



8/30/2021 near Mather house

Lecture 25

11/03/2021

Differential equations

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1) *Differential equations*

Differential equations

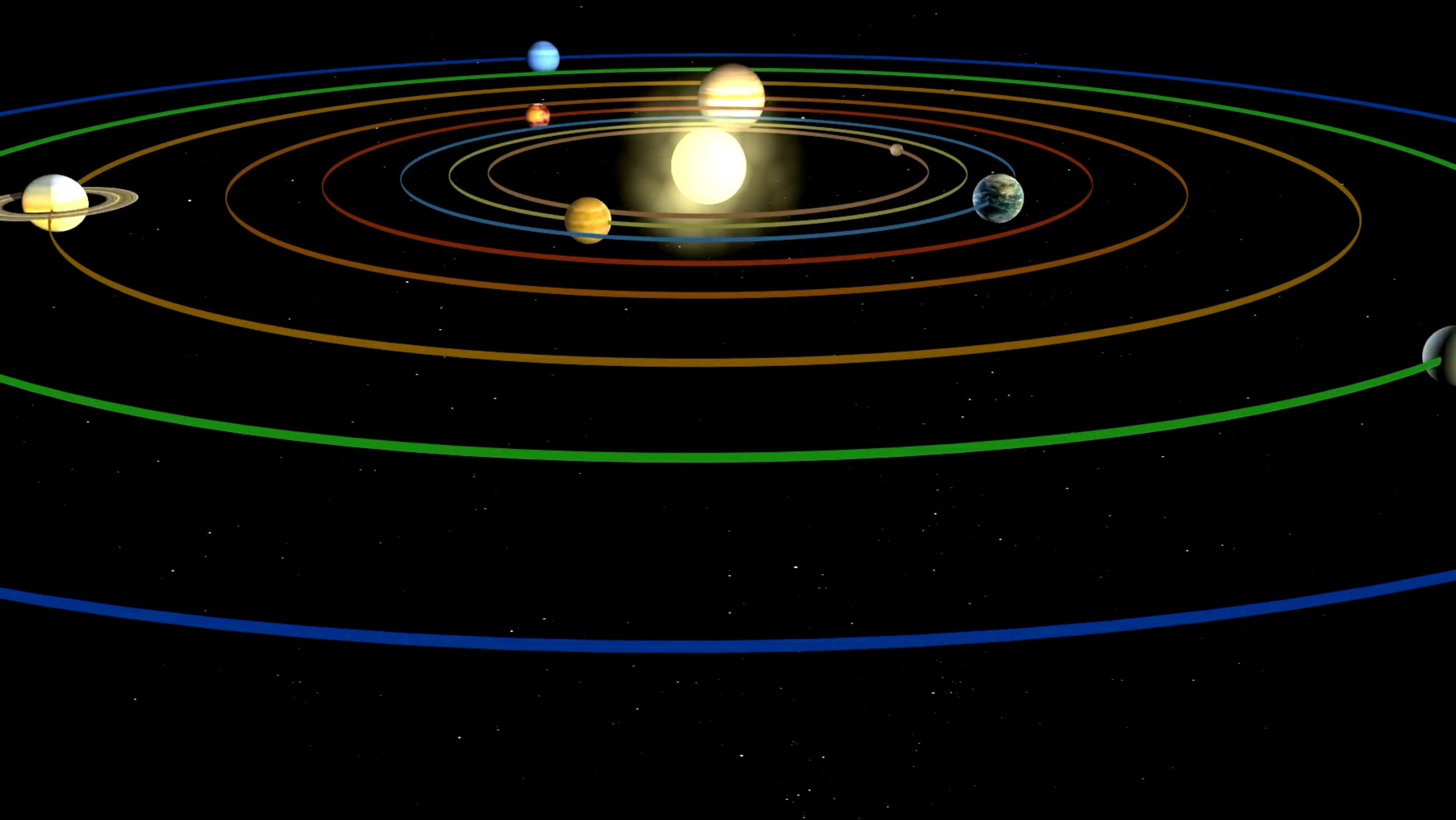
Differential equations provide the fabric we are made of and the rules of events we measure and experience. They relate quantities changing in time.

We use mostly the variable t for now.

Quantities changing with time are in this lecture called $y(t)$ or $M(t)$.

From Physics

Differential equations were first studied in the context of physics and astronomy. The goal was to understand where we come from and how things will continue.



From Biology

Also very complex processes like life are described by differential equations.



Adding epigenetic tag

From Chemistry

A famous reaction is the Belousov-Zhabotinski system. It is a non-linear chemical oscillator found in the 1950ies.



Still actively investigated

Here is an example using this process

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Article

How Chemistry Computes: Language Recognition by Non-Biochemical Chemical Automata. From Finite Automata to Turing Machines

Marta Dueñas-Díez^{1,2} and Juan Pérez-Mercader^{2,3,4,*}

SUMMARY

Every problem in computing can be cast as decision problems of whether strings are in a language or not. Computations and language recognition are carried out by three classes of automata, the most complex of which is the Turing machine. Living systems compute using biochemistry; in the artificial, computation today is mostly electronic. Thinking of chemical reactions as molecular recognition machines, and without using biochemistry, we realize one automaton in each class by means of one-pot, table top chemical reactors: from the simplest, Finite automata, to the most complex, Turing machines. Language acceptance/rejection criteria by automata can be formulated using energy considerations. Our Turing machine uses the Belousov-Zhabotinsky chemical reaction and checks the same symbol in an Avogadro's number of processors. Our findings have implications for chemical and general computing, artificial intelligence, bioengineering, the study of the origin and presence of life on other planets, and for artificial biology.

INTRODUCTION

Computation is everywhere around us (Moore and Mertens, 2011; Rich, 2008) and is central to life on Earth. Computation takes place not only in the myriad of electronic devices we use daily but also in living systems. In life, biochemistry implements computation via the chemical properties of "organic" matter (Conrad, 1972; Katz, 2012), i.e., using chemical support: inputs are chemical substances, the mechanical processing occurs via chemical reaction mechanisms, and the result is chemical before its transduction into specific functionalities, chemical or otherwise.

More specifically, a computation is (Rich, 2008; Katz, 2012) the process by which information in sequences belonging to a language and consisting of "symbols" in an alphabet are fed to a computing device ("automaton") that recognizes the symbols and is endowed with some rules that allow the automaton to process the symbols according to these rules to eventually deliver an output, such as Acceptance or Rejection of the sequence as belonging or not to the language recognized by the automaton. The pattern of symbols in the sequence is characteristic of the language to which the sequence belongs. We can interpret this process as a metaphor for a chemical reaction or combination of chemical reactions, as one can think of chemical reactions as the result of molecular recognition events that occur precisely, predictably, and repeatedly.

Then, we can ask if the power and complexity of biochemistry are necessary to carry out computations using only chemistry. To do so it is useful to recall that languages are classified into an inclusive hierarchy, the Chomsky hierarchy (Rich, 2008; Hopcroft et al., 2007; Chomsky, 1956; Sudkamp, 2006) (cf. Table 1), and that there is a direct correspondence between language complexity and the capabilities of the automata that recognize the language. The most powerful automata are the Turing machines (Turing, 1936).

We answer the aforementioned question by providing non-biochemical realizations of the automata using non-biochemical reactions running in a "one-pot reactor," that is, in a single well-mixed container where multiple rounds of reactions can take place. We do not need any intermediation from either external geometrical aids to channel and direct the chemical fluids or from reactions involving complex biomolecules. To carry out computations we rely fully on the power of molecular recognition associated with the occurrence of chemical reactions and the robustness provided by an Avogadro's number of "processors" working simultaneously. For this, we will introduce in our experimental examples the means for the chemical rendering (translation) of alphabet symbols, the chemical copy of the sequence (transcription), a means

¹Repsol Technology Lab, c/ Agustín de Betancourt s/n, Móstoles, Madrid 28935, Spain

²Department of Earth and Planetary Sciences, Harvard Origins of Life Initiative, Harvard University, 20 Oxford Street, Cambridge, MA 02138, USA

³Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA

⁴Lead Contact

*Correspondence: jperzmercader@fas.harvard.edu

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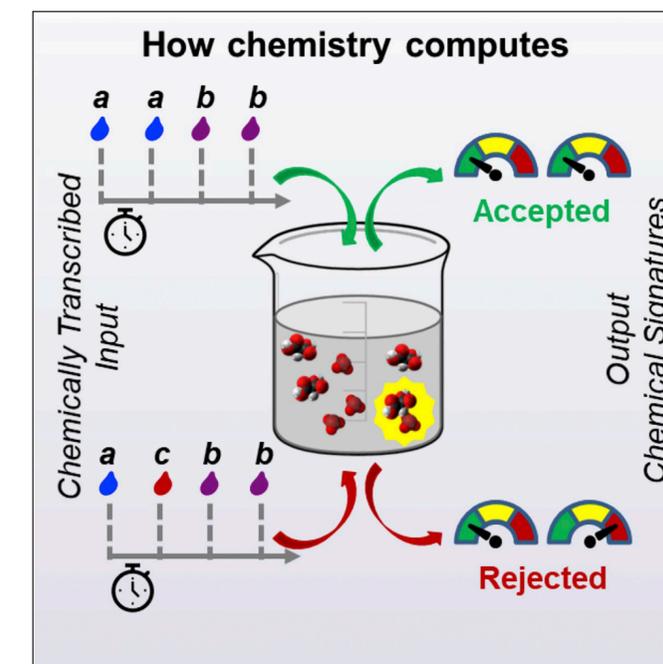


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jperzmercader@fas.harvard.edu

HIGHLIGHTS
Computations are language recognition events carried out by "computing automata"

Chemical reactions are molecular recognition events equivalent to automata

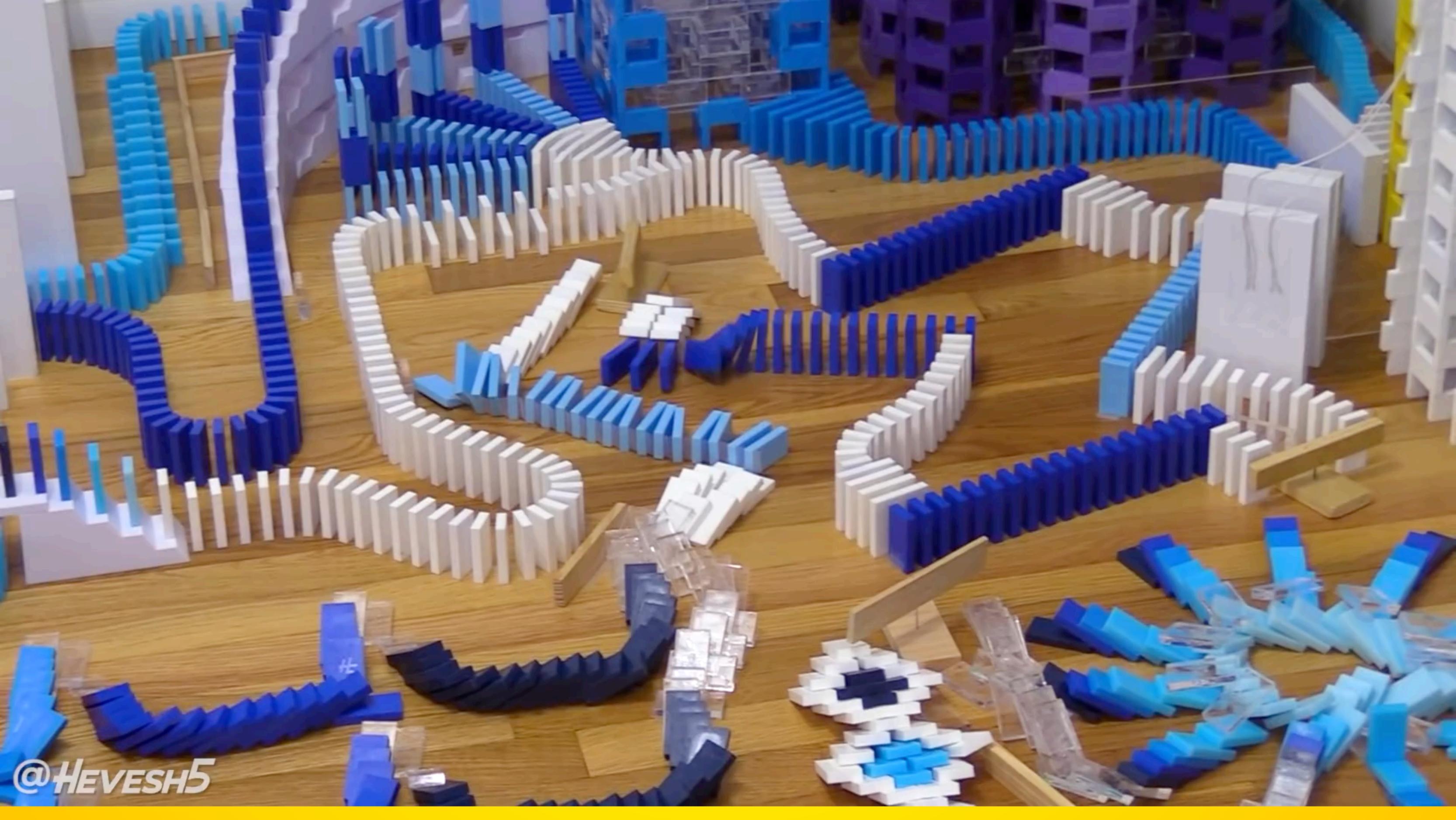
Words in a language can be represented by sequences of chemical reactants

Inorganic reactions like automata, including Turing machines, recognize languages

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Or simply having fun



@HEVESH5

2) Predicting the future

Past - Present - Future

Assume we know a quantity now,
and how it has changed in the past,
can we predict its behavior in the
future? Diff equations are a theory of time.

past: rates of changes

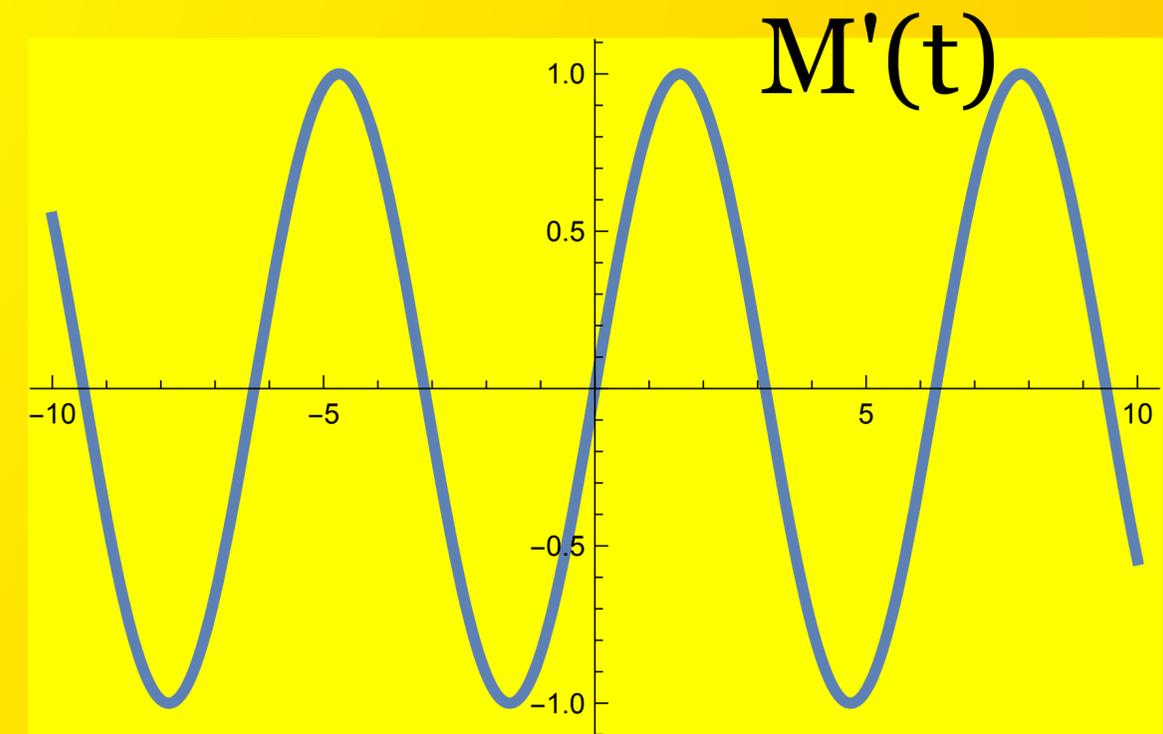
present: current state

future: integrate

Example

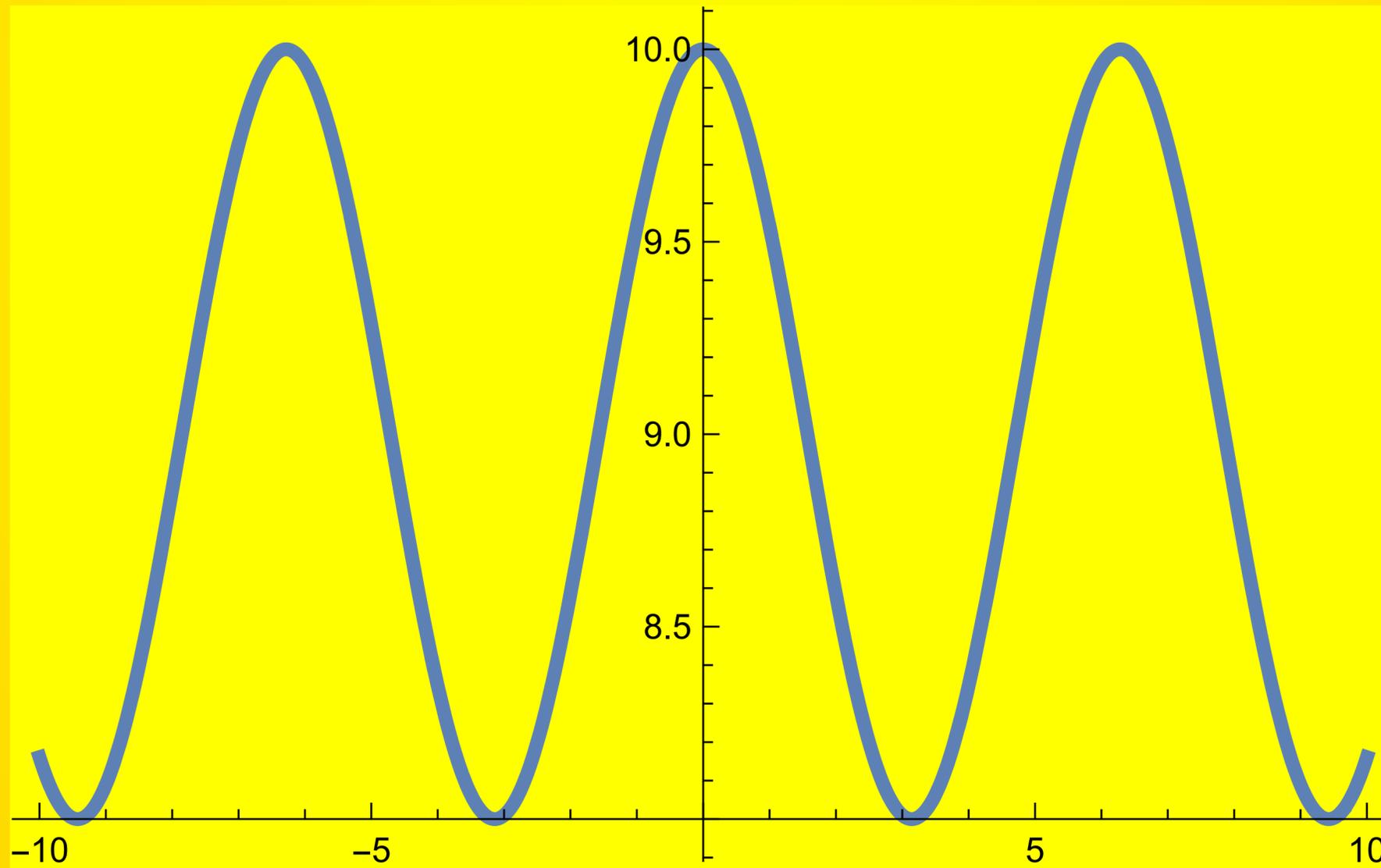
Assume we know that $M(0)=10$ and that $M'(t) = \sin(t)$. Can you predict $M(t)$ for all time t ?

past: $M'(t)$
present: $M(0)$
future: $M(t)$



The future is ...

Could you get $M(t)$? If yes, you predicted the future!



3) *Exponential growth*





4) *Exponential decay*



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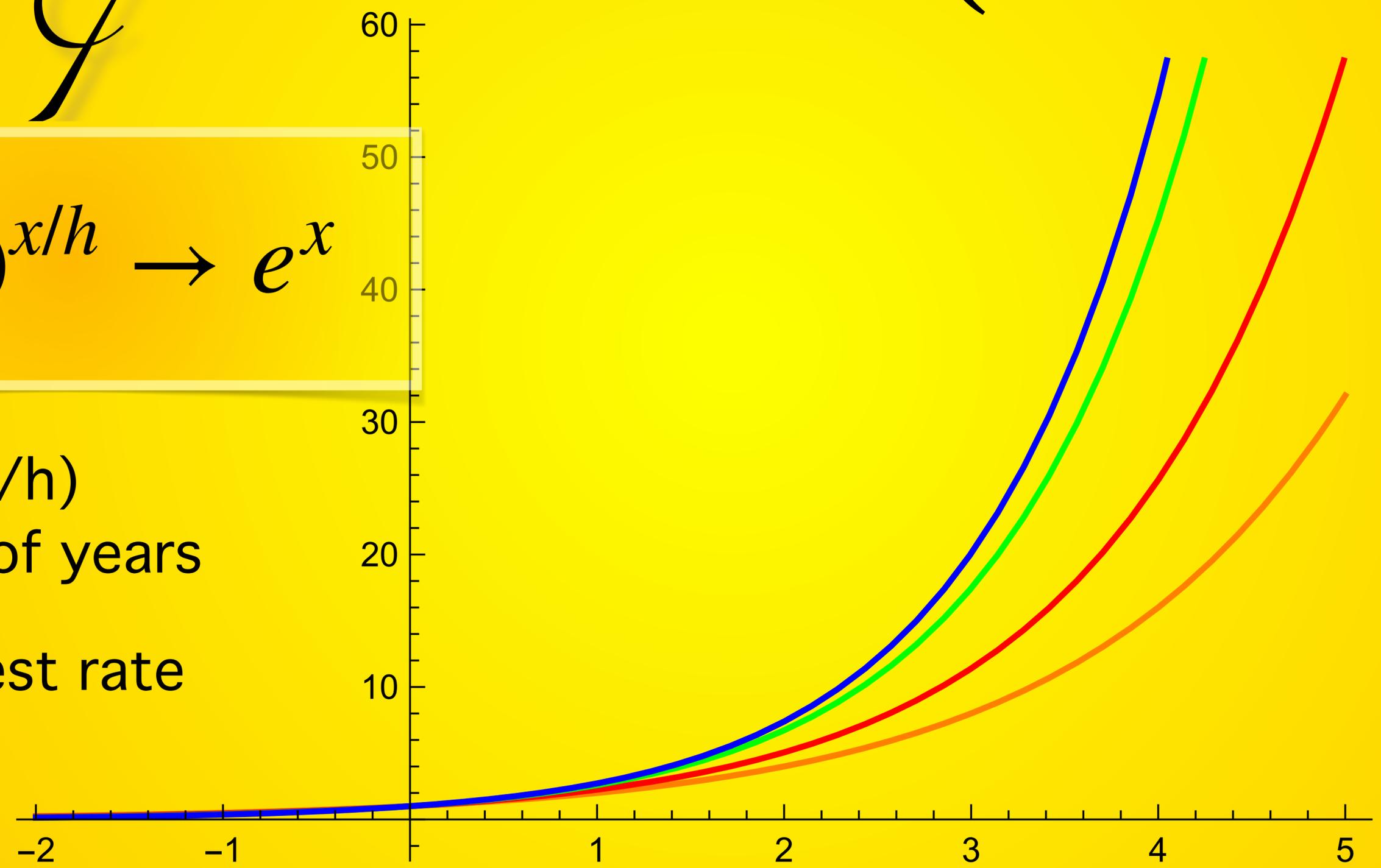
NOVA

5) *Banking problems*

Exponential $(1+h)^{x/h}$

$$(1+h)^{x/h} \rightarrow e^x$$

$n=(x/h)$
number of years
 h : interest rate



- $(1+1)^x$
- $(1+\frac{1}{2})^{2x}$
- $(1+\frac{1}{10})^{10x}$
- $\exp(x)$

Compound Interest



6) *Worksheet*

The End