8/30/2021 near Mather house

Lecture 23

10/28/2021

Frankenstein Review

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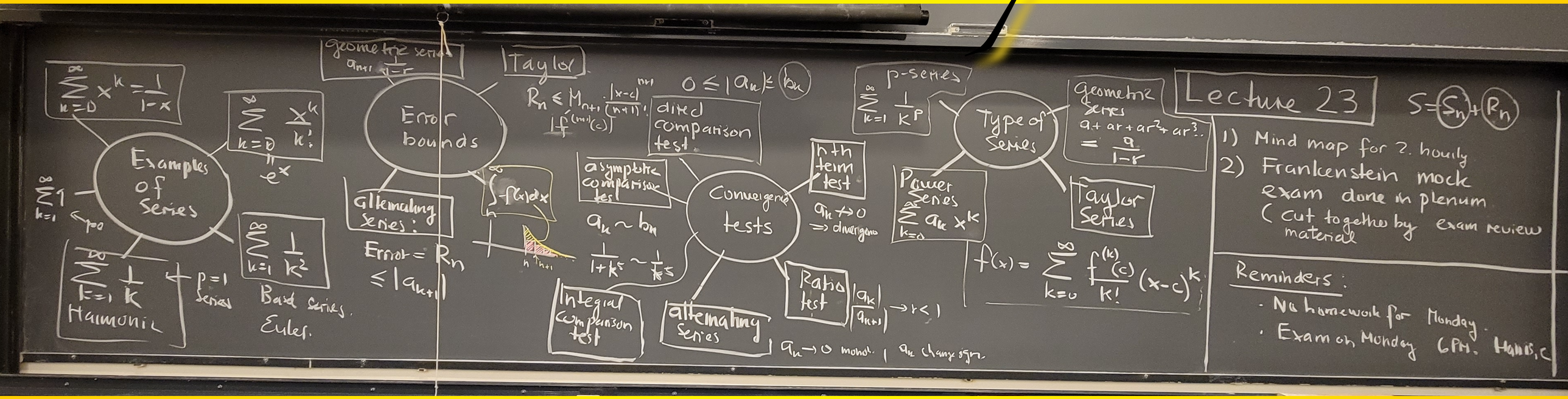
Mind Map


Examples of Series
Harmonic
p=1 series
Basel series. Euler.
Error bounds
Alternating series:
Error = $R_n \leq |a_{n+1}|$
Taylor
$R_n \in M_{n+1} \cdot \frac{|x-c|^{n+1}}{(n+1)!}$
$M = |f^{(n+1)}(c)|$
Convergence tests
$0 \leq |a_k| \leq b_k$
dited comparison test
asymptotic comparison test
$a_k \sim b_k$
$\frac{1}{1+k^5} \sim \frac{1}{k^5}$
integral comparison test
alternating series
$a_k \to 0$ monot. | a_k changes sign.
nth term test
$a_k \not\to 0 \Rightarrow$ divergence
Ratio test
$\left|\frac{a_k}{a_{k+1}}\right| \to r < 1$
Type of Series
p-series
$\sum_{k=1}^{\infty} \frac{1}{k^p}$
Power series
$\sum_{k=0}^{\infty} a_k x^k$
$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(c)}{k!}(x-c)^k$
Geometric series
$a + ar + ar^2 + ar^3 \ldots = \frac{a}{1-r}$
Taylor Series
Lecture 23
$S = S_n + R_n$
1) Mind map for 2. hourly
2) Frankenstein mock exam done in plenum (cut together by exam review material
Reminders:
- No homework for Monday.
- Exam on Monday 6PM. Hamblic

Tests

Ratio

n'th Term

Comparison

Integral

Alternating

p-series

Geometric

Which type of series ?

$$\sum_{k=1}^{\infty} \frac{1}{k^5}$$

$$\sum_{k=1}^{\infty} \frac{1}{5^k}$$

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln(5^k)}$$

$$\sum_{k=1}^{\infty} e^{-5k}$$

$$\sum_{k=1}^{\infty} \frac{x^k}{k!}$$

Which type of test ?

$$\sum_{k=1}^{\infty} (-k)^{-k}$$

$$\sum_{k=1}^{\infty} \frac{k^5}{5^k}$$

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{5+k^5}}$$

$$\sum_{k=1}^{\infty} \frac{1}{k(\ln(k^5))^5}$$

$$\sum_{k=1}^{\infty} \frac{k^5 e^{-5k}}{1+k}$$

$$\sum_{k=1}^{\infty} \frac{\sqrt{1+k^2}}{1+k^3}$$

Frankenstein problems

Cut together
from practice material

$\sum \frac{1}{n \ln(n)}$ diverges $\int \frac{1}{x \ln x}\,dx = \ln\ln(x)$

$\sum \frac{1}{n \ln(n)^2}$ converges $\int \frac{1}{x \ln(x)^2}\,dx = \frac{-1}{\ln(x)}$

(*)

$a_n = \frac{3^n}{n!}$ $\quad 3\left|\frac{1}{n+1}\right| < 1$

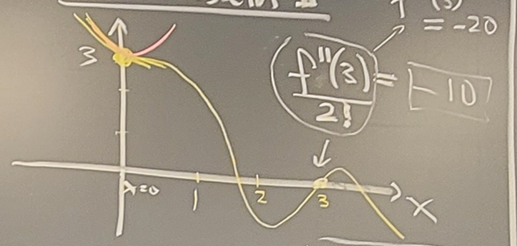
$\frac{a_{n+1}}{a_n} = \frac{3^{n+1}}{3^n} \cdot \frac{n!}{(n+1)!}$

Frankenstein III

	converge	diverge	Test used
$\sum \frac{1}{n+\ln(n)^4}$			$\frac{1}{n+\ln(n)^4} \sim \frac{1}{n}$
$\sum \frac{n^4}{4^n}$			$\left\|\frac{a_{n+1}}{a_n}\right\| = \frac{(n+1)^4}{n^4} \cdot \frac{4^n}{4^{n+1}} < 1$
$\sum \frac{3^n}{n!}$			Ratio test (*)
$\sum \frac{6n^2+4}{100n^2+1}$			nth term test $\sim \frac{6n^2}{100n^2} = \frac{6}{100}$
$\sum \frac{2^n}{n^n}$			Ratio $\frac{2^{n+1}}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n} \to 0$
$\sum (-1)^n \frac{n^2+2^n}{5^n-2}$			nth term test
$\sum \frac{(-1)^n 3^n}{2^{n^2}}$			alternating

$\frac{2^n}{(n+1)^{n+1}} \cdot \frac{n^{n+1}}{2^n} = \frac{n^n}{(n+1)^n} = \frac{1}{\left(\frac{n+1}{n}\right)^n} \to \frac{1}{e} < 1$

Frankenstein II



$f''(3) = -20$

$\frac{f''(3)}{2!} = -10$

a) $c = 0$ (ii)

b) $c = 3$ (i)

$f(0) = 3 > 0$
$f'(0)$ slope < 0
$f''(0)$ concave > 0

$f'(3) = 0$
$f''(3) \begin{cases} > 0 \\ < 0 \end{cases}$

Frankenstein I

geometric series

a) $\sum_{n=5}^{\infty} 3 \cdot (0.99)^n$ $\quad a = 3 \cdot 0.99^5$, $r = 0.99$, $\frac{3 \cdot 0.99^5}{1-r} = 300 \cdot 0.99^5$

b) $\sum_{n=0}^{\infty} a_n = 5$, $r = 3/5$, $a_0 = 2$, $\frac{2}{1-r} = 5$, $(1-r) = 2/5$

c) $\sum_{k=1}^{\infty} a_n$ converges, $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \ln(2)$, $\sum_{k=1}^{\infty} |a_n|$ does not converge absolutely

b) $\sum_{n=0}^{\infty} 2 \cdot \left(\frac{2}{5}\right)^n = 5$

Frankenstein IV

	Center	Interval
$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^{3/2}}$	1	$-1 \le x \le 1$
$\sum_{n=200}^{\infty} 4^n (x+3)^{2n}$	-3	$R = \frac{1}{2}$, $-3.5 < x < -2.5$
$\sum_{n=14}^{\infty} \frac{(1+e^{n^2})x^n}{1+10^{-100}e^{2n^2}}$	0	$-1 < x < 1$

$\frac{a_n}{a_{n+1}} = \frac{(x-1)^n}{(x-1)^{n+1}} \cdot \frac{n^{3/2}}{(n+1)^{3/2}} = \frac{1}{|x-1|} \cdot \frac{1}{\left(\frac{n+1}{n}\right)^{3/2}} = \frac{1}{|x-1|} > 1$

$|x-1| < 1$

$\left|\frac{(x+3)^{2n+2}}{(x+3)^{2n}} \cdot \frac{4^{n+1}}{4^n}\right| < 1$

$(x+3)^2 \cdot 4 < 1$

$|x+3|^2 < \frac{1}{2}$

$a_n \sim \frac{1}{e^{n^2} \cdot 10^{-100}}$

Frankenstein V

Error: $\frac{M_3 |b-a|^3}{3!} = \frac{M_3}{3!}$

$f(x) = \frac{1}{x}$ $\quad f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{1!}(x-1)^2$

For any $|x| < 1$ the ratio test gives convergence.

Ratio test

$\frac{e^{n^2}}{e^{(n+1)^2}} |x| = \frac{e^{n^2}}{e^{n^2+2n+1}} |x| = \frac{1}{e^{2n+1}} |x| < 1$

The End