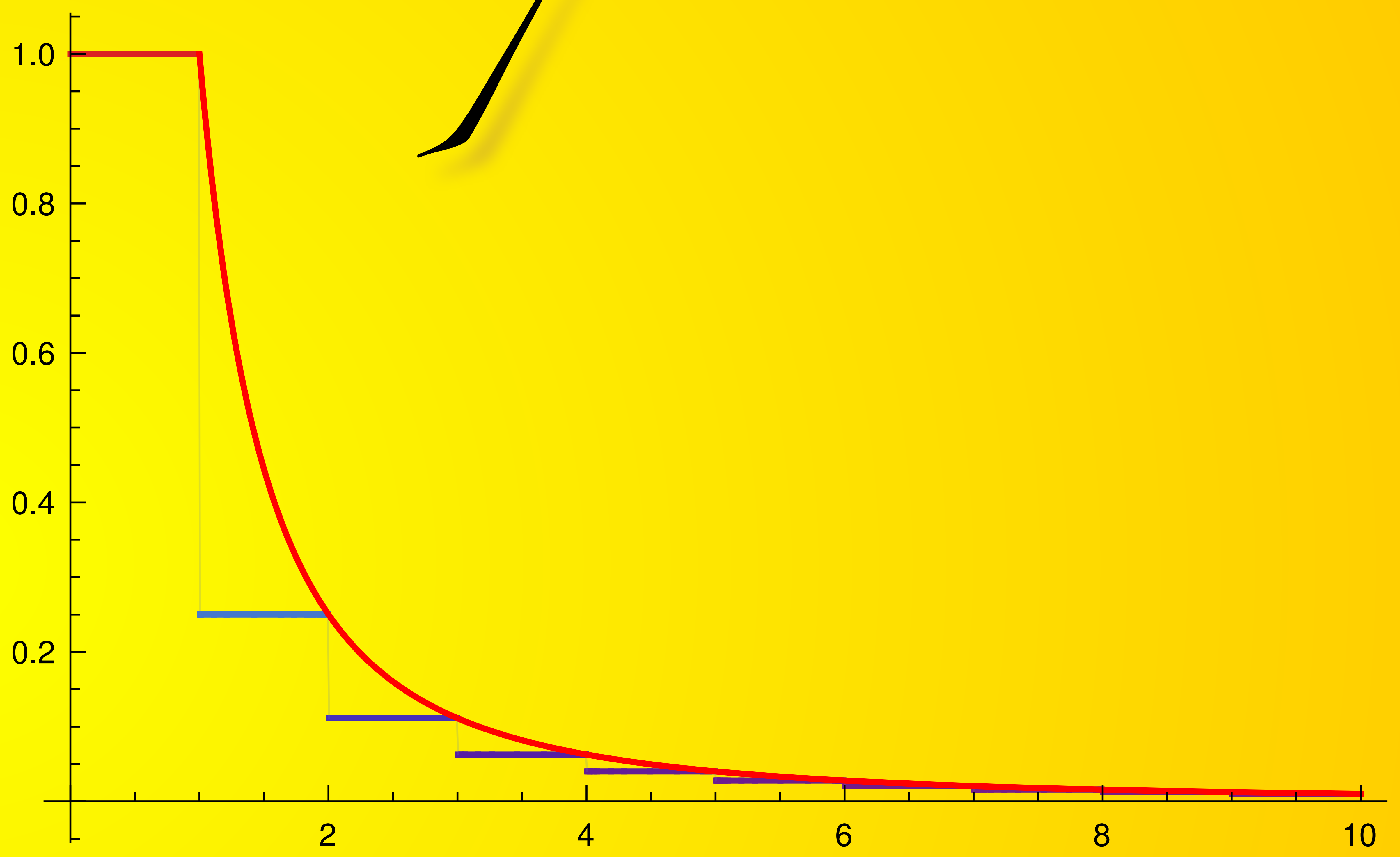
The exact sum is $S = \pi^2/6 = 1.6449\ldots$

$p = 1$



diverges

$p = 2$



converges

Worksheet

The End