8/30/2021 near Mather house

Lecture 24

11/01/2021

Power series as functions

Table of Contents

1) Power series reminder

2) Useful for finding Taylor series

3) Useful for integration

4) Worksheet

Power Series

$$S = \sum_{k=0}^{\infty} a_k (x - c)^k$$

power series

Taylor series:

$$a_k = \frac{f^{(k)}(c)}{k!}$$

2) Finding Taylor series

Example

Find the Taylor series of arctan(x)

$$\sum_{k=0}^{\infty} (-1)^k x^{2k} = \frac{1}{1-x^2}$$

$$\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} = \arctan(x)$$

In dot dot dot notation

$$1 - x^2 + x^4 - x^6 + x^8 - \ldots = \frac{1}{1 - x^2}$$

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots = \arctan(x)$$

3) Integration

Example

Find $\int_0^1 e^{-x^2} dx$

$$e^{-x^2} = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{k!}$$

$$\int_0^1 e^{-x^2} dx = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)k!} \Big|_0^1$$

Good luck at the exam!

Remember the tests

Ratio

n'th Term

Comparison

Integral

Alternating

p-series

Geometric

Which type of test ?

$$\sum_{k=1}^{\infty} (-k)^{-k}$$

$$\sum_{k=1}^{\infty} \frac{k^5}{5^k}$$

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{5+k^5}}$$

$$\sum_{k=1}^{\infty} \frac{1}{k(\ln(k^5))^5}$$

$$\sum_{k=1}^{\infty} \frac{k^5 e^{-5k}}{1+k}$$

$$\sum_{k=1}^{\infty} \frac{\sqrt{1+k^2}}{1+k^3}$$

The End