8/30/2021 near Mather house

Lecture 28

11/10/2021

Compartmental analysis

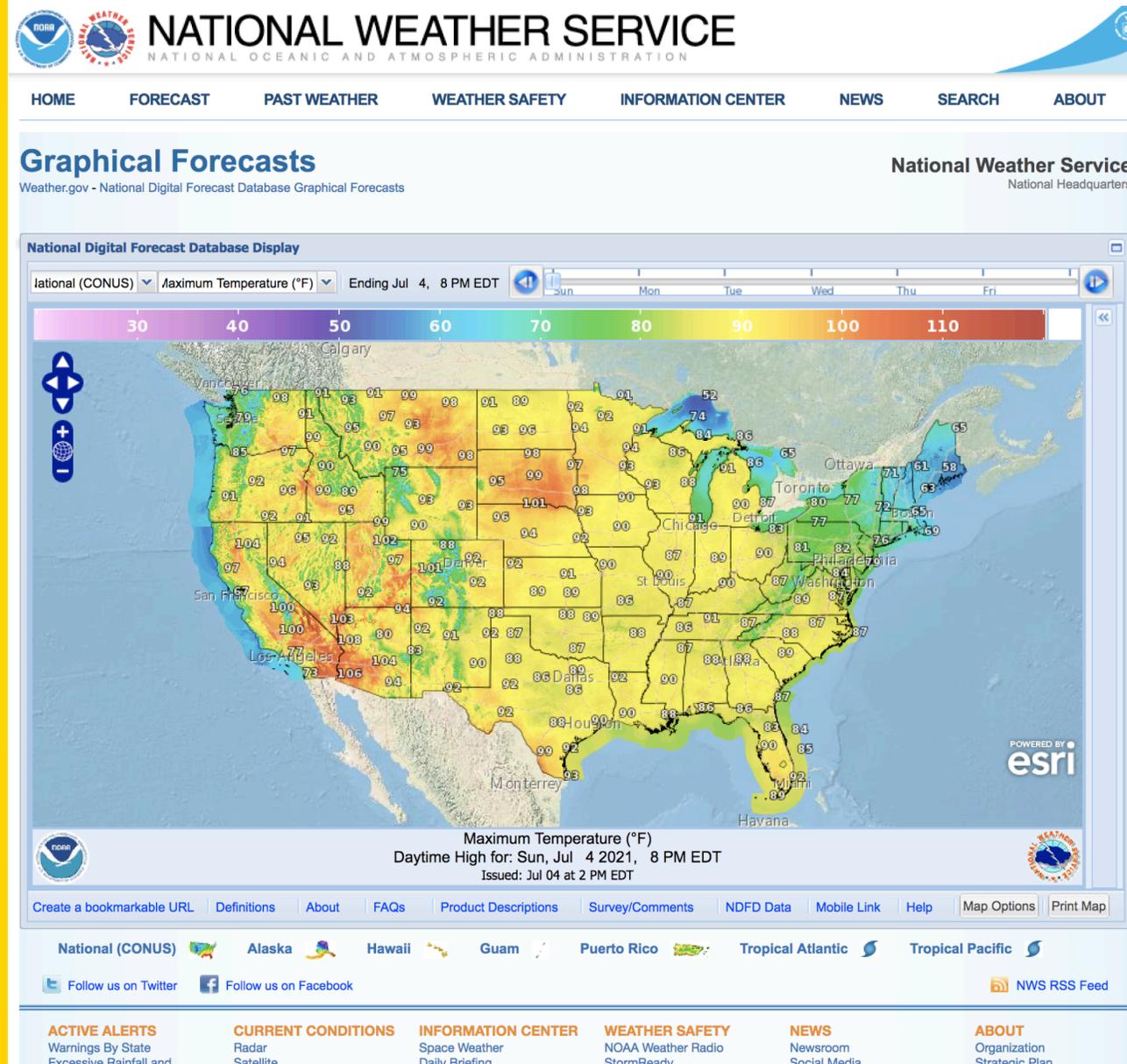# Table of Contents

1) Eggnog review: 2 pictures!

2) Input-Container-Output

3) Example

4) Worksheet

Weather in real time

     The US Weather service runs an interesting website that allows you to view weather forecast data at any place in the United States. Here is the url for this website: https://digital.weather.gov/. Go to this page and here is what you see:

I am writing this on July 7 in Cambridge MA, so the weather there interests me. I can focus in on the forecasts for that day using the arrows and ± on the left side near the top of the map. You can see the screen shot of the temperature forcasts for 8pm in
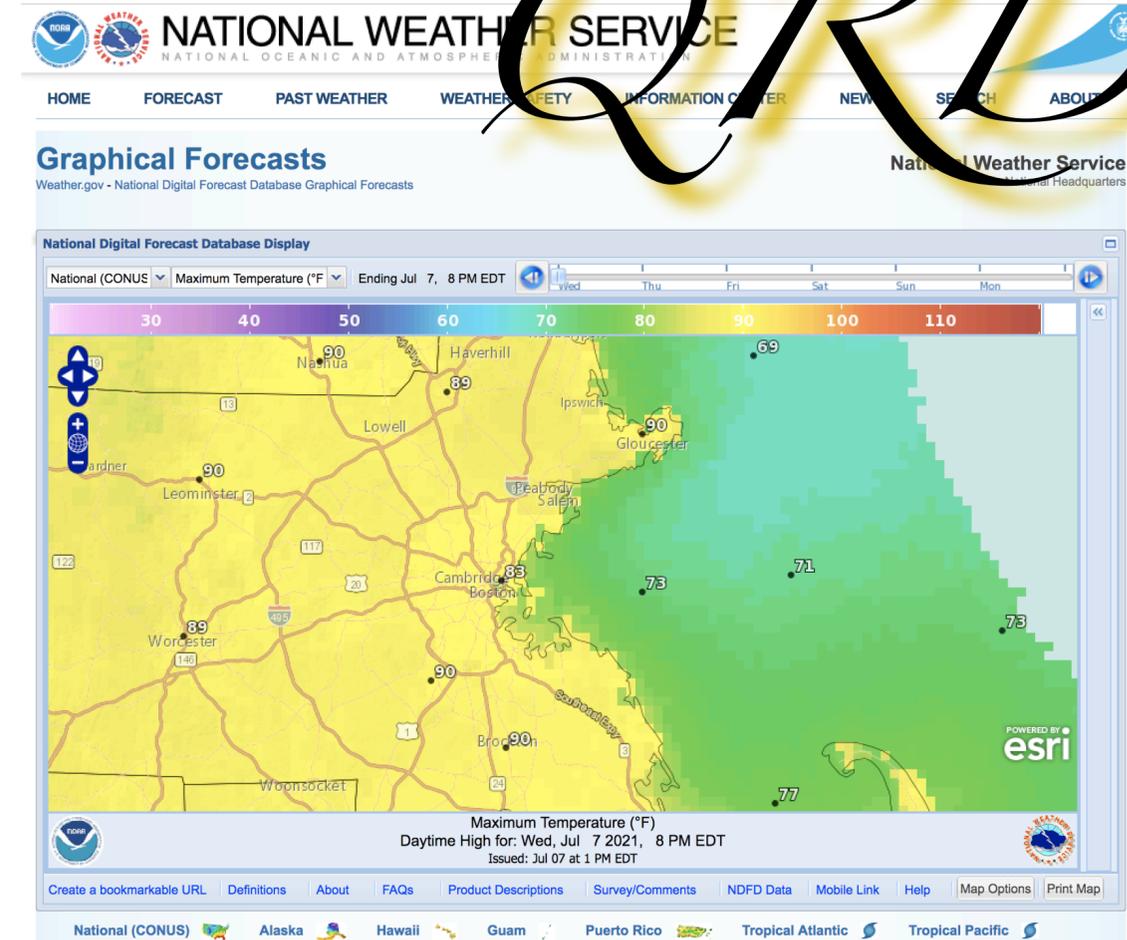
Notice that this depicts the *maximum* temperature for the Boston area up until 8pm tonight. It says this under the temperature map--it also says there when the forcast was issued.

     Tempertures in the 90º's are nice!! Most people don't like it that hot but I do because I save the memory of hot days in a specal place for the upcoming grim days of January when I am out with the flying snow, the temperature below zero (Fahrenheit) and a gale blowing long and evil from the northeast. That's when I use up those memories of 90º days.
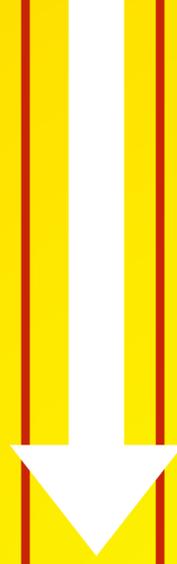
     The weather service is very good at explaining how they produce their graphics. You get this information and the raw data too by clicking on the button under the map that says 'NDFD Data' (see the screen shot below).
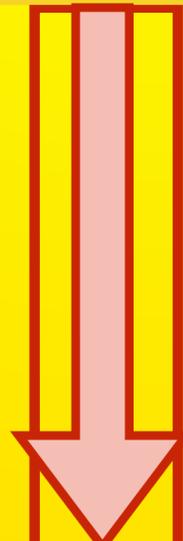
Eggnog

input : sugar milk
4 Gallons/min
1/2 concentration

100 Gallon tank

S(t) amount of sugar

milk
cream
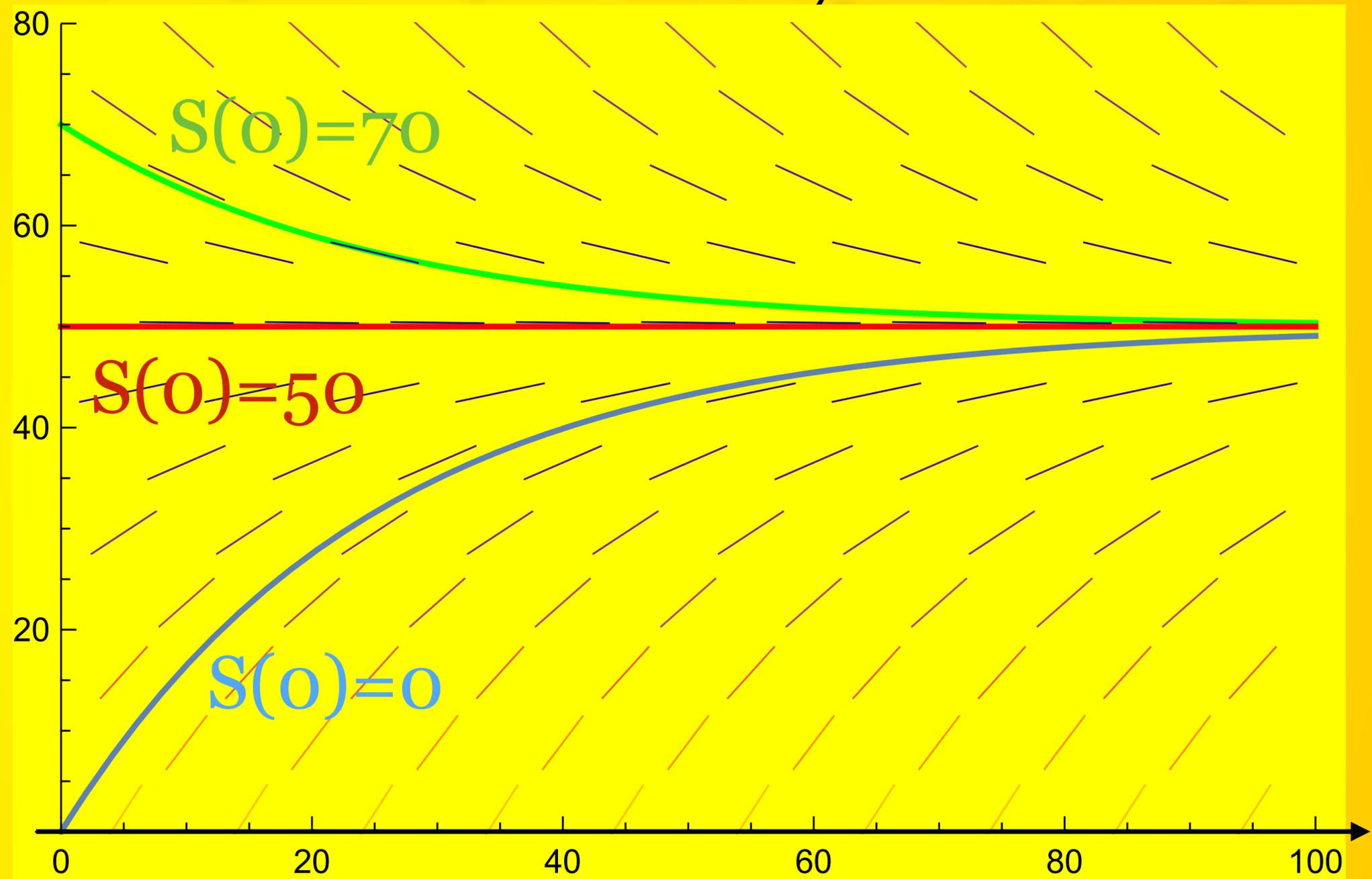egg whites

output:
Eggnog

4 Gallons/min

Slopefield picture (t-S plane)

$$S'(t) = 2 - \frac{S(t)}{25}$$

S(0)=70

S(0)=50

S(0)=0

$$S(t) = 50 - 50e^{-t/25}$$

t

S-S′ picture

$$S'(t)=f(S(t))$$

$$= 2-\frac{S(t)}{25}$$

S′

S

S=50 is
equilibrium
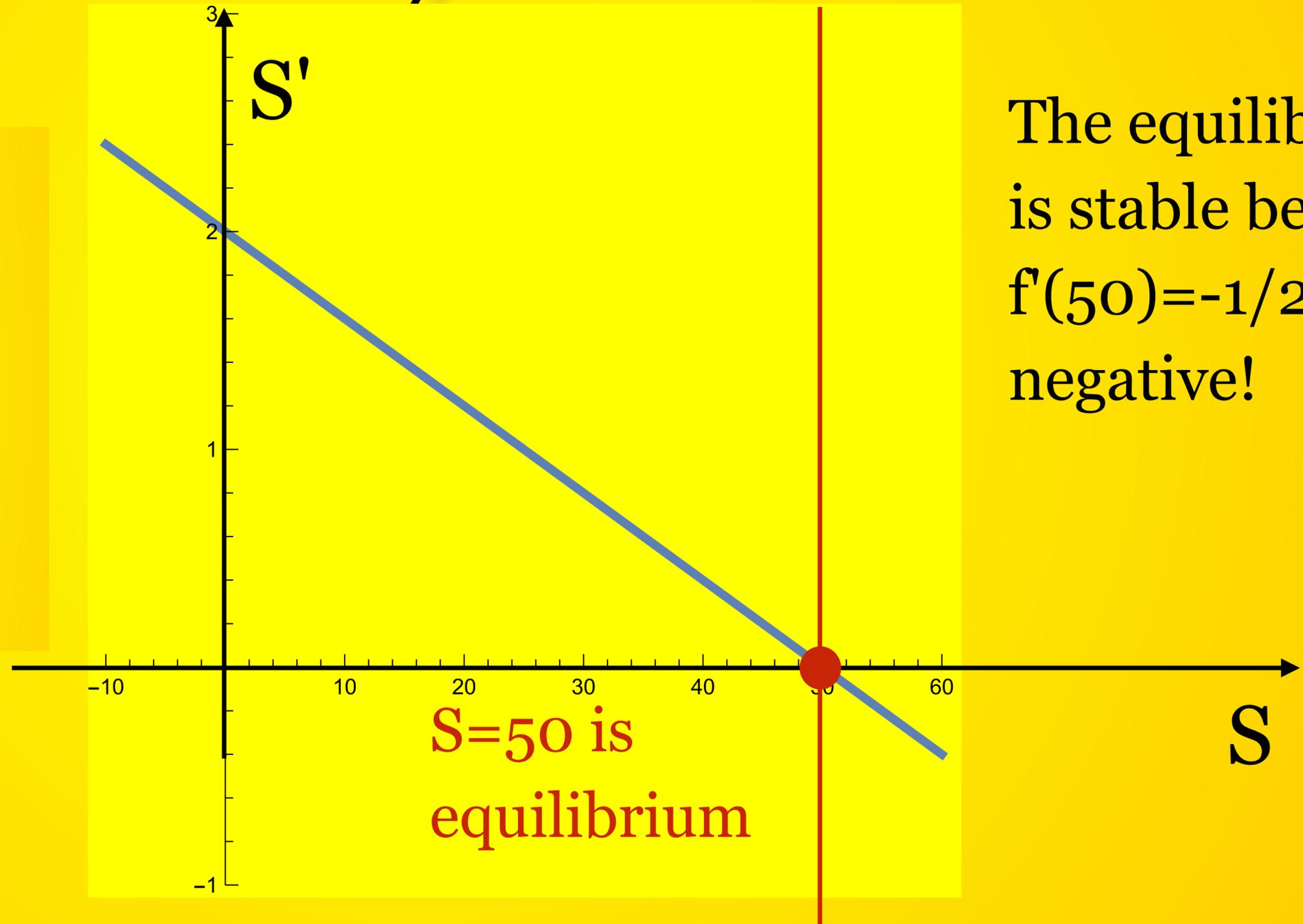
The equilibrium
is stable because
f′(50)=-1/25 is
negative!

Input-Container-Output

$$y'(t) = \text{input} - \text{output}(y)$$

container

input $\longrightarrow$ output

both input and output depend on
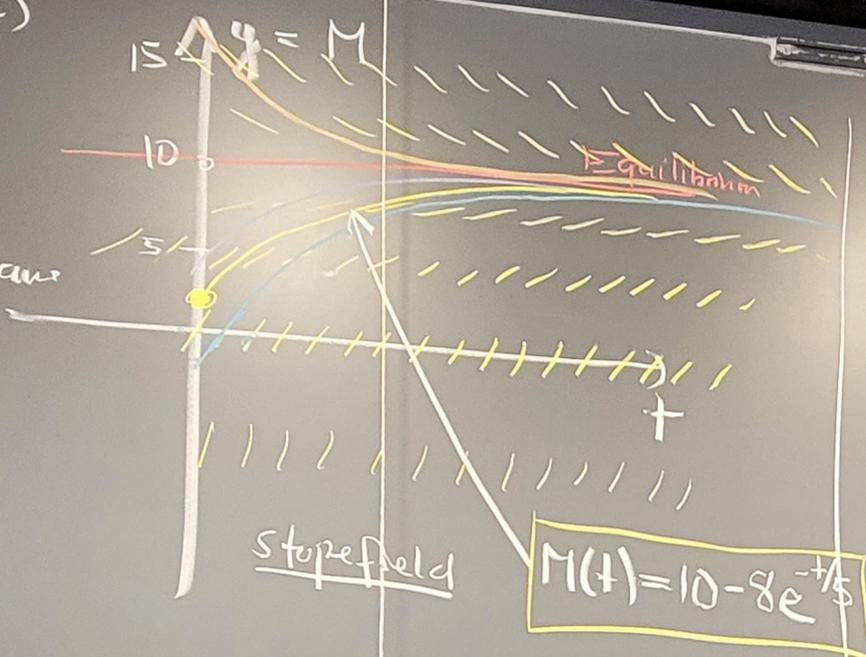concentration. Only output depends
on the quantity y.

Worksheet problems

$$M' = 2 - \frac{M}{5} = f(M,t)$$
$$M(0) = 2$$

Draw the slope field picture in the t-M plane
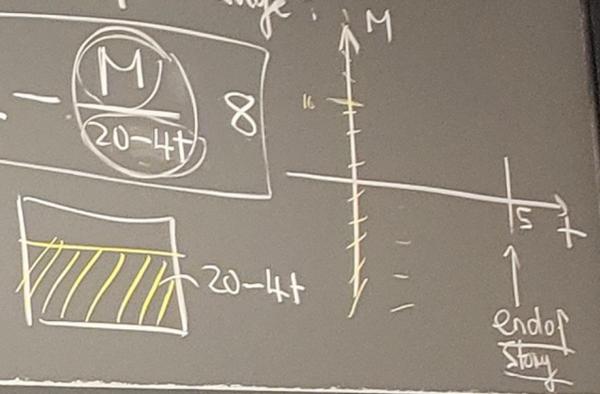
<u>equilibrium</u>: $M = 10$

slope field:

$y = M$

15

10 — Equilibrium

5

slopefield

$$M(t) = 10 - 8e^{-t/5}$$

Find the solution $M(t)$ satisfying $M(0) = 2$

$$\| \\ 10 + ce^{-t/5}$$

How do we find c?
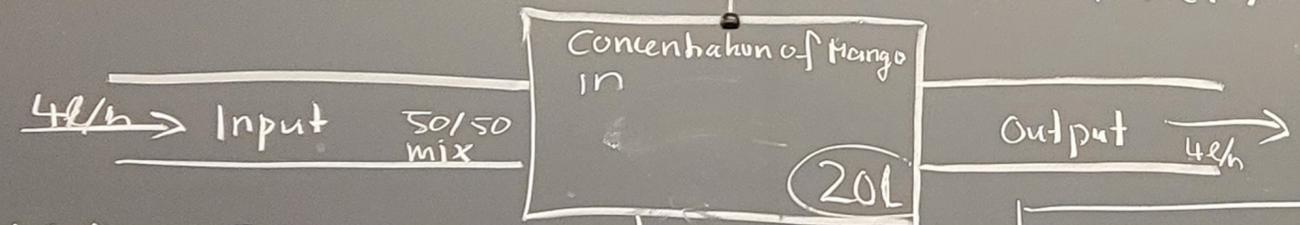
$$M(0) = 10 + ce^{-0t} = 2$$
$$= 10 + c = 2$$
$$c = -8$$

Assume: Output changes to $8 \ell/\text{hour}$. How does the diff equ. change?

$$M' = 2 - \frac{\textcircled{M}}{20 - 4t} \cdot 8$$

no more autonomous!

20 − 4t

end of story

<u>Mango - Cranberry - Orange.</u>

$M(t) = $ Mango in Liters

Concentration of Mango in

$4\ell/h \to$ Input   50/50 mix

20 L

Output $4\ell/h \to$

a) $M(0) = 2$

b) Input: 2

c) $\dfrac{\text{Mango in Mixer}}{\text{Total of juice in Mixer}} = \dfrac{M(t)}{20}$

d) Output: $4 \cdot \dfrac{M(t)}{20} = \dfrac{M(t)}{5}$
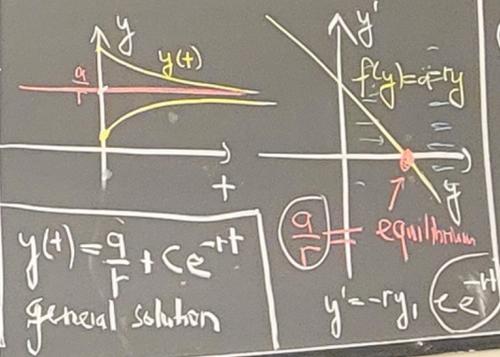
$$M'(t) = \text{Input} - \text{Output}$$
$$= 2 - \frac{M(t)}{5}$$
$$M(0) = 2$$

$$y' = a - ry$$
input   output   Story

Review: 2 pictures

$y(t) = \dfrac{a}{r} + ce^{-rt}$

general solution

$y' = -ry$, ce^{-rt}

$\dfrac{a}{r} = $ equilibrium

Lecture 28

Compartmental Analysis
Input - mixer - output analysis

① Review: $y' = f(y,t)$
   Stability, autonomous, (y,y') and (t,y) pictures
② Orange-Cranberry - Mango problem
③ General strategy to solve input-output prob.
④ Salt water problem (workshed)
⑤ Huuuuuhs! problem.

Reminders

1) Recovery Exam 1 due Tomorrow

2) Homework 26 due Friday

3) QRD project on Weather due Friday

The End