8/30/2021 near Mather house

Lecture 19

10/20/2021

Asymptotics

Table of Contents

1) Growth rates and P-NP

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Notation

$$a_k \sim b_k \quad \text{if} \quad \frac{a_k}{b_k} \to 1$$

$$a_k \ll b_k \quad \text{if} \quad \frac{a_k}{b_k} \to 0$$

Examples

$$3k^5 + 2k^3 + 5k \sim 3k^5$$

$$k^2 \ll 2^k$$

$$k! \ll k^k$$

$$\ln(k) \ll \sqrt{k}$$

$$e^{k^2} + k^2 \sim e^{k^2}$$

True or False?

$$e^{k^2+k} \sim e^{k^2}$$

HOSPITAL

Guillaume de l'Hospital

The origin of L'Hôpital's rule

by D. J. Struik, Massachusetts Institute of Technology, Cambridge, Massachusetts

The so-called rule of L'Hôpital, which states that

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{f'(a)}{g'(a)}$$

when $f(a)=g(a)=0$, $g'(a)\neq 0$, was published for the first time by the French mathematician G. F. A. de l'Hôpital (or De Lhospital) in his *Analyse des infiniment petits* (Paris, 1696) [1].* The Marquis de l'Hôpital was an amateur mathematician who had become deeply interested in the new calculus presented to the learned world by Leibniz in two short papers, one of 1684 and the other of 1686. Not quite convinced that he could master the new and exciting branch of mathematics all by himself, L'Hôpital engaged, during some months of 1691–92, the services of the brilliant young Swiss physician and mathematician, Johann Bernoulli, first at his Paris home and later at his château in the

* Numerals in brackets refer to the notes at the end of this article.

Johann Bernoulli

"LITTLE O"

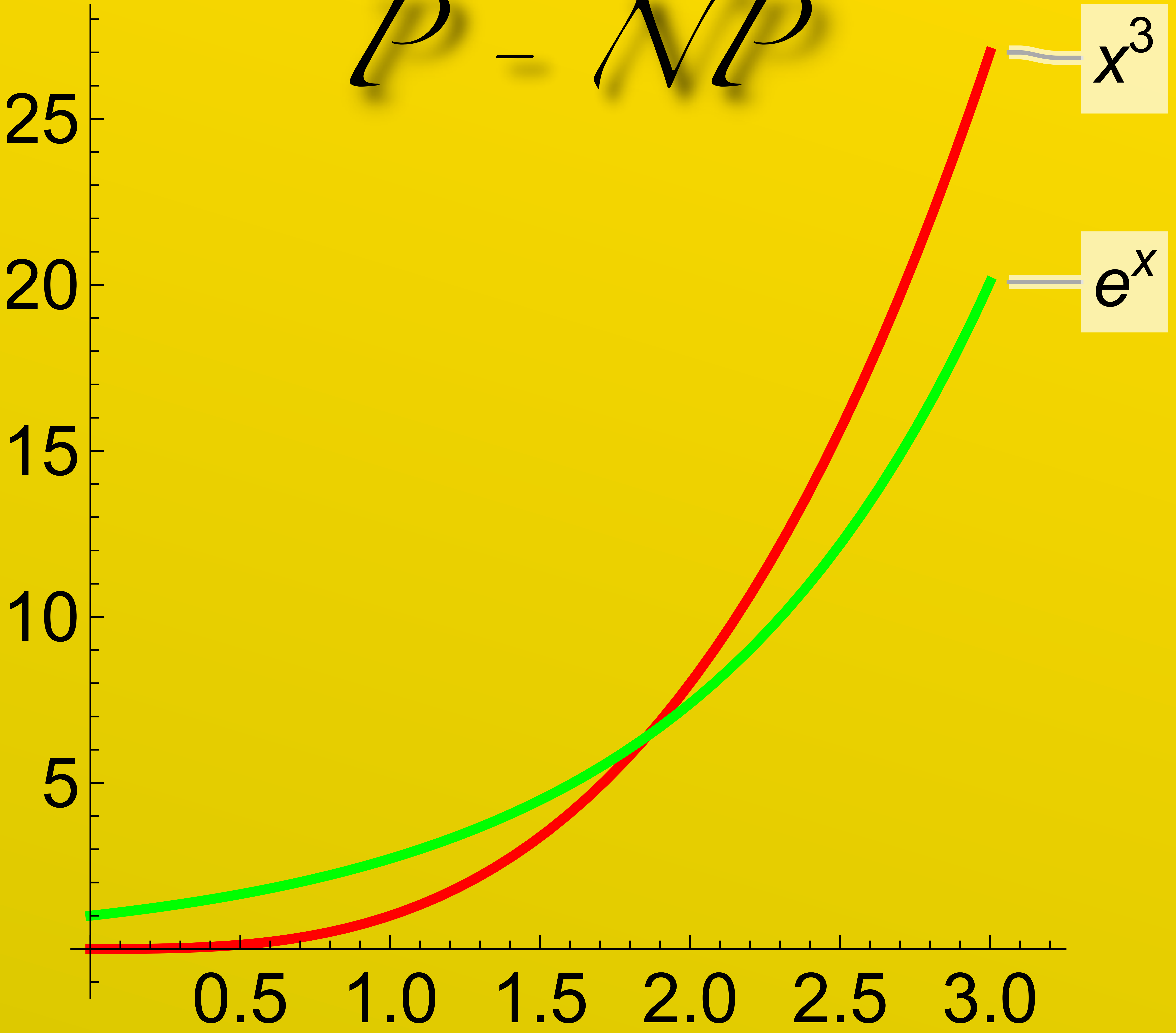
$$a_k \ll b_k$$

Landau's little o notation

$$a_k = o(b_k)$$

Oliver

$\mathcal{P}$ - $\mathcal{NP}$



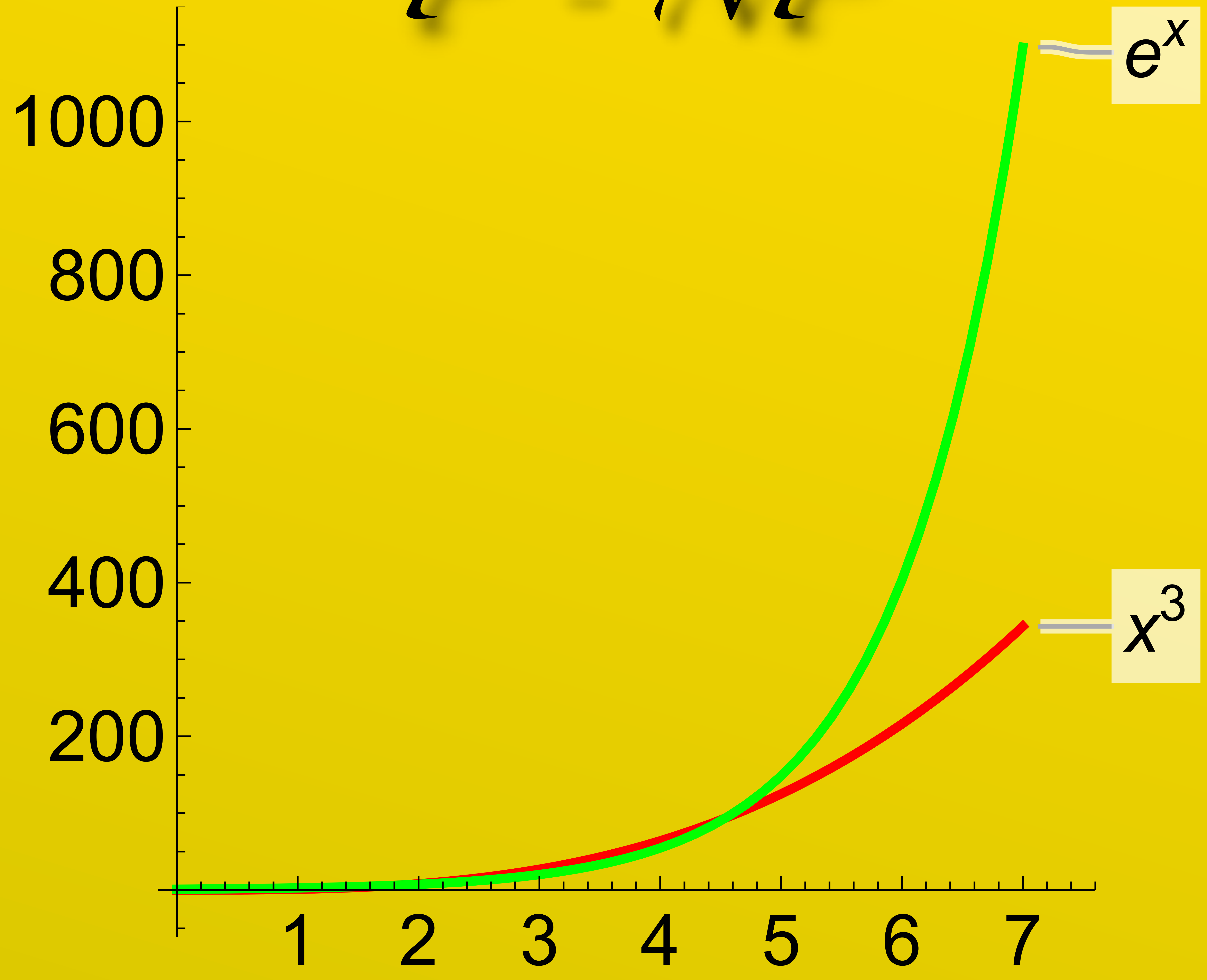
What grows faster?

$$e^x \quad >> \quad x^3$$

or

$$e^x \quad << \quad x^3$$

$\mathcal{P}$ - $\mathcal{NP}$



Use Hopital on

$$x^3/e^x$$

to see it!

$\mathcal{P}$ - $\mathcal{NP}$

polynomial exponential

$$r^k \ll k^r$$

for every positive r

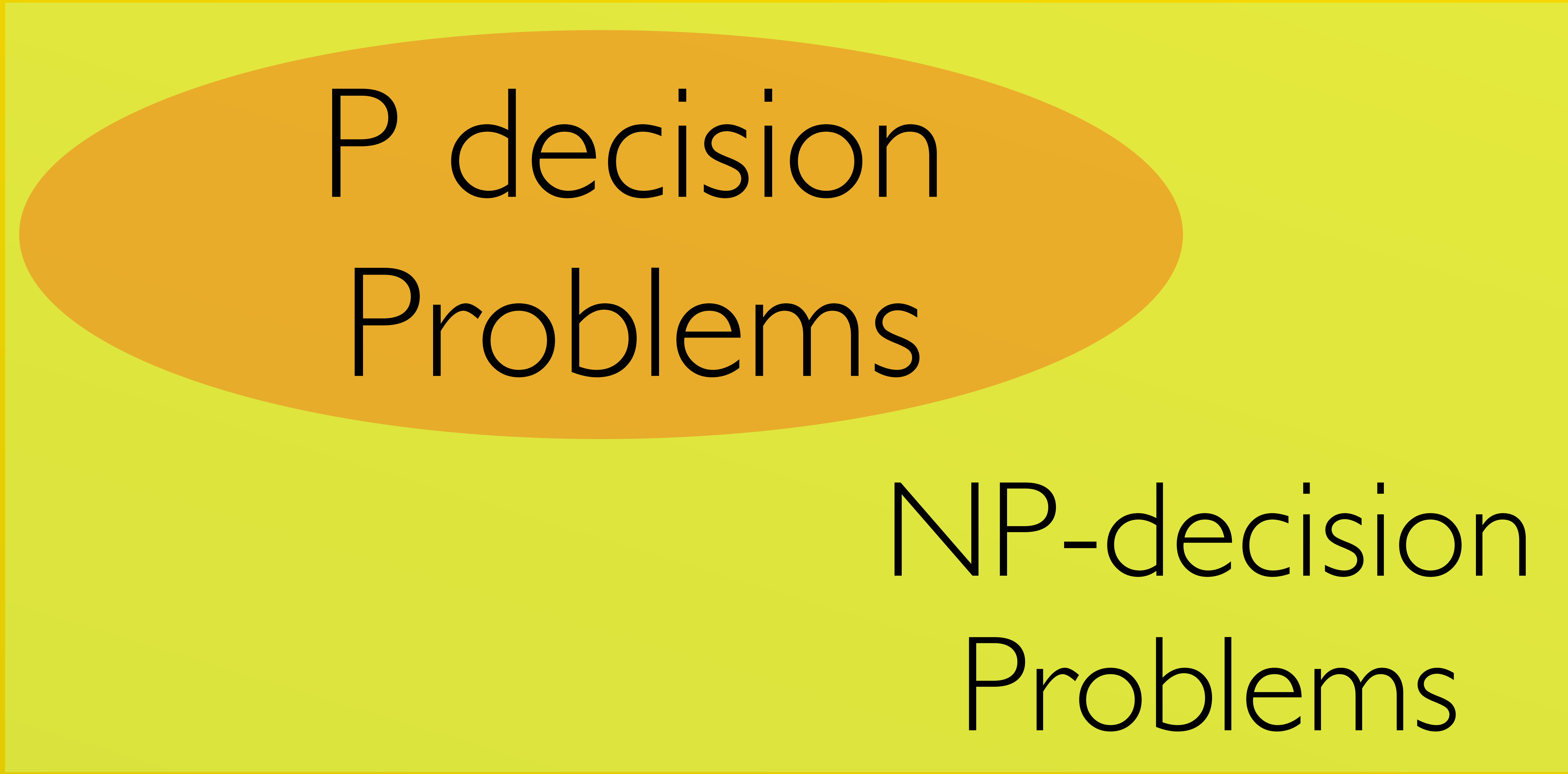
P - NP PROBLEM

P=NP?

Stephen Cook
1971

P: can be solved in polynomial time

NP: can be verified in polynomial time



P stands for for Polynomial
NP for Nondeterministic Polynomial

Travelling Salesman movie (2012)

LOG - ROOTS

$$\ln(k) \ll \sqrt{k}$$

Can you prove this with Hopital?

Clever:

replace k with k^2 $\qquad \ln(k^2) << k$

then apply exp on both sides $\qquad k^2 << e^k$

LOG-ROOTS

Logs grow slower than roots

COMPARE

What grows faster?

$$k + 10^{100}\ln(k)$$

$$k + 10^{100}\sqrt{k}$$

to find out compute

$$\lim_{k\to\infty} \frac{k + 10^{100}\ln(k)}{k + 10^{100}\sqrt{k}}$$

Worksheet

The End