



Lecture 30

11/15/2021

*Euler's
formula*

8/30/2021 near Mather house

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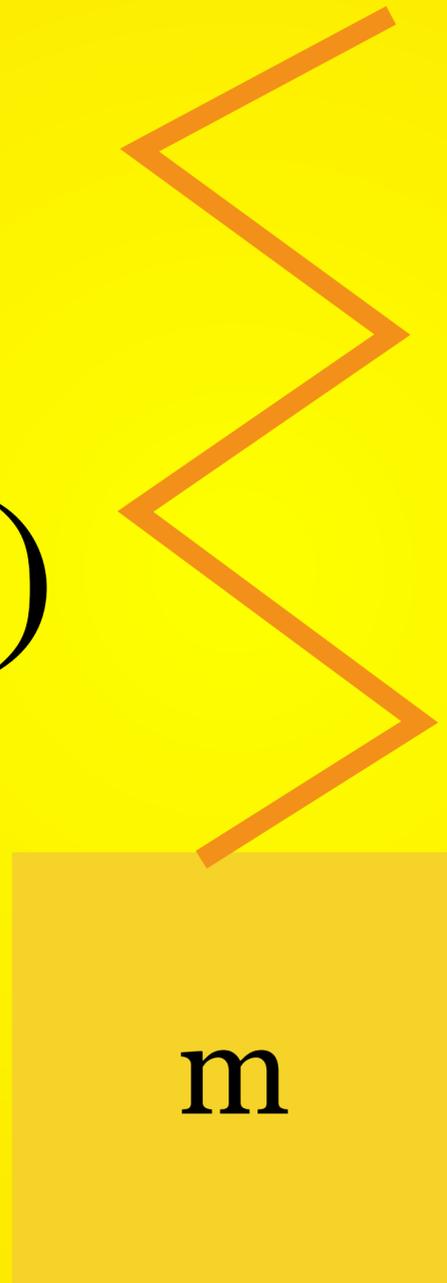
Second order systems

Harmonic Oscillator

spring constant

$$mx''(t) = -cx(t)$$

mass



$x(t)$

m

With Damping

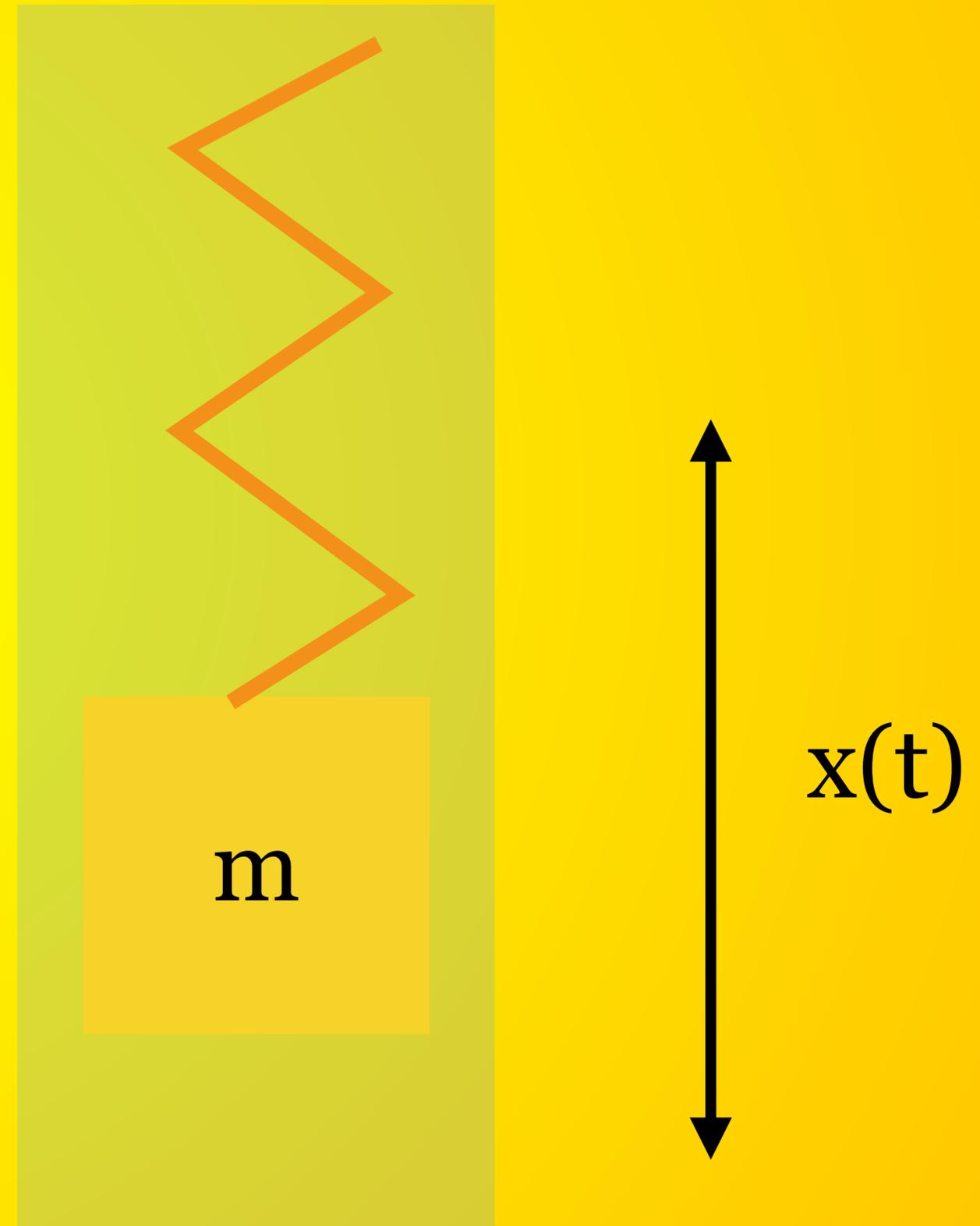
spring constant

$$mx''(t) = -bx'(t) - cx(t)$$

mass

damping

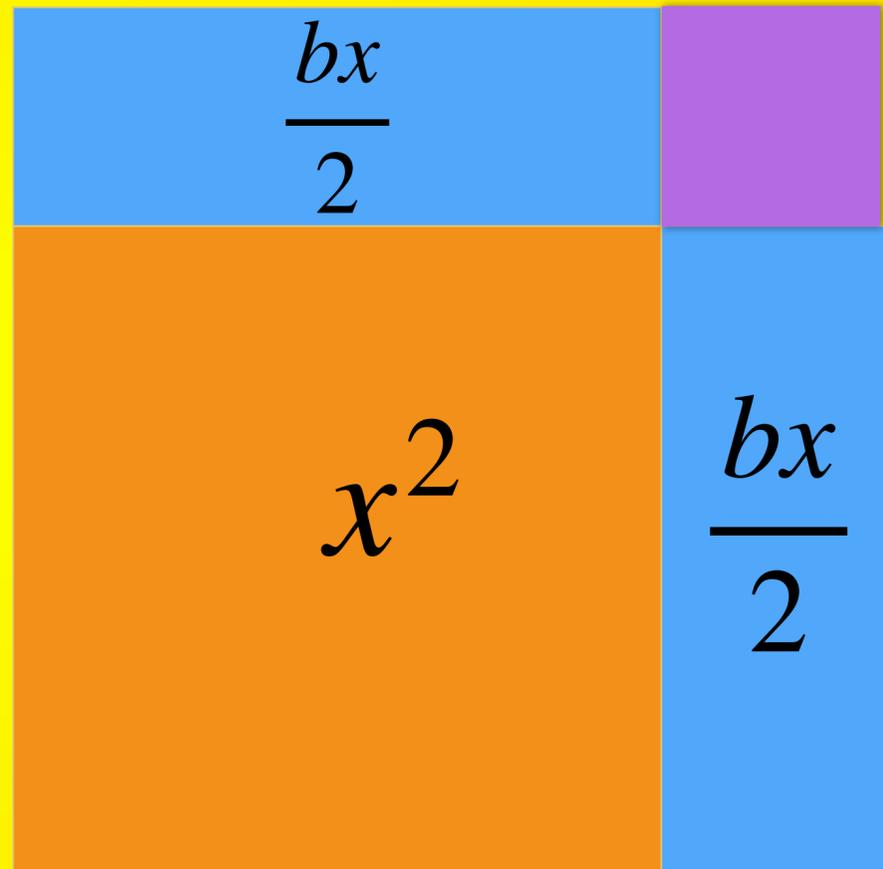
-> Monday lecture



Quadratic equation

Completion of the Square

$$x^2 + bx + c + \frac{x^2}{4}$$

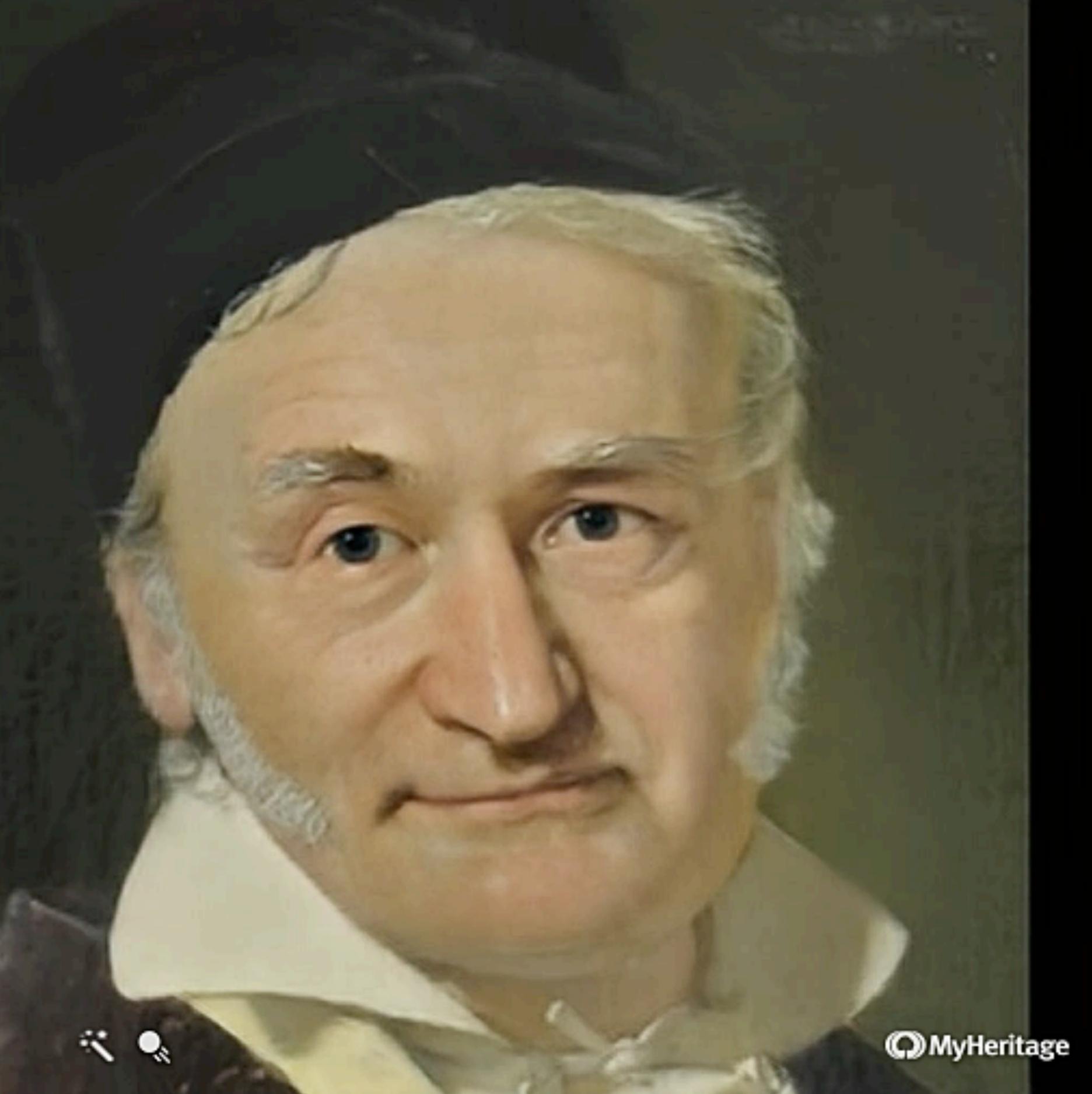


$$= \left(x + \frac{b}{2}\right)^2 + \frac{x^2}{4}$$



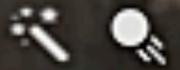
The Physician

Complex Numbers



$$\sqrt{-1} = i$$

Gauss in 1825 :
“The true
metaphysics of the
square root of -1 is
elusive”.





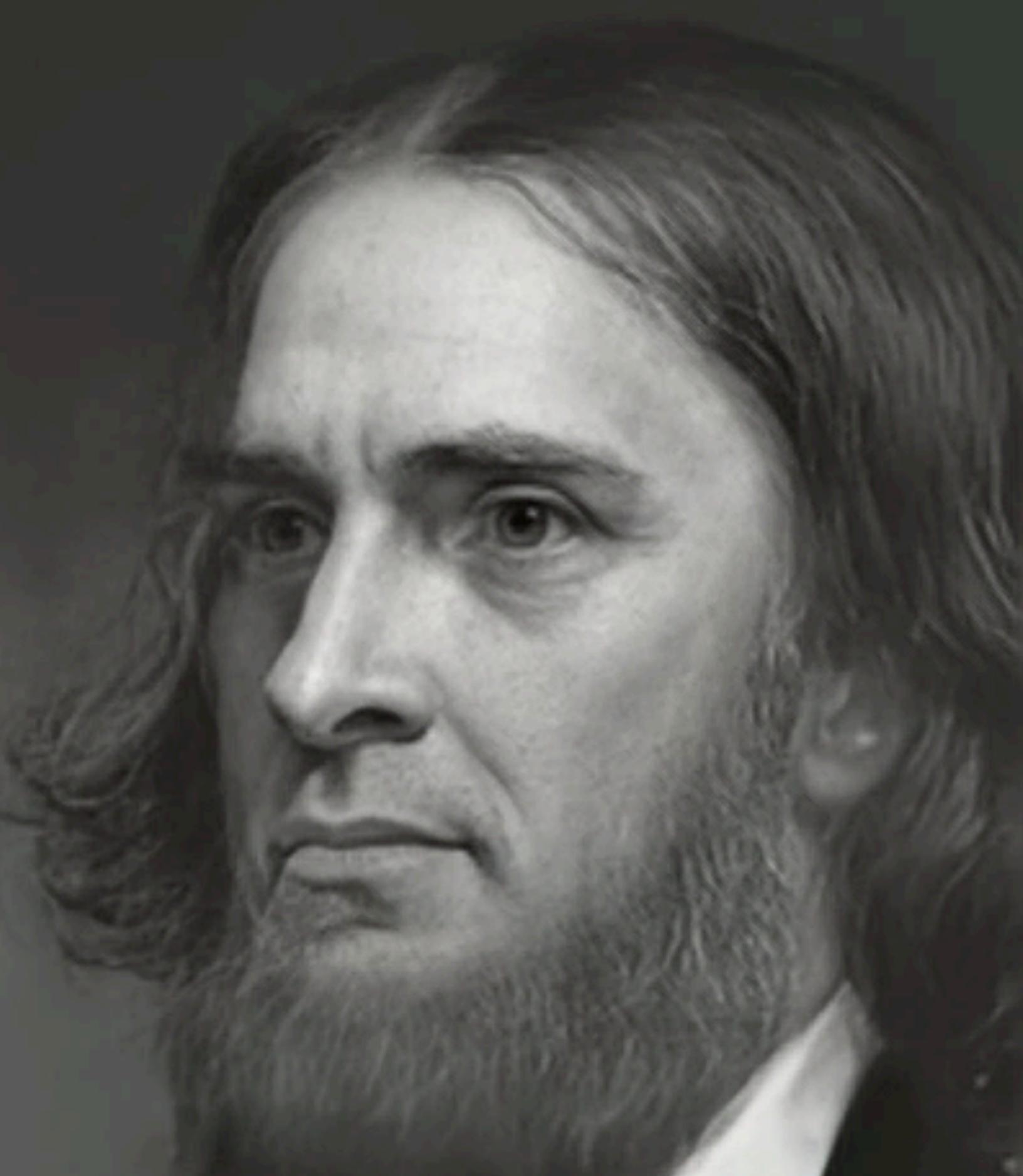
Euler's Formula

Algebra

Geometry

$$1 + e^{i\pi} = 0$$

analysis



Peirce

Benjamin Peirce: It is absolutely paradoxical, we cannot understand it, and we don't know what it means. but we have proved it, and therefore, we know it is the truth.

Atiyah



*The equivalent of Hamlet's line
"To be or not to be" in literature
"Combining depth with brevity"*

Sergio Albeverio (Bonn University, Germany)

$$e^{i\pi} = -1$$

$$V - E + F = \chi$$

$$dF = 0, \quad -\delta F = J$$

Sir Michael Atiyah (Edinburgh University, UK)

$$e^{i\pi} = -1$$

$$V - E + F = 2 \quad (\text{Euler formula for triangulation of polyhedron})$$

$$x^2 + y^2 = z^2 \quad (\text{Pythagoras})$$

Simplicity + Depth = Beauty.

I once explained in a general talk that formula $e^{i\pi} = -1$ was the equivalent of the Hamlet line "To be or not to be" in literature, combining depth with brevity.



Worksheet

Reminders

1) Point Recovery for second midterm

2) HW 28 due Wednesday

The End