8/30/2021 near Mather house

Lecture 31

11/17/2021

The general case

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Four Cases

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

Two real different

Double real

Purely complex

Mixed

Examples

Find the general Solution

$$x''(t) - 49x(t) = 0$$

$$x''(t) - 14x'(t) + 49x(t) = 0$$

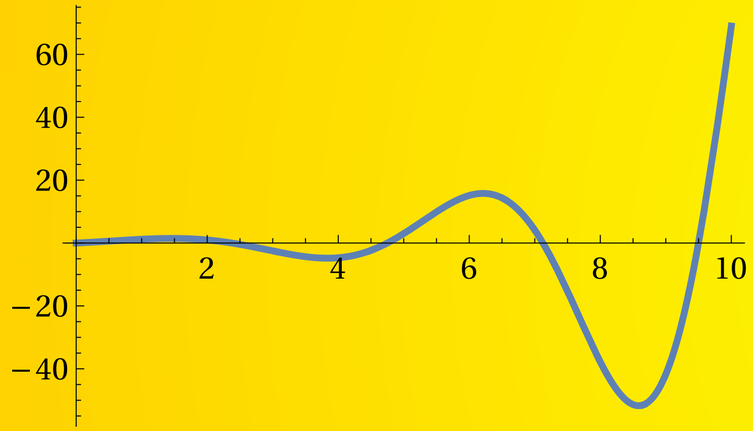
$$x''(t) + 49x(t) = 0$$

$$x''(t) + 2x'(t) + 10x(t) = 0$$

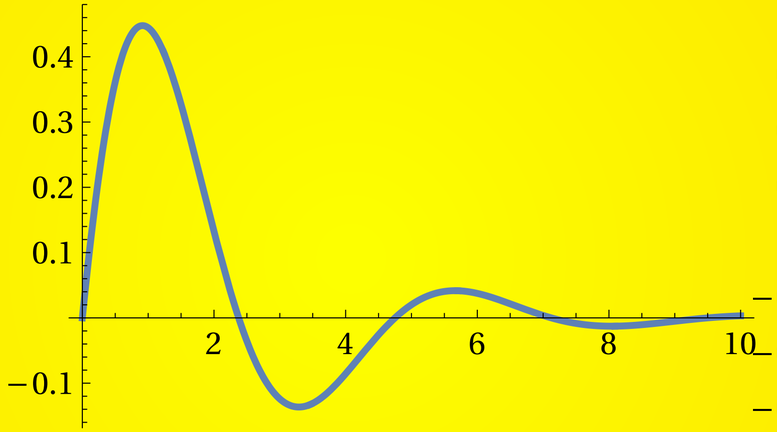
Matching Problem

Diffeq	A-F	Diffeq	A-F
$x'' + 6x = 0$		$x'' - x' + 6x = 0$	
$x'' + x' + 2x = 0$		$x'' + x' + 2x = 0$	
$x'' = 0$		$x'' + 2x = 0$	

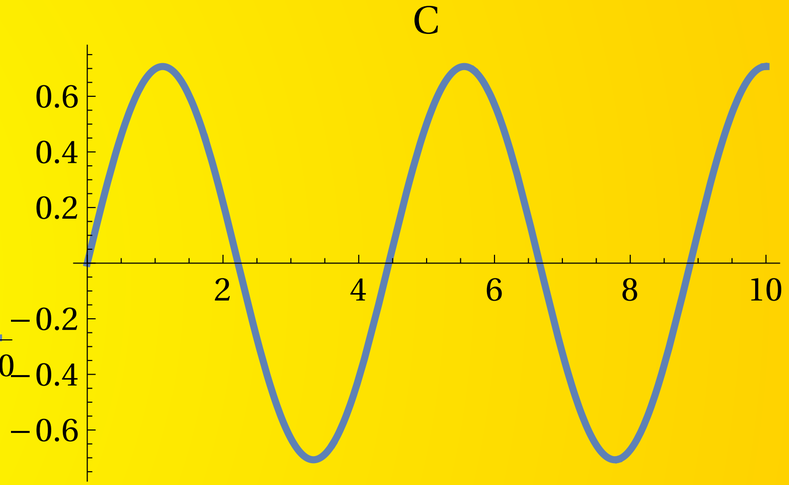
A



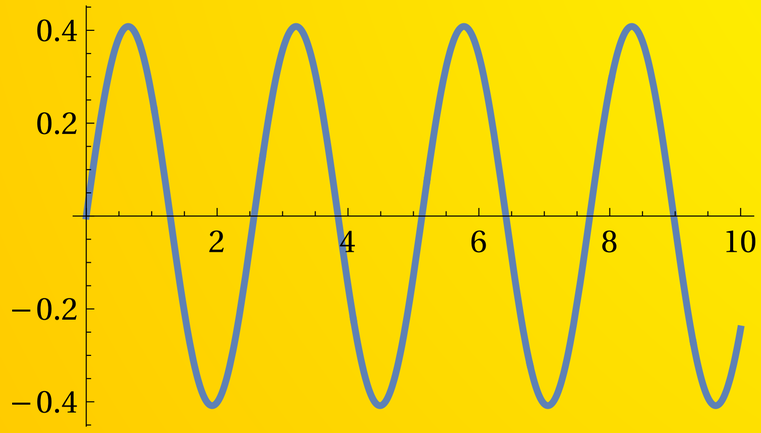
B



C



D



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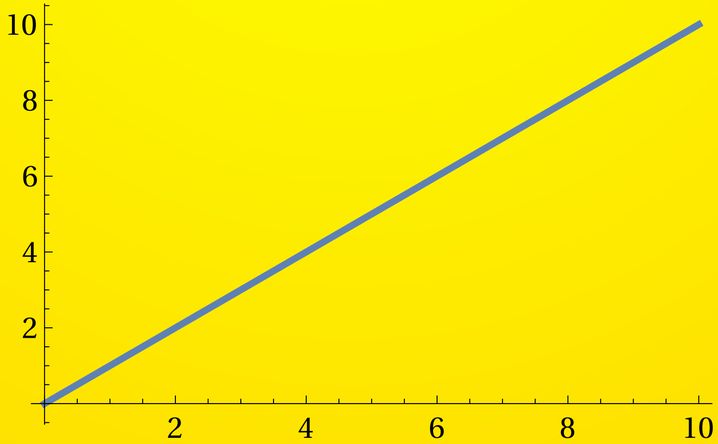
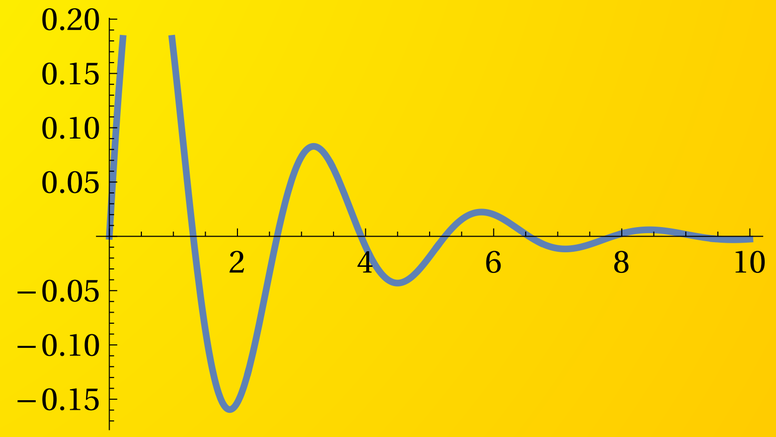
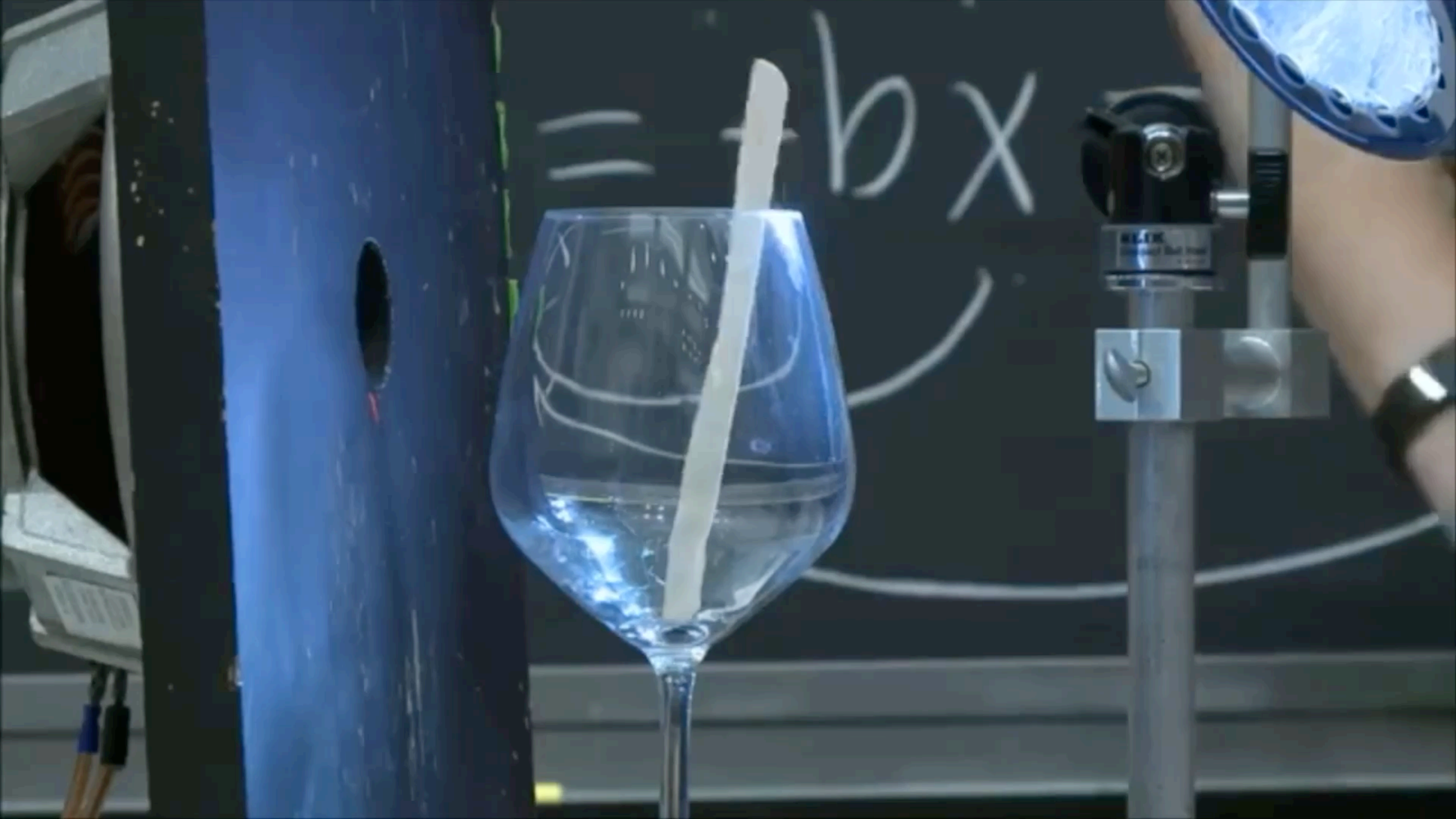


F



Resonance


bx

Worksheet

1)

$$x'' + 6x' + 13 = 0$$

$$x(0) = 7$$

$$x'(0) = -1$$

1) Solution

$$e^{-3t}[7\cos(2t) + 10\sin(2t)]$$

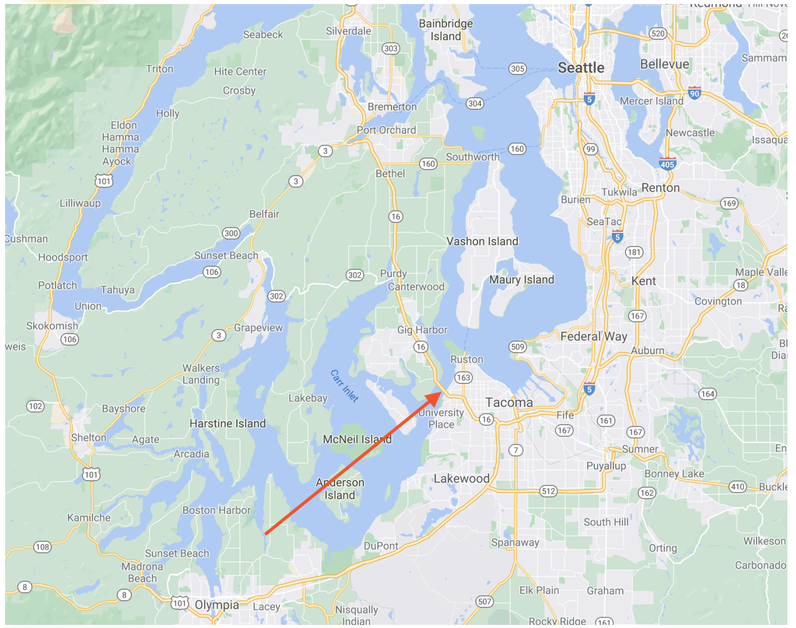
Reminders

QRD

The Tacoma Narrows bridge

Probably you have seen a video of the collapse of the Tacoma Narrows bridge (for example https://www.youtube.com/watch?v=j-zczJXSxnw). Here is a screen shot from the video of the bridge collapsing:

By way of background, the Tacoma Narrows bridge connects the city of Tacoma in Washington State to Gig Harbor and other towns on the opposite side of a branch of Puget Sound. The original version of the bridge opened in July of 1940; and that version collapsed four months later, on November 7, 1940. The red arrow in the image below points to the bridge that crosses Tacoma Narrows today.



Even if you have seen the video of the collapse before, please watch it now because we are going to do some Math 1b analysis of what is depicted there. But be careful, there are many videos of the collapse on the web. Please watch this one:

https://www.youtube.com/watch?v=j-zczJXSxnw .

At around 2:00 in the video the camera is aimed down the central yellow line of the bridge and you see it oscillating side to side in a periodic, sinusoidal motion. This is reminiscent of some of the second order differential equations that we explored in Math 1b. Remember that some of them did have solutions that were periodic with respect to time. In particular, remember that if ω is a positive number, then a differential equation for a function of a real variable t having the form

$$\frac{d^2}{dt^2} a + \omega^2 a = 0 \qquad (1)$$

has the general solution

1) Point Recovery for second midterm

2) HW 29 due Friday

3) QRD problem

The End