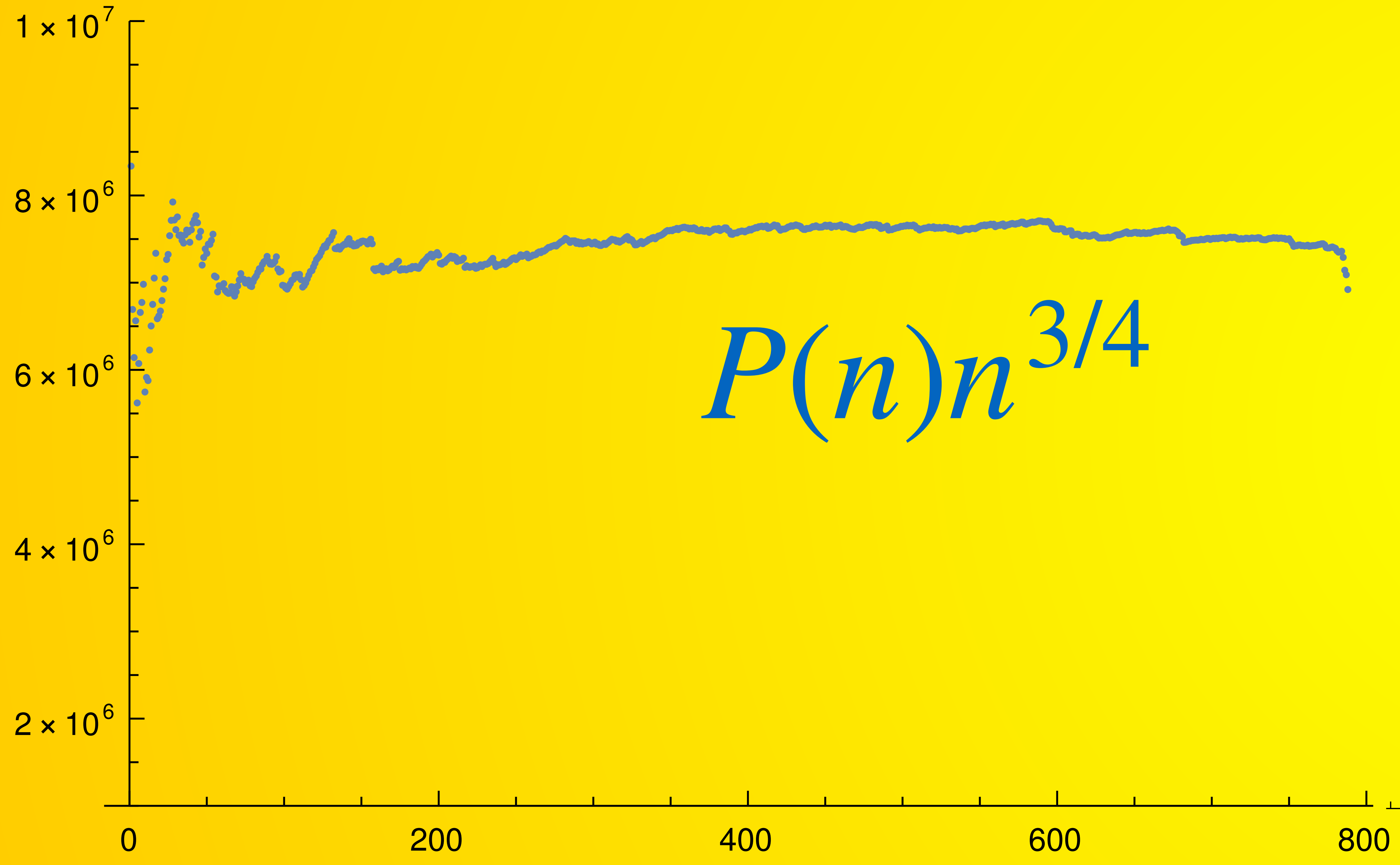
8/30/2021 near Mather house

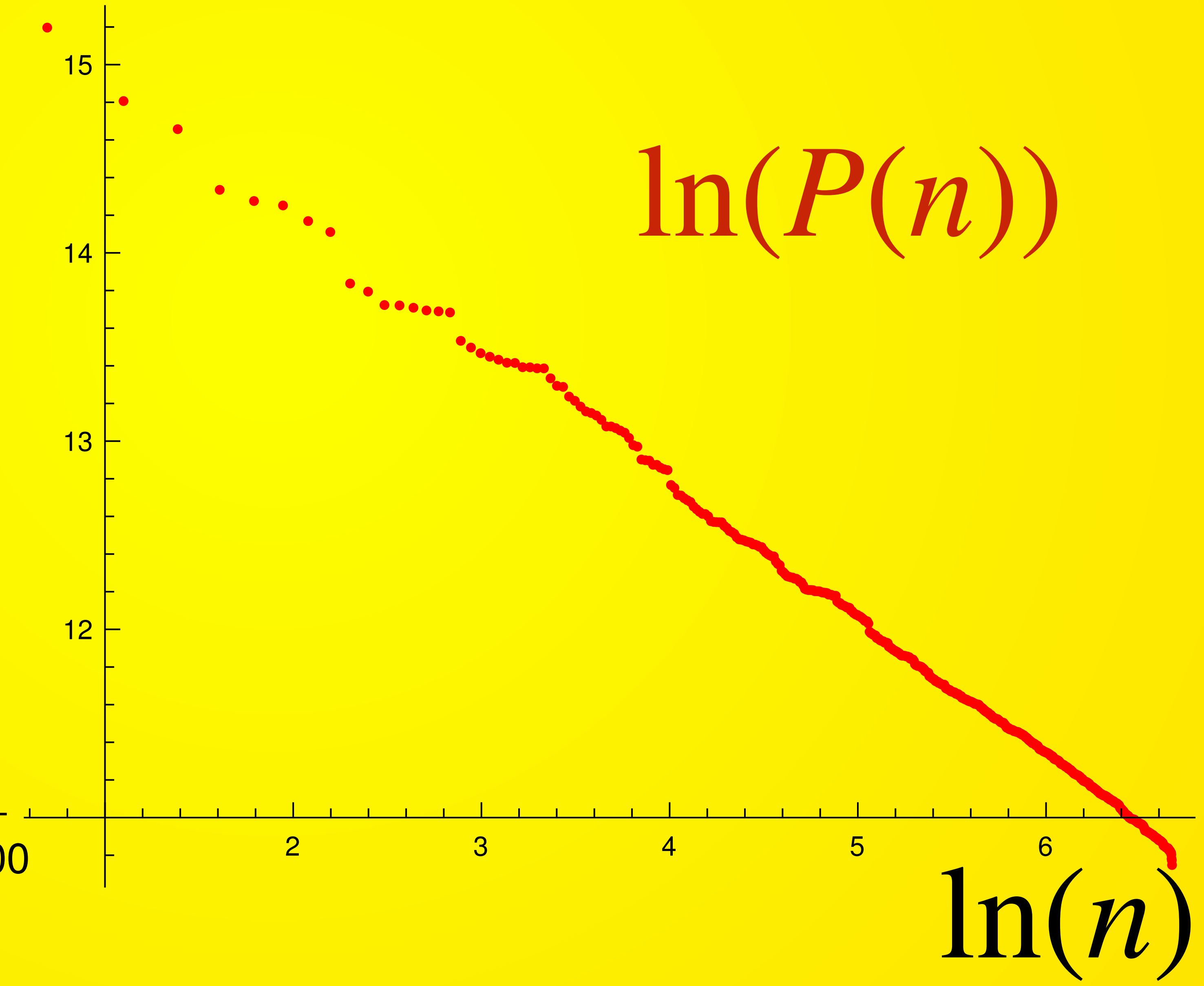
Lecture 22

10/26/2021

Power Series

QRD Reminder





George Zipf
1902-1950

Zipf was a linguist who investigated the frequency of a word in dependence of its rank.

Zipf's law

Table of Contents

Power Series

A series

$$S = \sum_{k=0}^{\infty} a_k(x-c)^k$$

is called a **power series**

Power series

Example:

$$\sum_k kx^k$$

In this case, $a_k = k$

Example: Taylor series

Examples are Taylor series where

$$a_k = \frac{f^{(k)}(c)}{k!}$$

Power Series or not?

$$\sum_{k=1}^{\infty} \frac{k}{2^k}(x-1)^k$$

$$\sum_{k=1}^{\infty} k^2(2-x)^k/k!$$

$$\sum_{k=1}^{\infty} k(x+5)^{1-k}$$

$$\sum_{k=1}^{\infty} \ln(k)(6-x)^{k+1}$$

Radius of Convergence

Interval of convergence: largest (c-R,c+R) on which the series converges.

R is called radius of convergence:

$$R = \lim_{k\to\infty} \frac{|a_k|}{|a_{k+1}|}$$ if the limit exists

Problem A

What is the Taylor series of $5/(1+x)$ centered at x=0?

What is the radius of convergence at x=0

Problem B

What is the Taylor series of $\sin(x^2)/x$ centered at x=0?

What is the radius of convergence at x=0

Problem C

What is the Taylor series of $x^5/(1 - x^4)$ centered at x=0?

What is the radius of convergence at x=0

Why ??

Use Calc

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = -\ln(1-x)$$

Power series in Stats

$$M_X(x) = \sum_n a_n x^n \qquad a_n = \frac{E[X^n]}{n!}$$

Moment generating function
of a random variable X.

$$M_X(x)M_Y(y) = M_{X+Y}(x)$$

for independent
X,Y

Coolest Power series ever:

$$\sum_{p \text{ prime}} x^p$$

$$= x^2 + x^3 + x^5 + x^7 + x^{11} + \ldots.$$

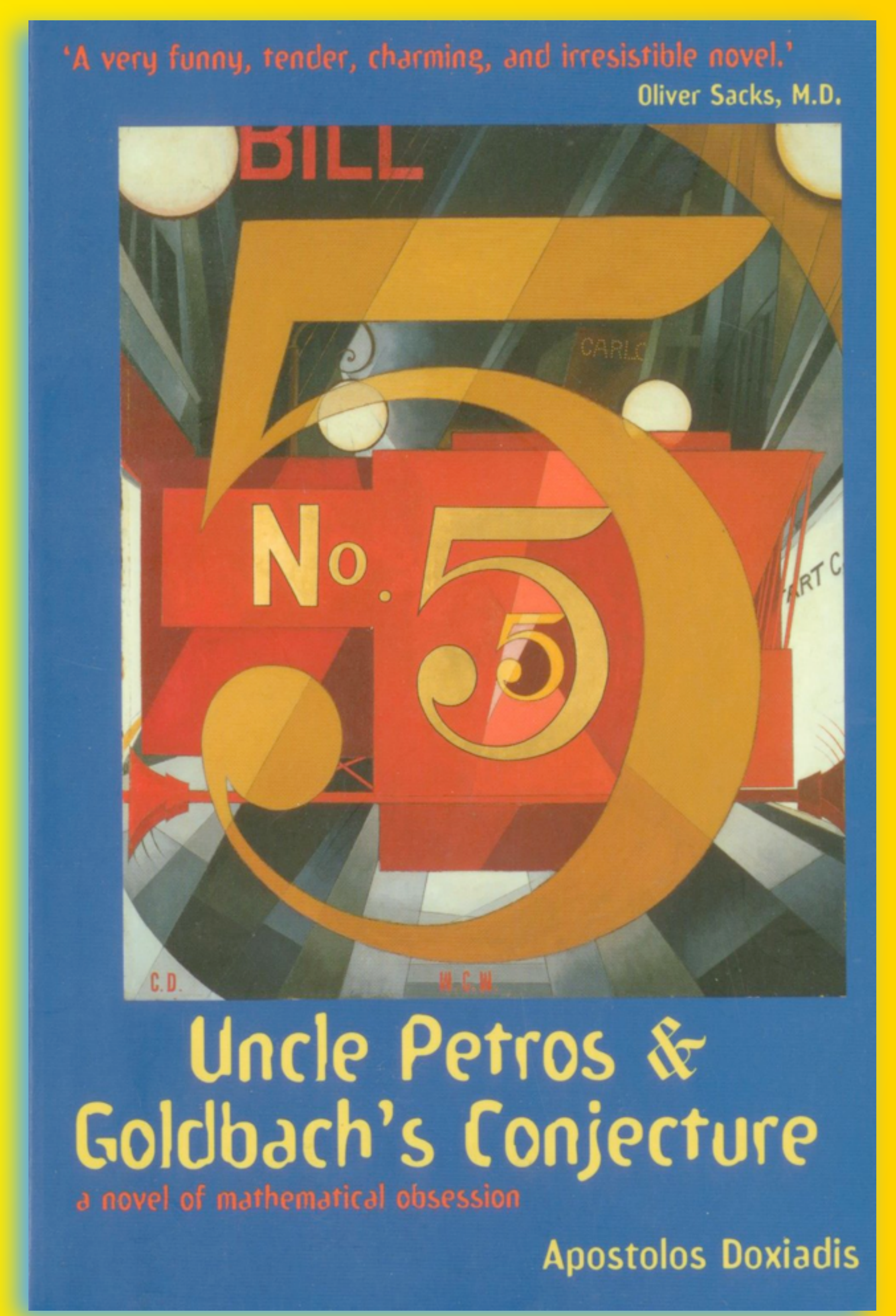
Goldbach conjecture

$$S(x)^2 = \sum_k b_k x^k$$

has non-zero b_{2k}

Goldbach

Every even integer larger than 2 is a sum of two primes



Worksheet

The End