



Lecture 20

10/22/2021

*Alternating
Series*

8/30/2021 near Mather house

Marzipan



Luebeck
in Germany

Zuerich
Oliver's
birth town

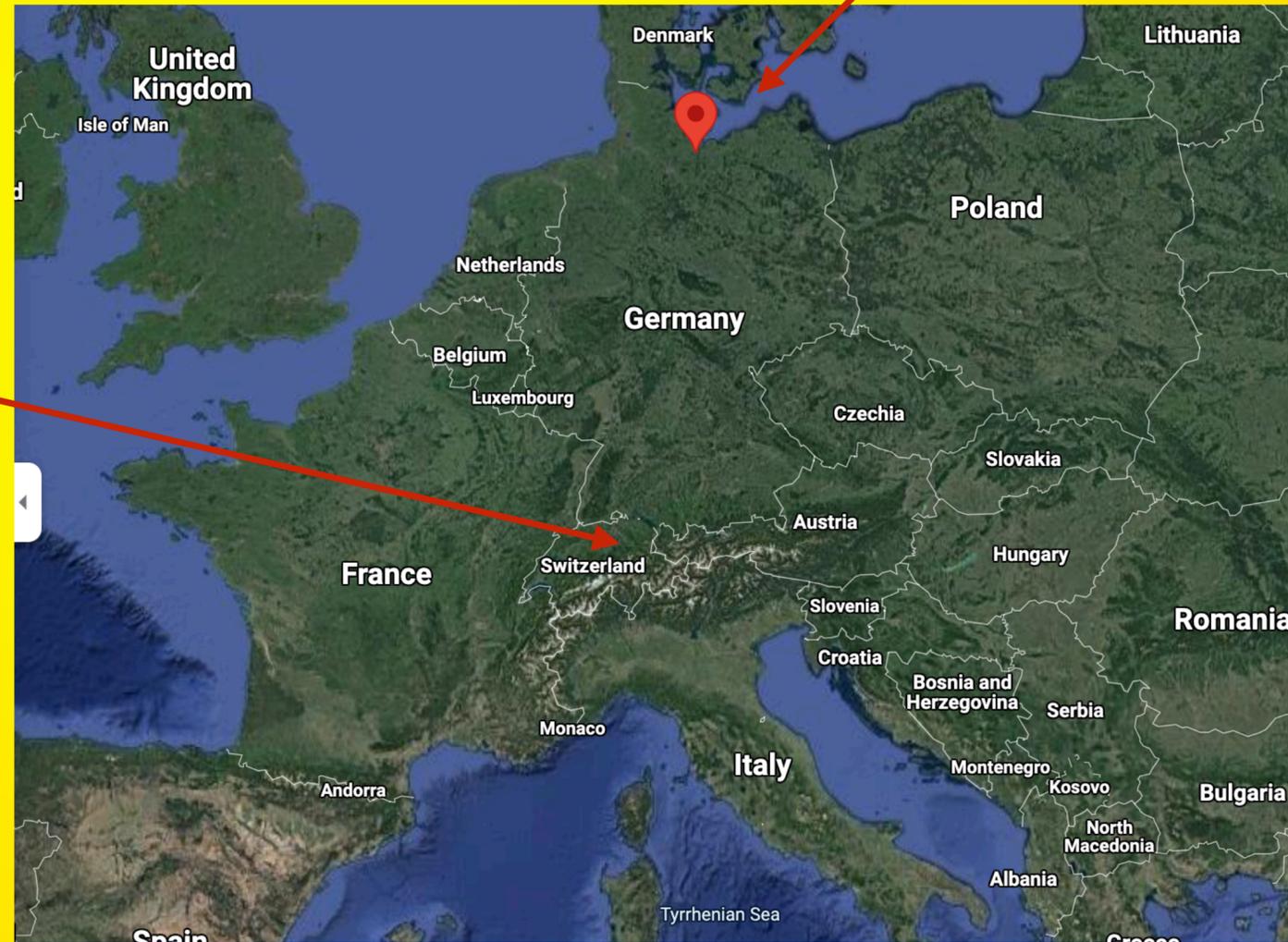


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Alternating Series

A series $\sum_k a_k$ is alternating if

$a_{k+1}a_k$ is negative for all k

Examples

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

Which are alternating?

$$\sum_{k=1}^{\infty} \cos(k\pi)$$

$$\sum_{k=1}^{\infty} \cos(k)$$

$$\sum_{k=1}^{\infty} \cos\left(\frac{k\pi}{2}\right)$$

$$\sum_{k=1}^{\infty} (-1)^{2k+1}$$

Which are alternating?

Taylor series
of $\sin(x)$ at $c=0$
for $x=1$

Taylor series of
 $\cos(x)$ at $c=0$
for $x=1$

Taylor series
of $\exp(x)$ at $c=0$
for $x=1$

Taylor series of
 $\ln(1+x)$ at $c=0$
for $x=1$

Alternating Series Test

If $\sum_k a_k$ is alternating and $|a_k| \rightarrow 0$

monotonically, then $\sum_k a_k$ converges.

This is also called the Leibniz criterion.

Leibniz

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k+1}$$
$$= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$
$$= \frac{\pi}{4}$$

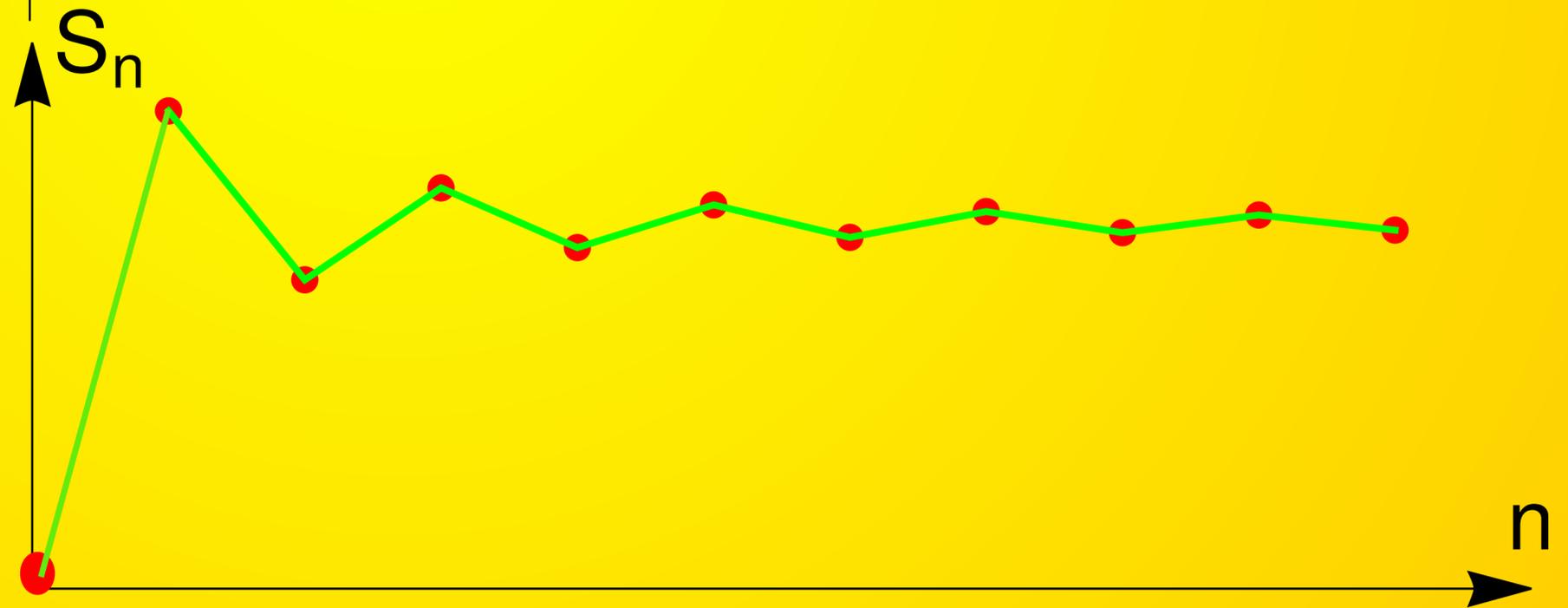


GODEFROI GUILLAUME
LEIBNITZ,
Né le 3 Juillet 1646 mort le 14 Novembre 1716.

Sequence



Sum



Which do converge?

$$\sum_{k=1}^{\infty} \frac{\cos(k\pi)}{\ln(k+1)}$$

$$\sum_{k=1}^{\infty} (-1)^k$$

$$\sum_{k=1}^{\infty} \sin(k\pi/2)/k$$

$$\sum_{k=1}^{\infty} a_k \quad \begin{array}{l} a_{2k} = 1/k \\ a_{2k+1} = -1/2^k \end{array}$$

Which do converge absolutely?

$$\sum_{k=1}^{\infty} \frac{\cos(k\pi)}{\ln(k+1)}$$

$$\sum_{k=1}^{\infty} (-1)^k$$

$$\sum_{k=1}^{\infty} \sin(k\pi/2)/k$$

$$\sum_{k=1}^{\infty} a_k \quad \begin{array}{l} a_{2k} = 1/k \\ a_{2k+1} = -1/2^k \end{array}$$

Error bound

If $S = \sum_k a_k$ is alternating and $|a_k| \rightarrow 0$ monotonically, then

$$|S_n - S| \leq |a_{n+1}|$$

The error is bound by the next unused term.

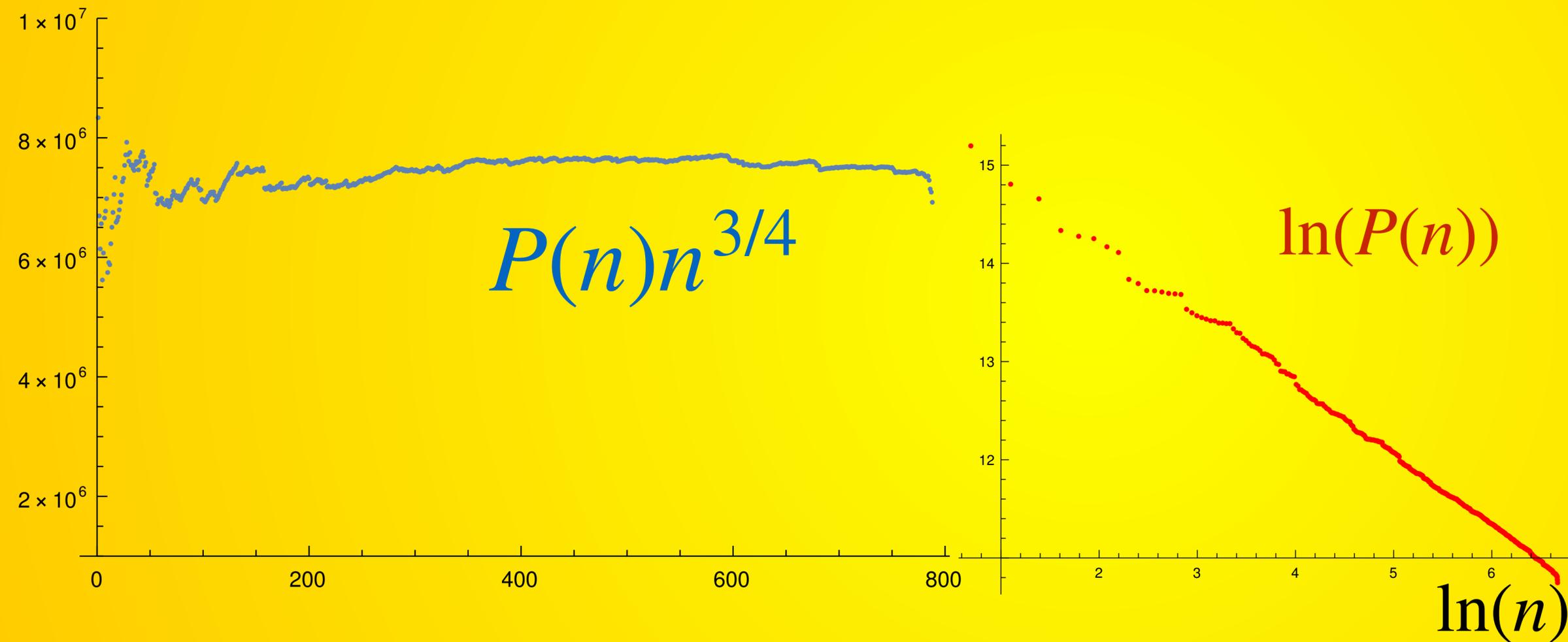
What is the error?

$$\sum_{k=1}^{1000} \frac{\cos(k\pi)}{k}$$

$$\sum_{k=1}^{10} \frac{(-1)^k}{k^2}$$

Worksheet

QRD Reminder



George Zipf
1902-1950

Zipf was a linguist who investigated the frequency of a word in dependence of its rank.

Zipf's law

The End