
8/30/2021 near Mather house

Lecture 32

11/19/2021

Systems of Diff eq

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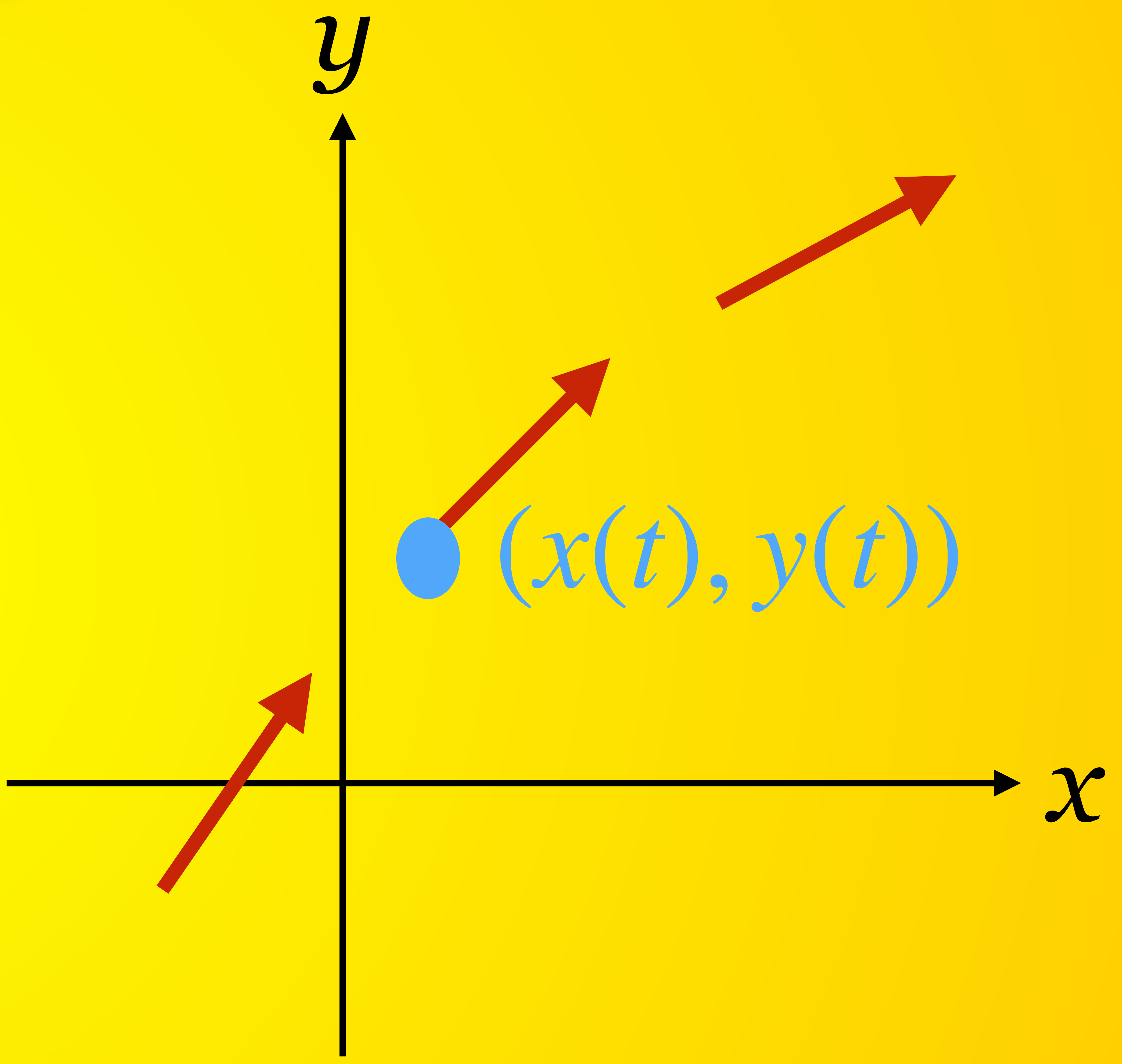
Systems

$$x'(t) = f(x, y)$$
$$y'(t) = g(x, y)$$



tracks two quantities x,y
in time t.

3 Cases we have seen

A)

$$x'(t) = y$$
$$y'(t) = F(x)$$

equivalent

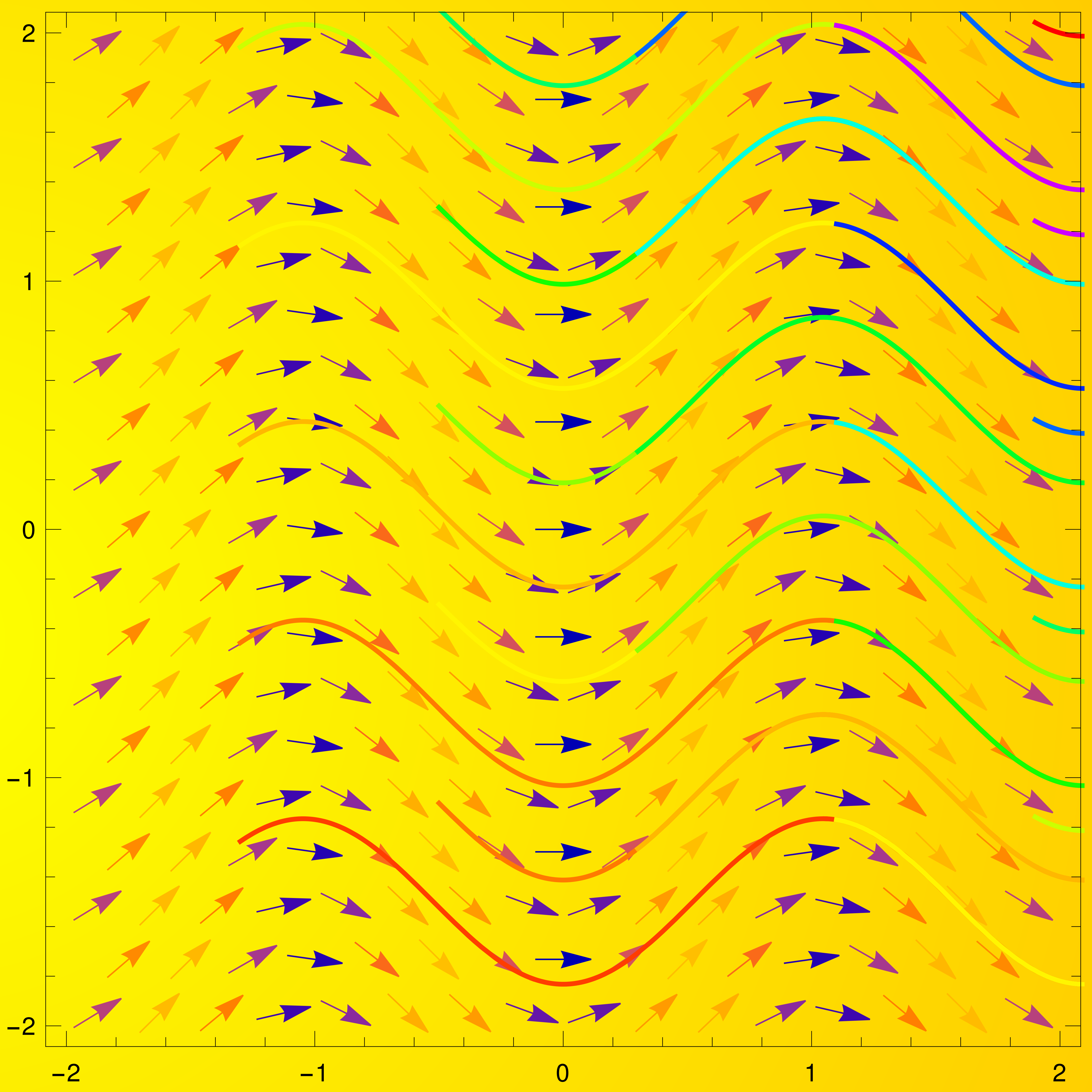
$$x''(t) = F(x)$$

we called this a homogeneous differential equation. Example: x''=9x

$\mathcal{B}$)

$$x'(t) = 1$$
$$y'(t) = g(x)$$

$$x'(t) = g(t)$$

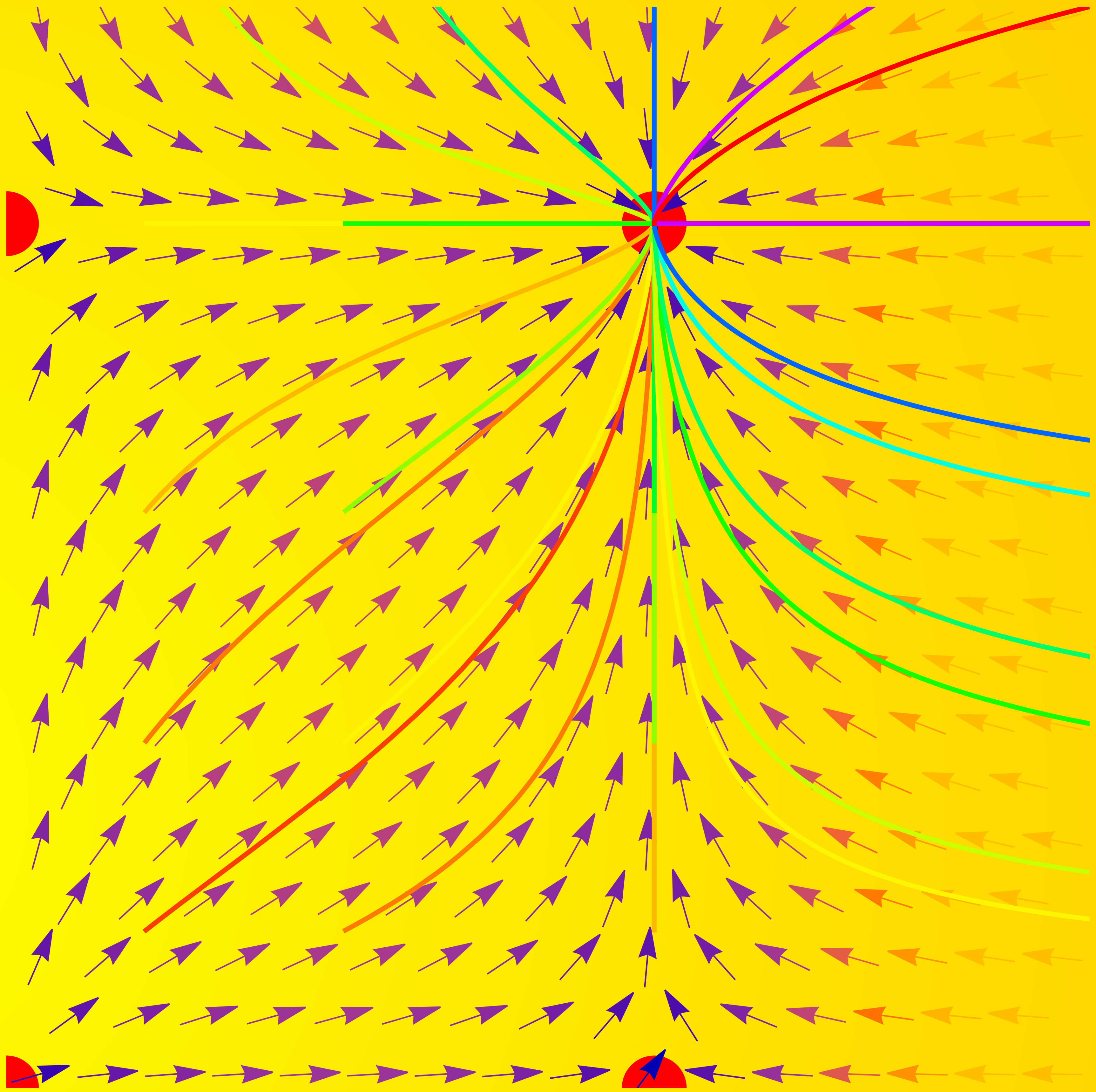


this was the case of slope fields.

C)

$$x'(t) = f(x)$$

$$y'(t) = g(y)$$



decoupled systems. Each variable evolves separately.

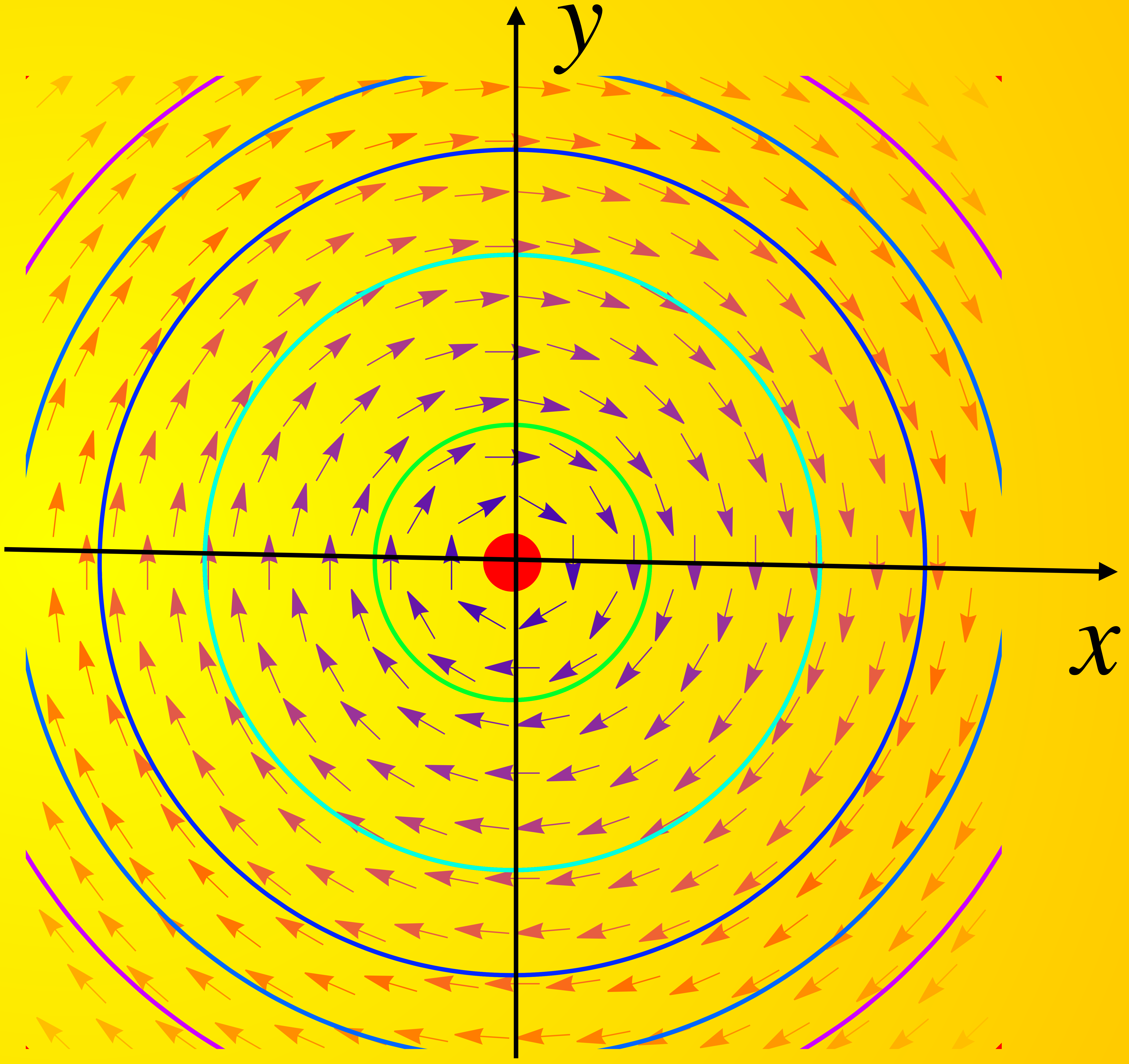
Harmonic Oscillator

we know that already as $x''=-x$,
we look at it now as a system

Harmonic Oscillator

$$x'(t) = y$$

$$y'(t) = -x$$



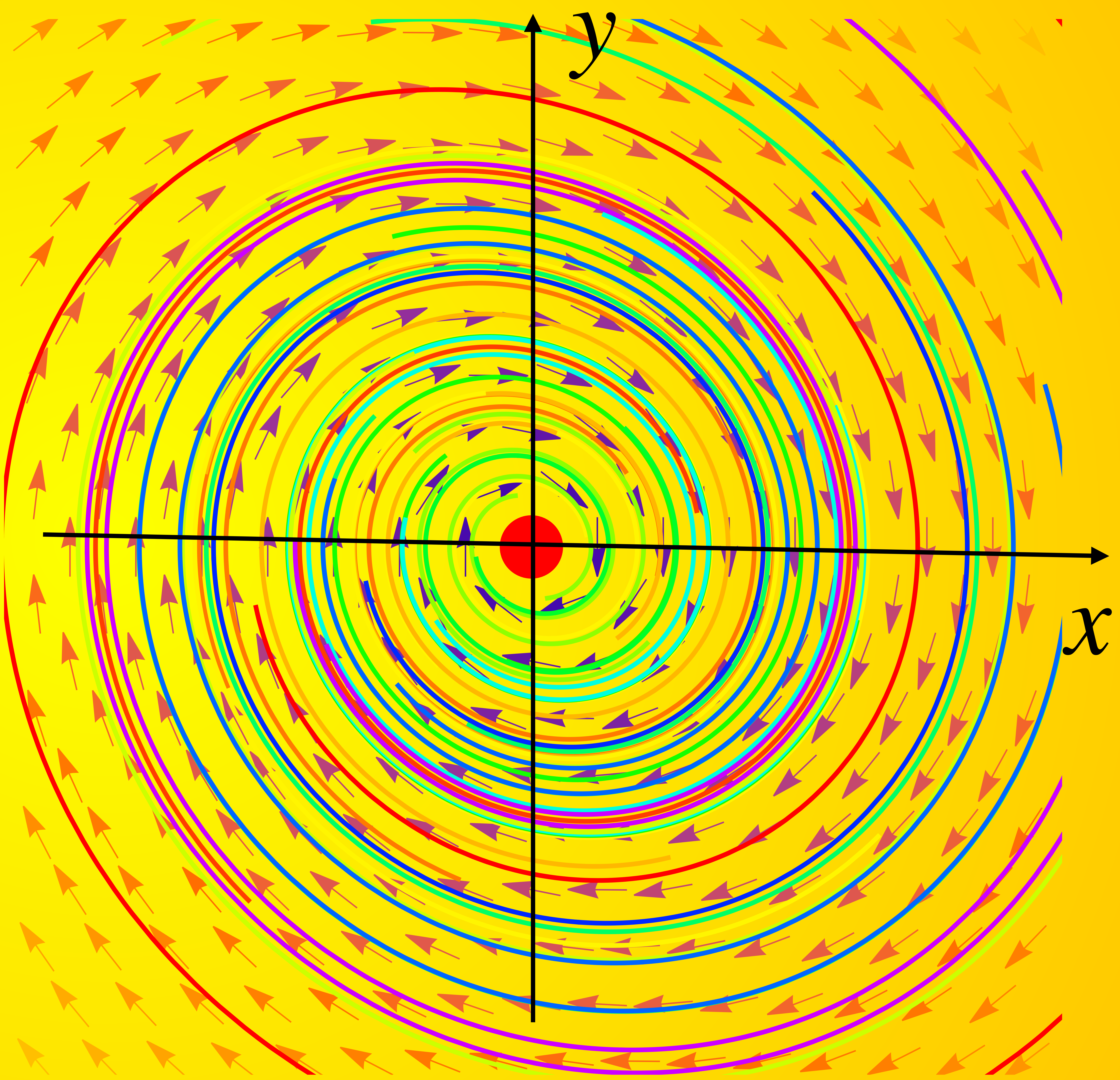
Oscillator with damping

we know that already as $x''=-x$,
we look at it now as a system

Damped Oscillator

$$x'(t) = y$$

$$y'(t) = -x - by$$



Population models

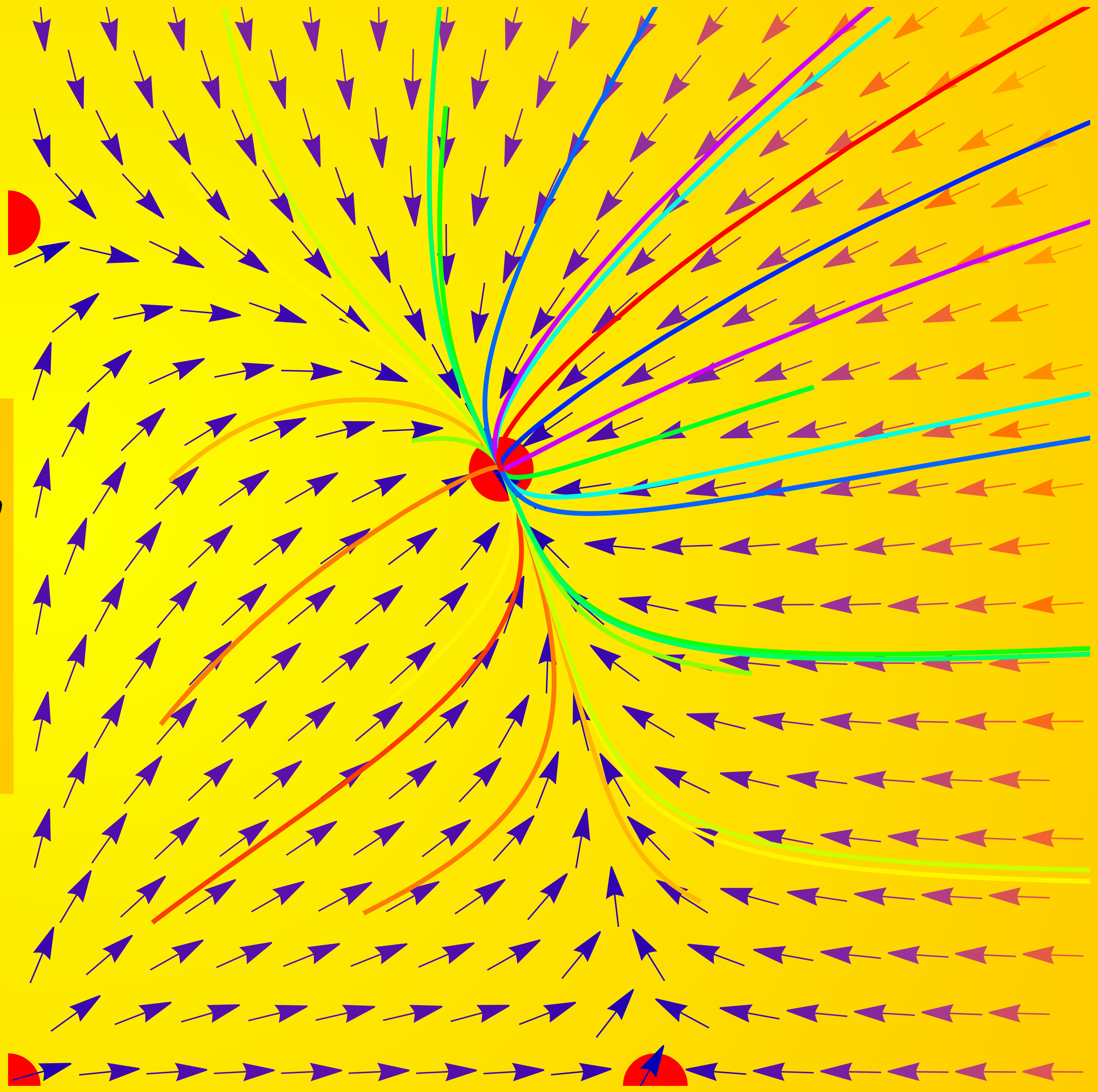
Competing

$$x'(t) = x(6 - 2x) - xy$$

$$y'(t) = y(4 - y) - xy$$

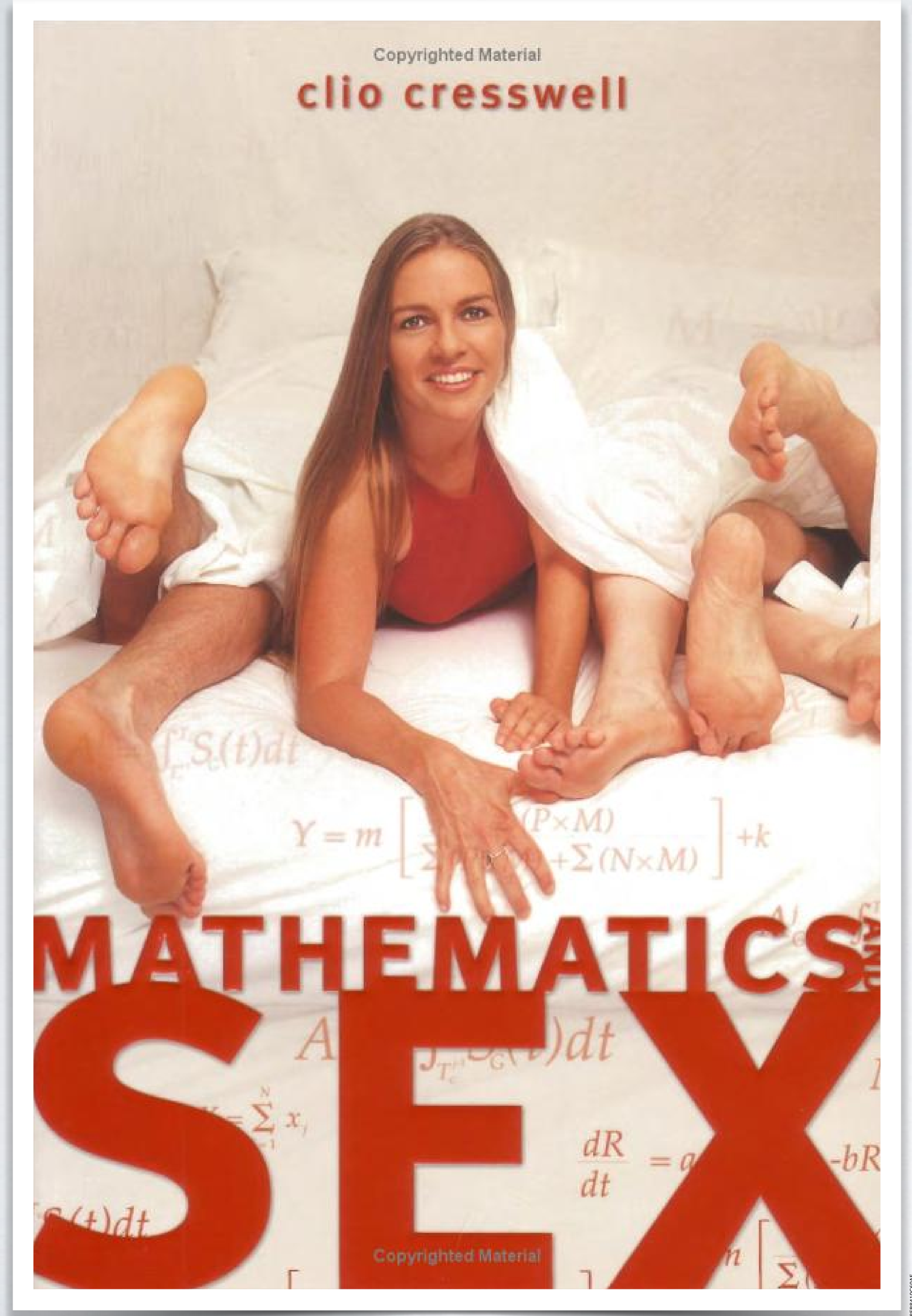
growth of x
gets slowed
with large y

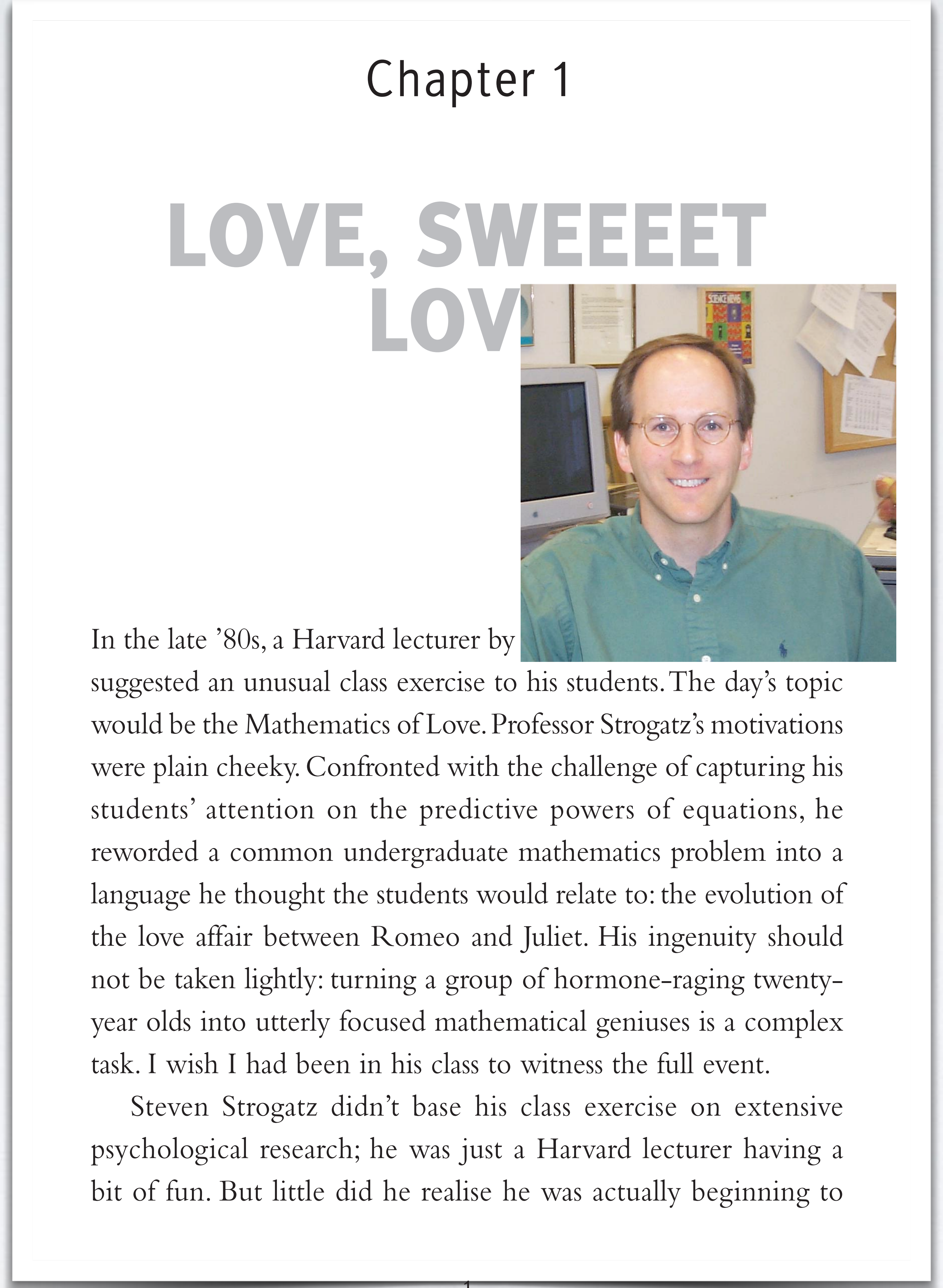
growth of y
gets slowed
with large x



Source of the Strogatz story

SOURCE





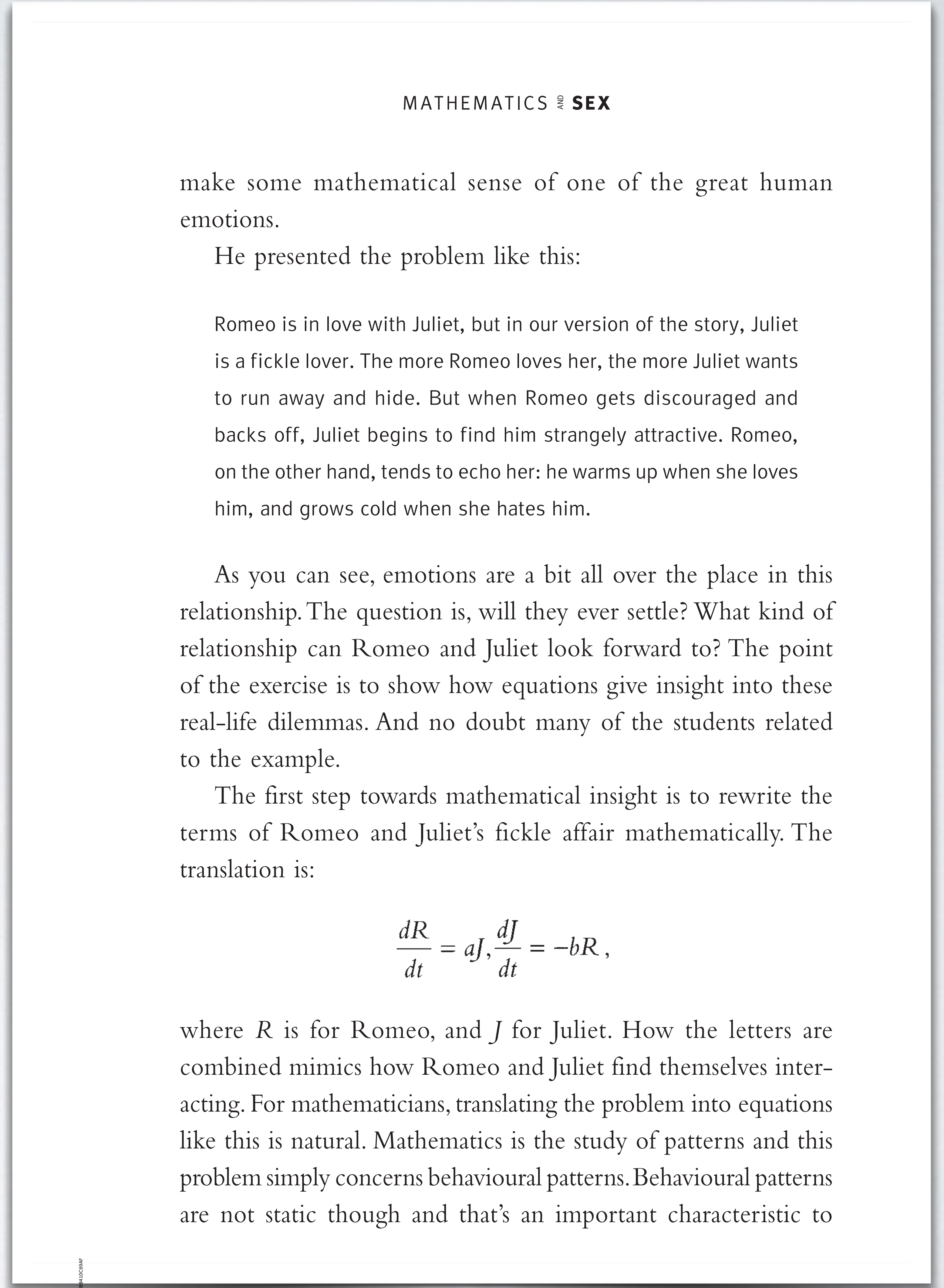
Chapter 1

LOVE, SWEEEET LOV

In the late '80s, a Harvard lecturer by suggested an unusual class exercise to his students. The day's topic would be the Mathematics of Love. Professor Strogatz's motivations were plain cheeky. Confronted with the challenge of capturing his students' attention on the predictive powers of equations, he reworded a common undergraduate mathematics problem into a language he thought the students would relate to: the evolution of the love affair between Romeo and Juliet. His ingenuity should not be taken lightly: turning a group of hormone-raging twenty-year olds into utterly focused mathematical geniuses is a complex task. I wish I had been in his class to witness the full event.

Steven Strogatz didn't base his class exercise on extensive psychological research; he was just a Harvard lecturer having a bit of fun. But little did he realise he was actually beginning to

1



MATHEMATICS AND SEX

make some mathematical sense of one of the great human emotions.

He presented the problem like this:

> Romeo is in love with Juliet, but in our version of the story, Juliet is a fickle lover. The more Romeo loves her, the more Juliet wants to run away and hide. But when Romeo gets discouraged and backs off, Juliet begins to find him strangely attractive. Romeo, on the other hand, tends to echo her: he warms up when she loves him, and grows cold when she hates him.

As you can see, emotions are a bit all over the place in this relationship. The question is, will they ever settle? What kind of relationship can Romeo and Juliet look forward to? The point of the exercise is to show how equations give insight into these real-life dilemmas. And no doubt many of the students related to the example.

The first step towards mathematical insight is to rewrite the terms of Romeo and Juliet's fickle affair mathematically. The translation is:

$$\frac{dR}{dt} = aJ, \frac{dJ}{dt} = -bR,$$

where R is for Romeo, and J for Juliet. How the letters are combined mimics how Romeo and Juliet find themselves interacting. For mathematicians, translating the problem into equations like this is natural. Mathematics is the study of patterns and this problem simply concerns behavioural patterns. Behavioural patterns are not static though and that's an important characteristic to

2

Some of Oliver's slides from 22b, ~ 2006

In the late '80s, a Harvard lecturer by the name of Steven Strogatz suggested an unusual class exercise to his students. The day's topic would be the Mathematics of Love. Professor Strogatz's motivations were plain cheeky. Confronted with the challenge of capturing his students' attention on the predictive powers of equations, he reworded a common undergraduate mathematics problem into a language he thought the students would relate to: the evolution of the love affair between Romeo and Juliet. His ingenuity should not be taken lightly: turning a group of hormone-raging twenty-year olds into utterly focused mathematical geniuses is a complex task. I wish I had been in his class to witness the full event.

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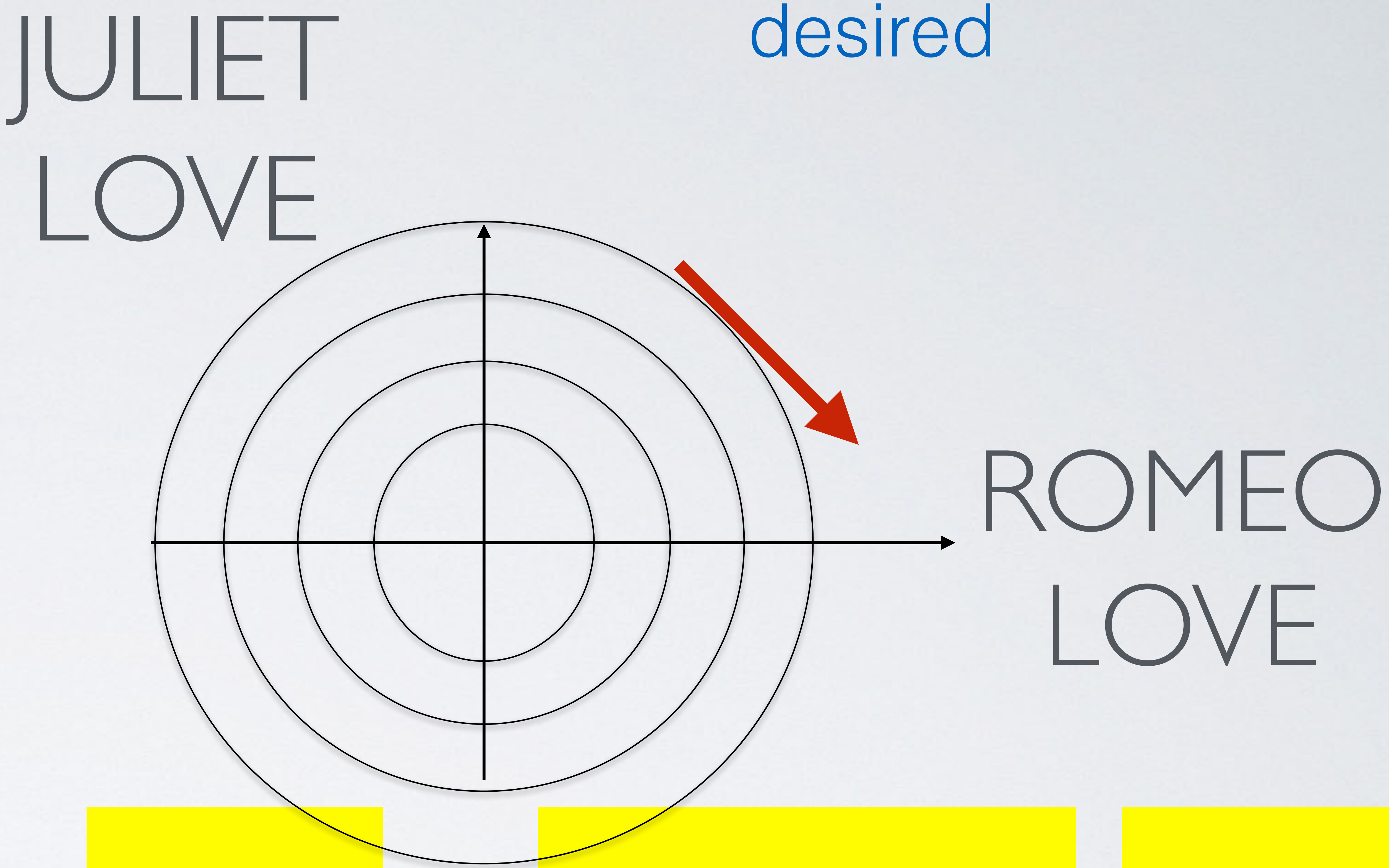
Romeo warms up when given love
Juliet runs away when being desired



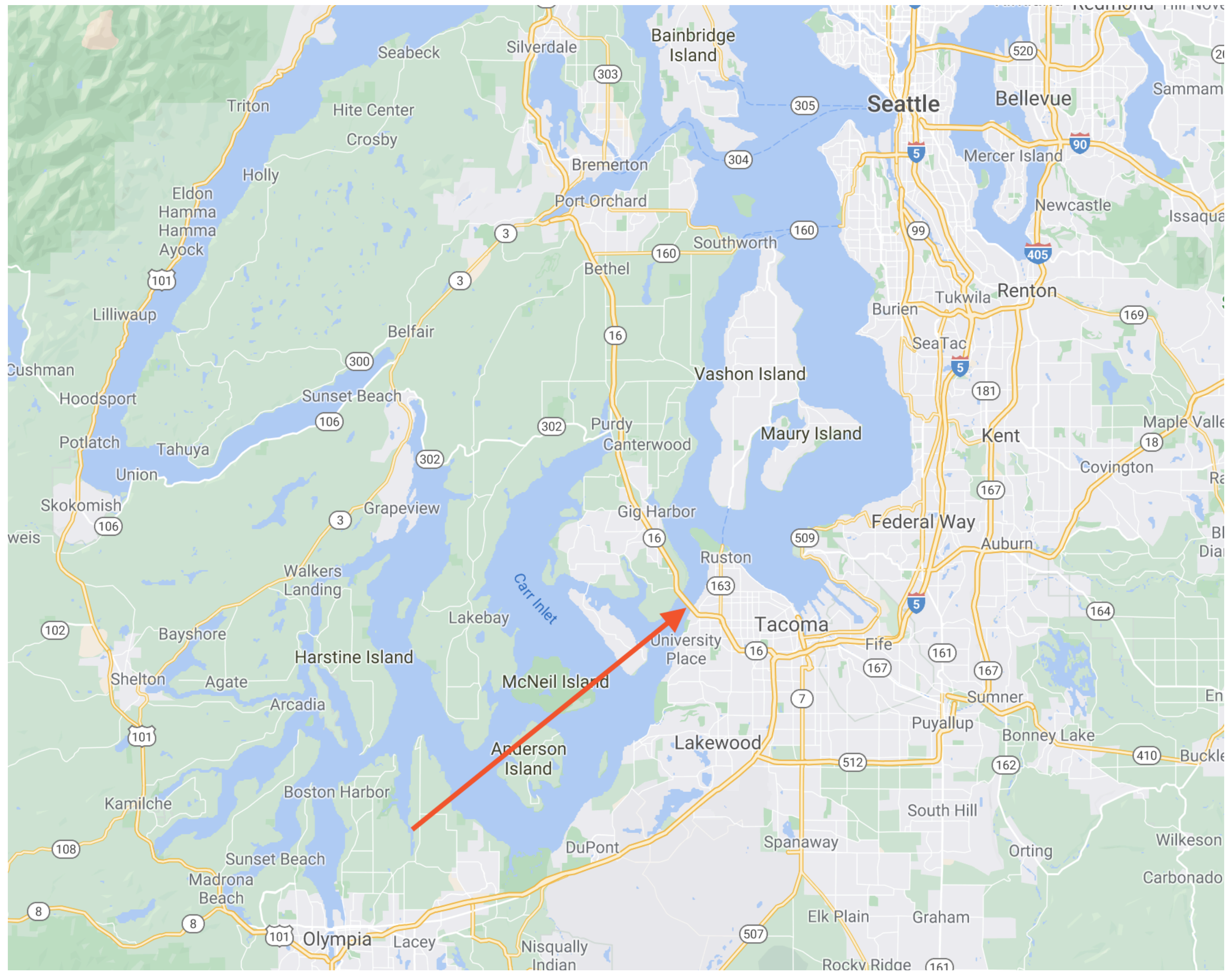
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Reminders

The Tacoma Narrows bridge

Probably you have seen a video of the collapse of the Tacoma Narrows bridge (for example https://www.youtube.com/watch?v=j-zczJXSxnw). Here is a screen shot from the video of the bridge collapsing:

By way of background, the Tacoma Narrows bridge connects the city of Tacoma in Washington State to Gig Harbor and other towns on the opposite side of a branch of Puget Sound. The original version of the bridge opened in July of 1940; and that version collapsed four months later, on November 7, 1940. The red arrow in the image below points to the bridge that crosses Tacoma Narrows today.



Even if you have seen the video of the collapse before, please watch it now because we are going to do some Math 1b analysis of what is depicted there. But be careful, there are many videos of the collapse on the web. Please watch this one:

https://www.youtube.com/watch?v=j-zczJXSxnw .

At around 2:00 in the video the camera is aimed down the central yellow line of the bridge and you see it oscillating side to side in a periodic, sinusoidal motion. This is reminiscent of some of the second order differential equations that we explored in Math 1b. Remember that some of them did have solutions that were periodic with respect to time. In particular, remember that if ω is a positive number, then a differential equation for a function of a real variable t having the form

$$\frac{d^2}{dt^2} a + \omega^2 a = 0 \tag{1}$$

has the general solution

2) HW 30 due Monday

3) QRD problem due Monday

The End