8/30/2021 near Mather house

Lecture 35

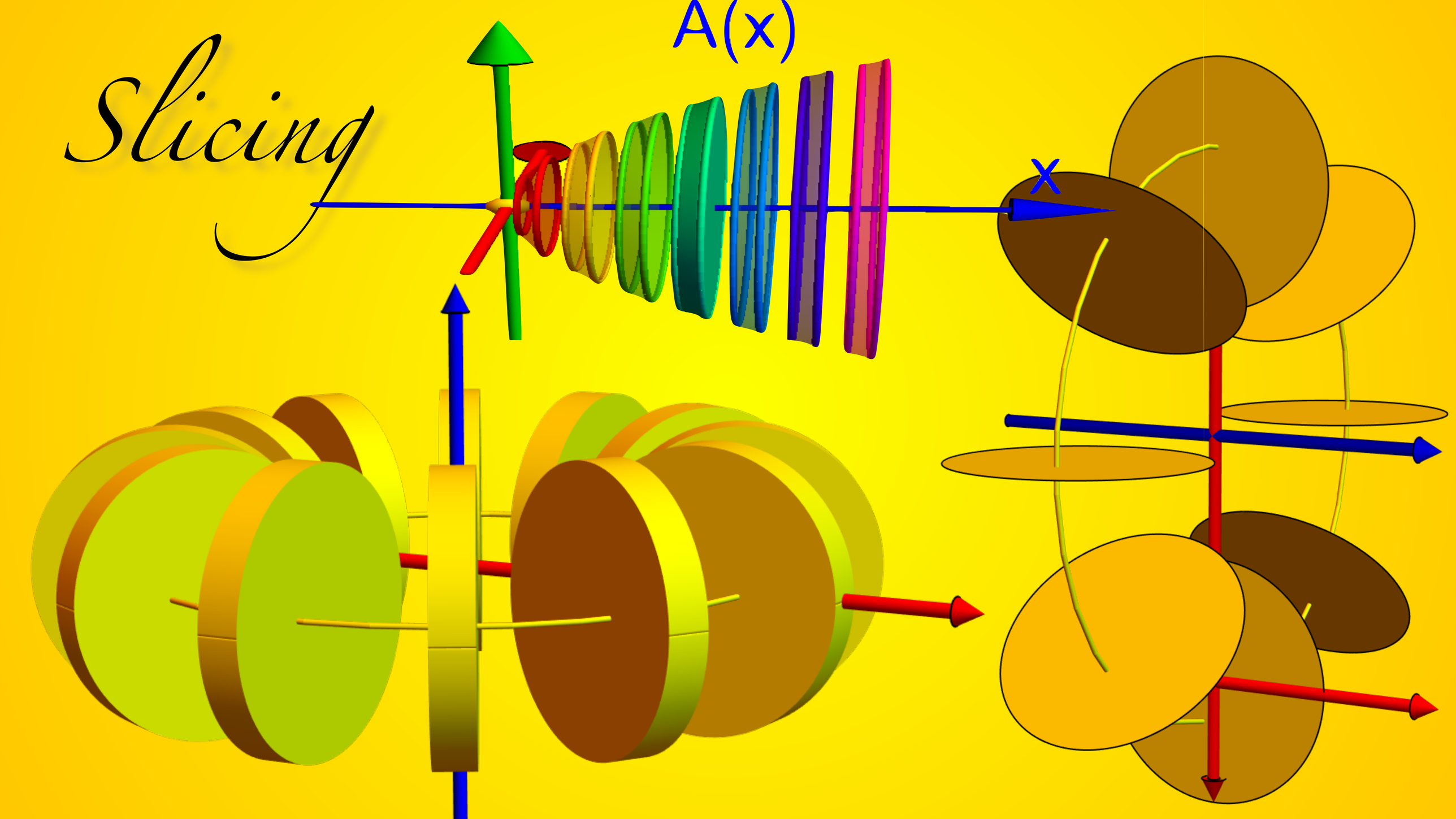
12/01/2021

Overview

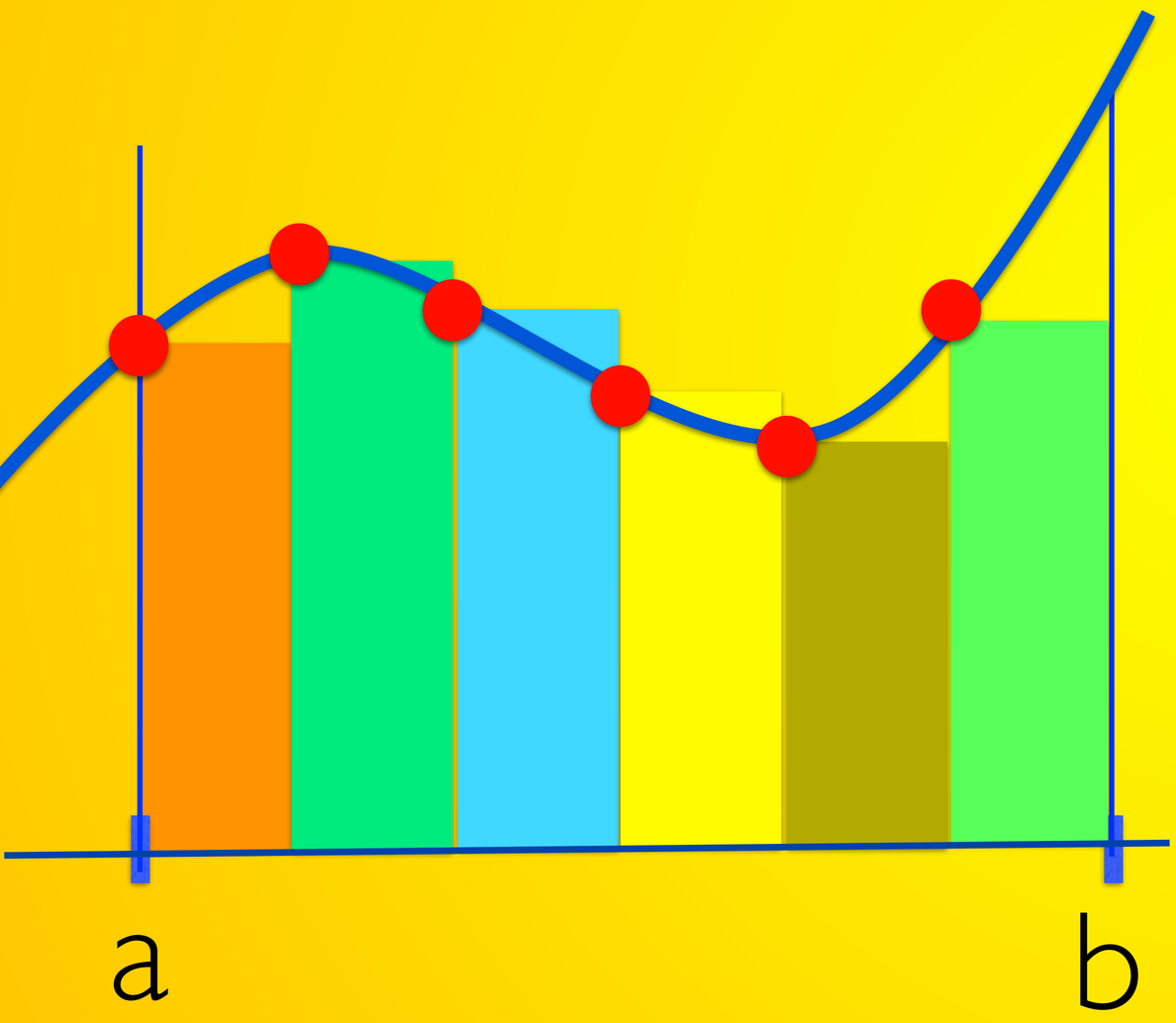
Table of Contents

I Topics

Slicing



Riemann Sums



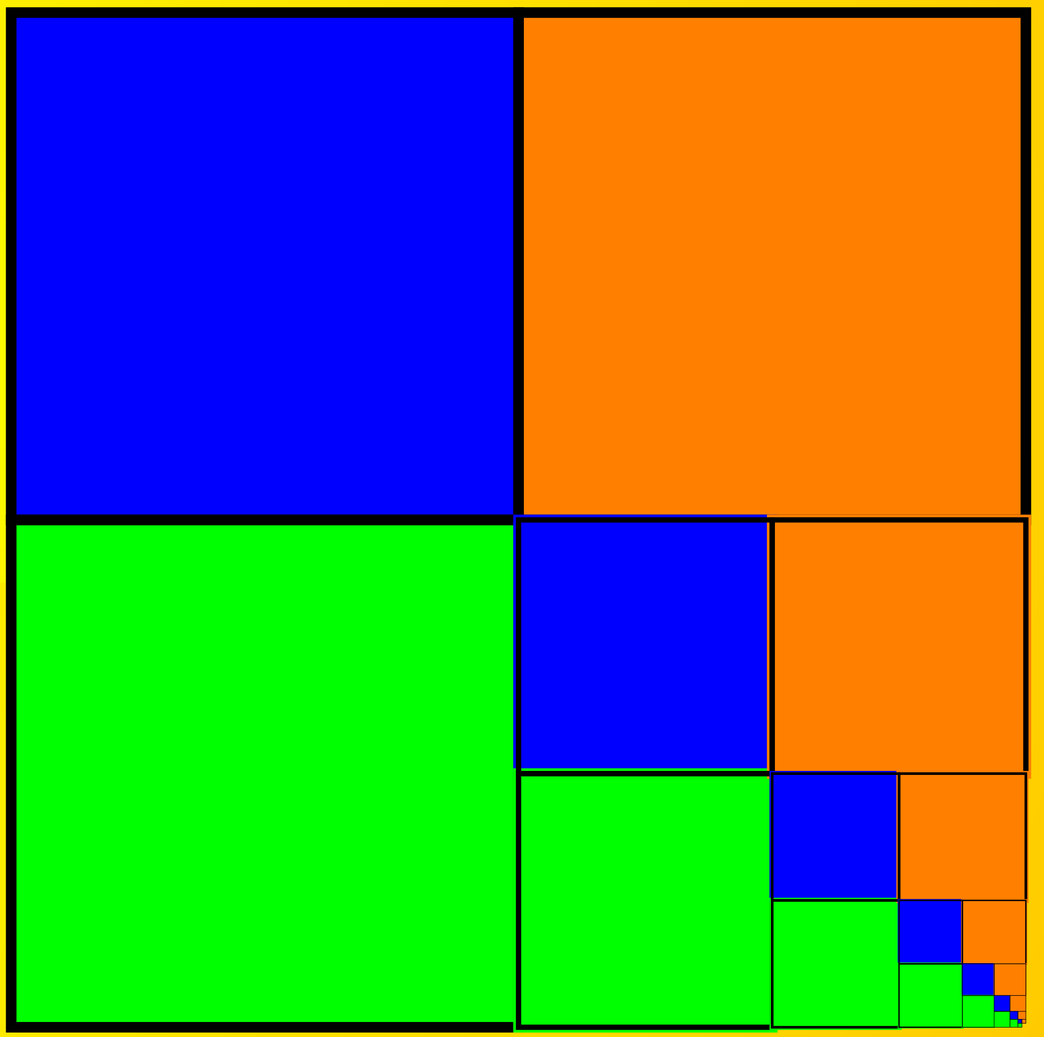
assume: $|f'(x)| \leq M$

$$\frac{M(b-a)^2}{2n}$$

Power Series

$$S = \sum_{k=0}^{\infty} a_k (x - c)^k$$

1/4+1/16+1/64 + ...=1/3



Convergence tests

Ratio

n'th Term

Comparison

Asymptotic

Alternating

Integral

Geometric

p-series

Error Term

$$P_n(x) = f(c) + f'(c)(x-c) + f''(c)\frac{(x-c)^2}{2} + \ldots + f^{(n)}(c)\frac{(x-c)^n}{n!}$$

$$|f(x) - P_n(x)| = |R_n(x)| \leq M_{n+1}\frac{|x-c|^{n+1}}{(n+1)!}$$

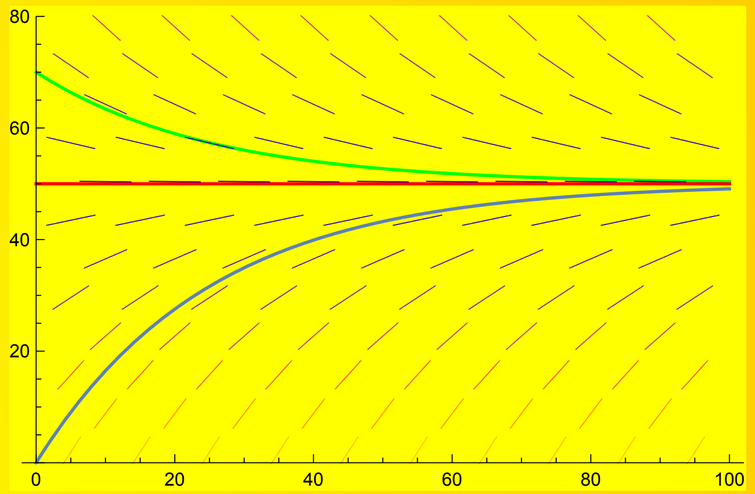
M_{n+1} = maximum of n+1'th derivative of f on [x,c]

Magic

$$\sqrt{101}$$

$$\sqrt{100+x} = 10 + \frac{x}{2} - \frac{x^2}{8 * 10^3}$$

Input-Output



input:

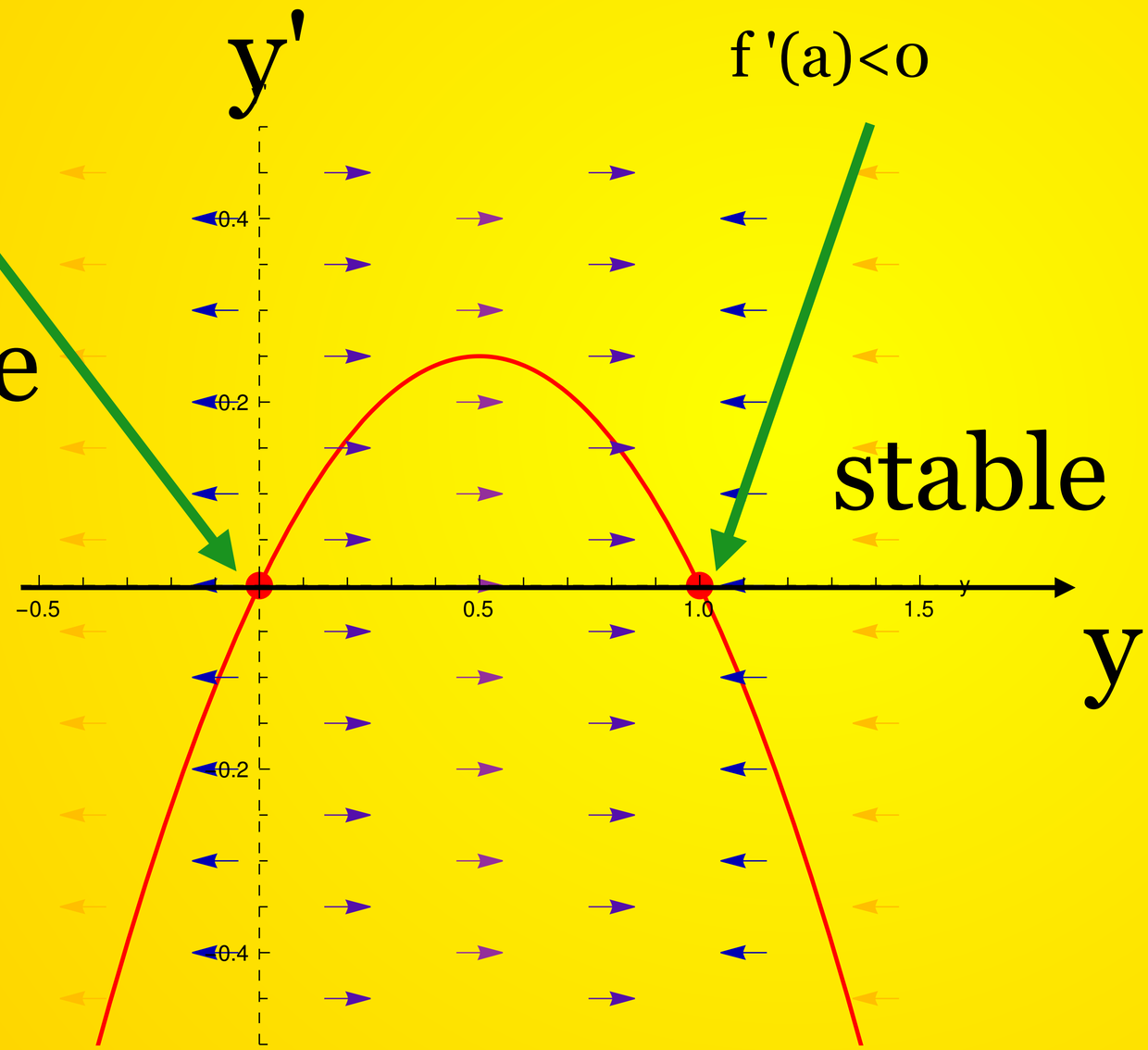
tank

output:

Autonomous

f '(a)>o

f '(a)<o



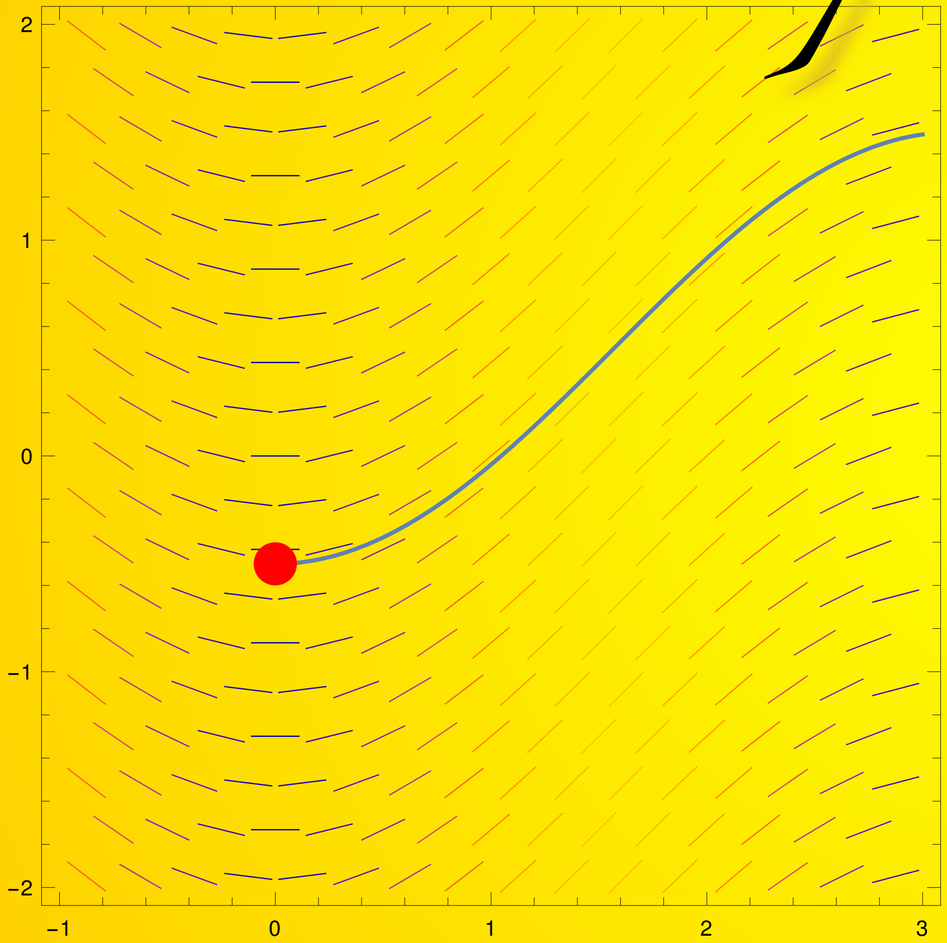
unstable

stable

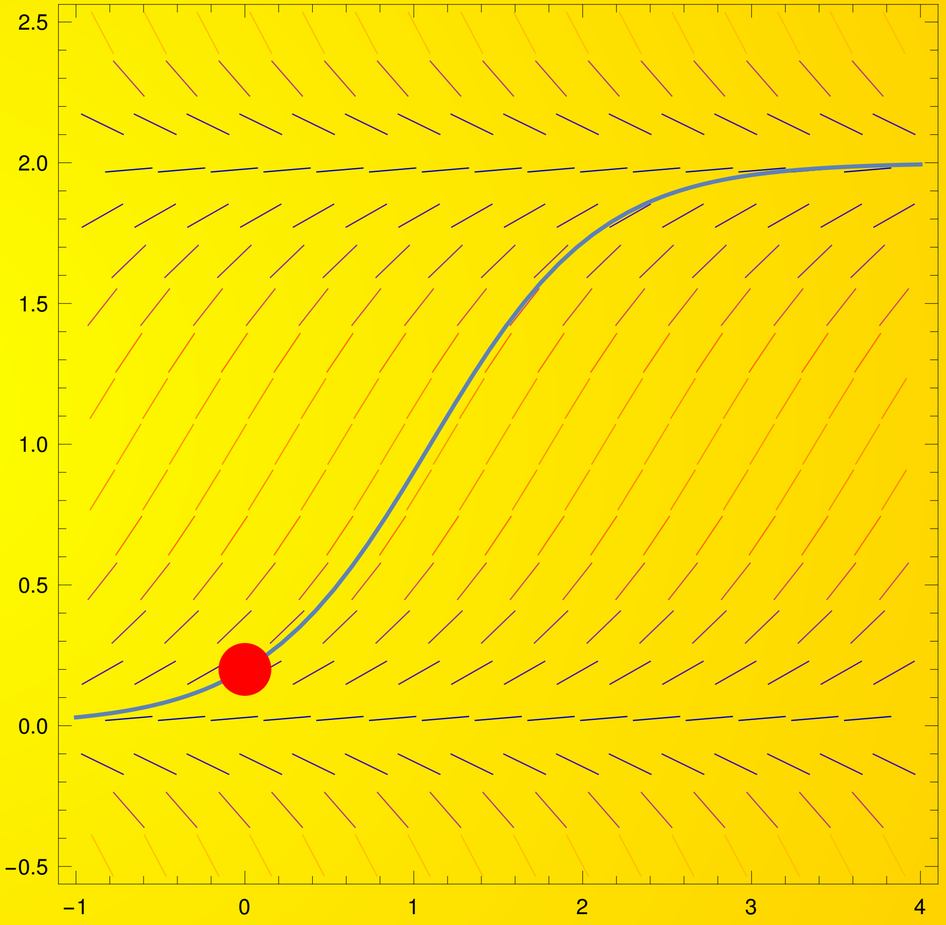
$$y'=f(y)$$

$$=y(1-y)$$

Slope fields



$y' = f(t)$



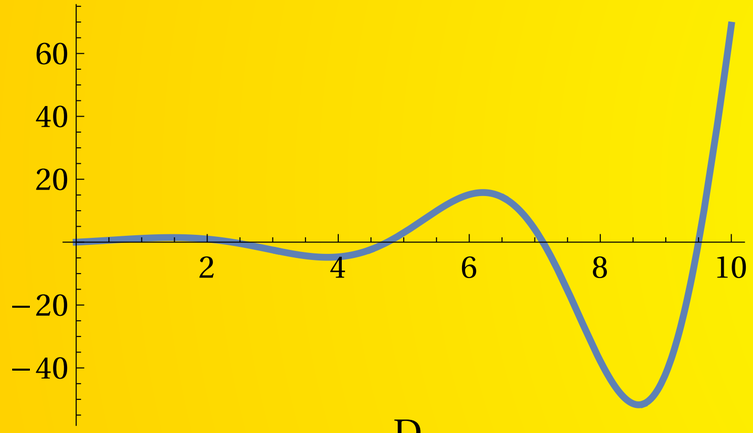
$y' = f(y)$

Second order Diff equ.

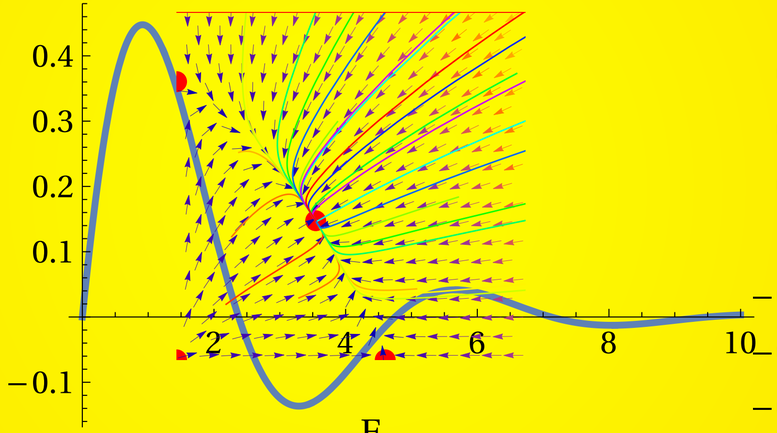
$x'' + 6x = 0$	
$x'' - x' + 2x = 0$	
$x'' = 0$	

$x'' - x' + 6x = 0$	
$x'' + x' + 2x = 0$	
$x'' + 2x = 0$	

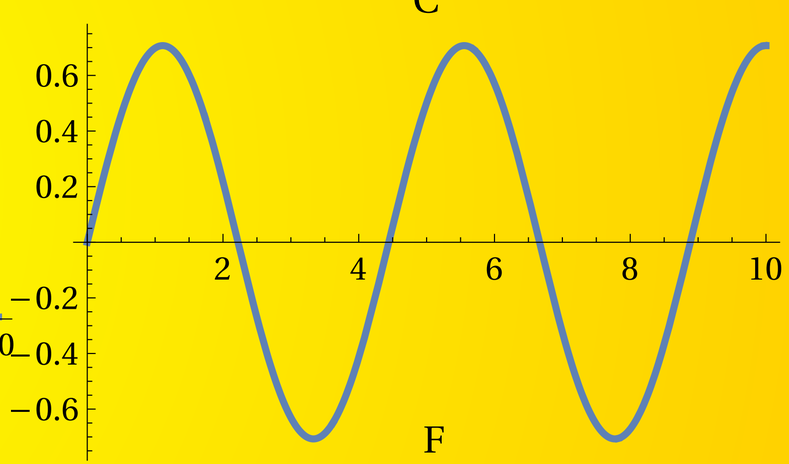
A



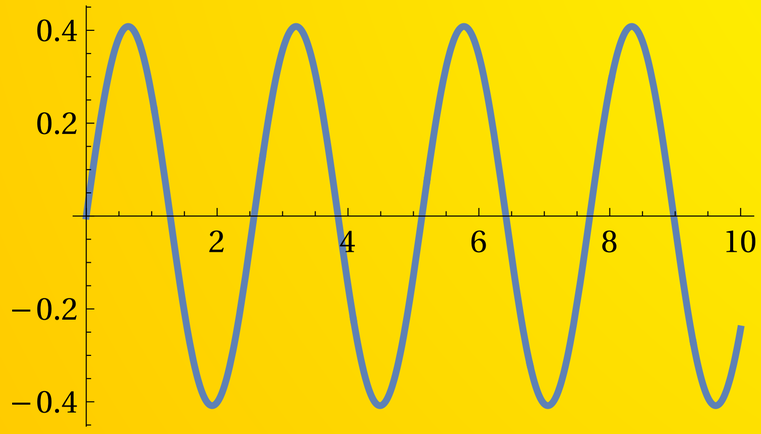
B



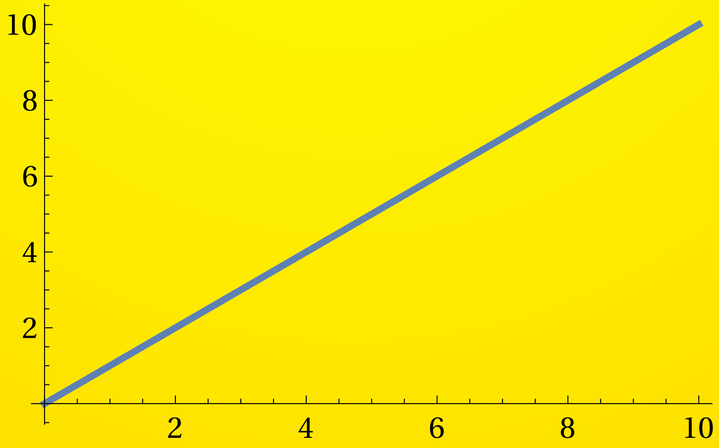
C



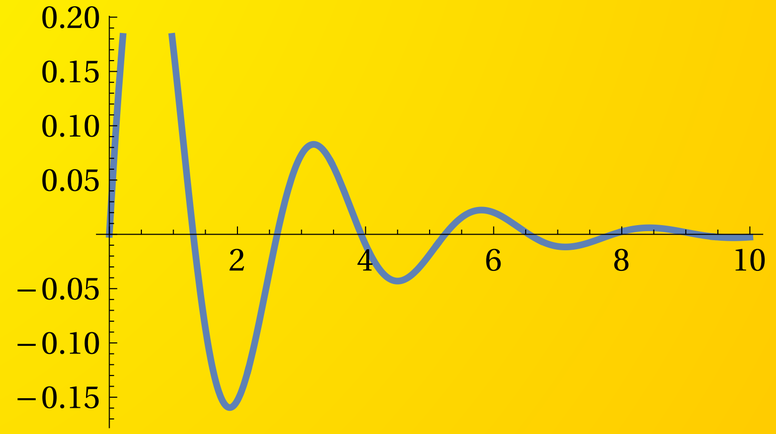
D



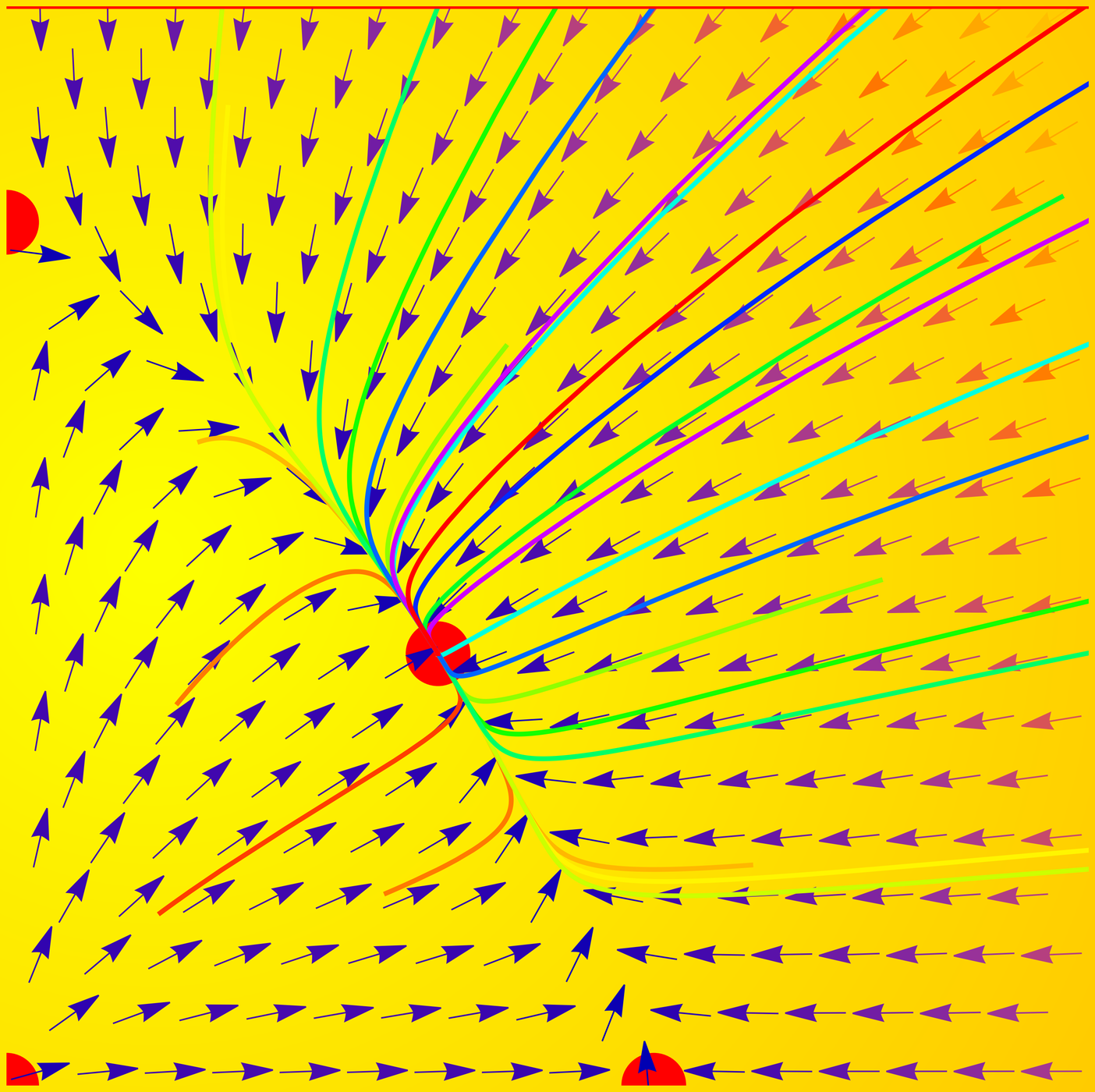
E



F



Systems of equations



Euler's Formula

$$1 + e^{i\pi} = 0$$

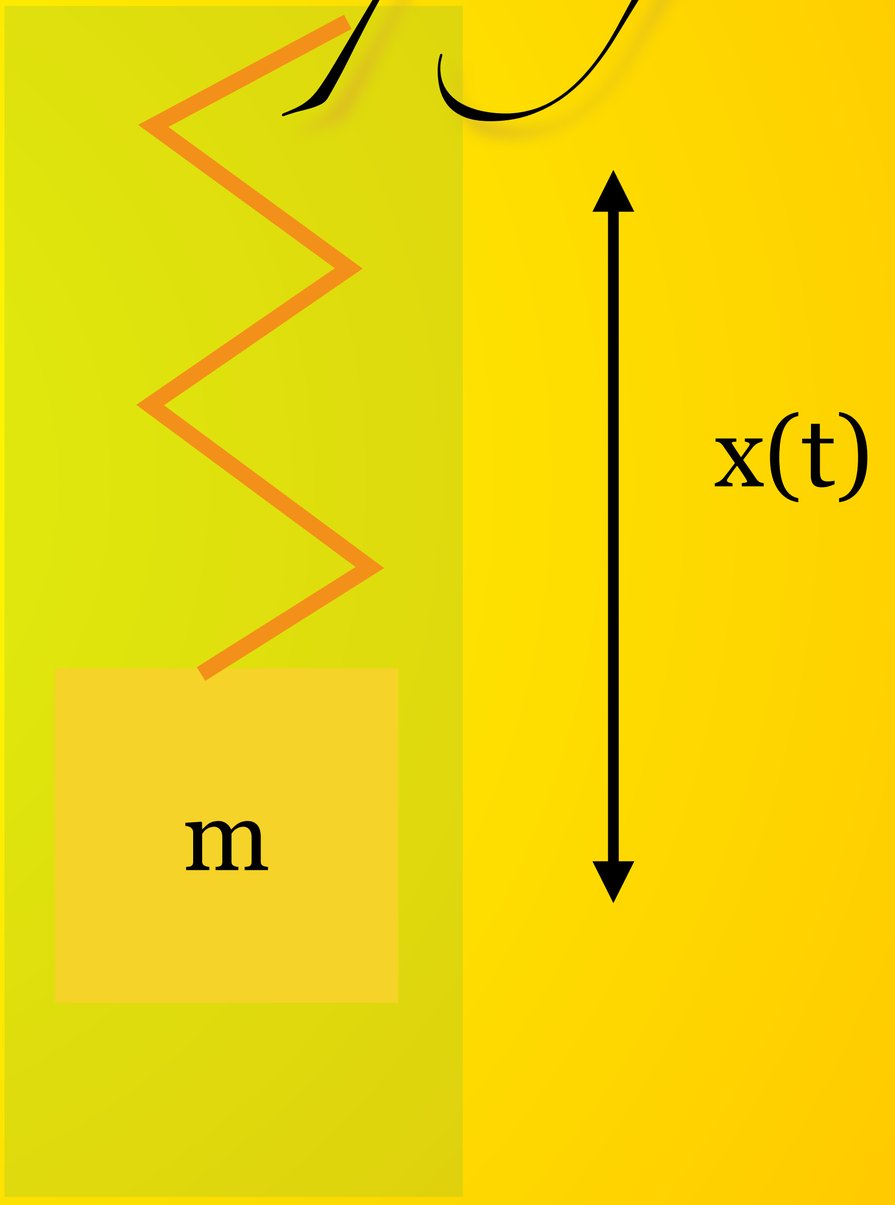
Oscillator with Damping

$$x''(t) = -bx'(t) - cx(t)$$

Acceleration Friction Push-Back

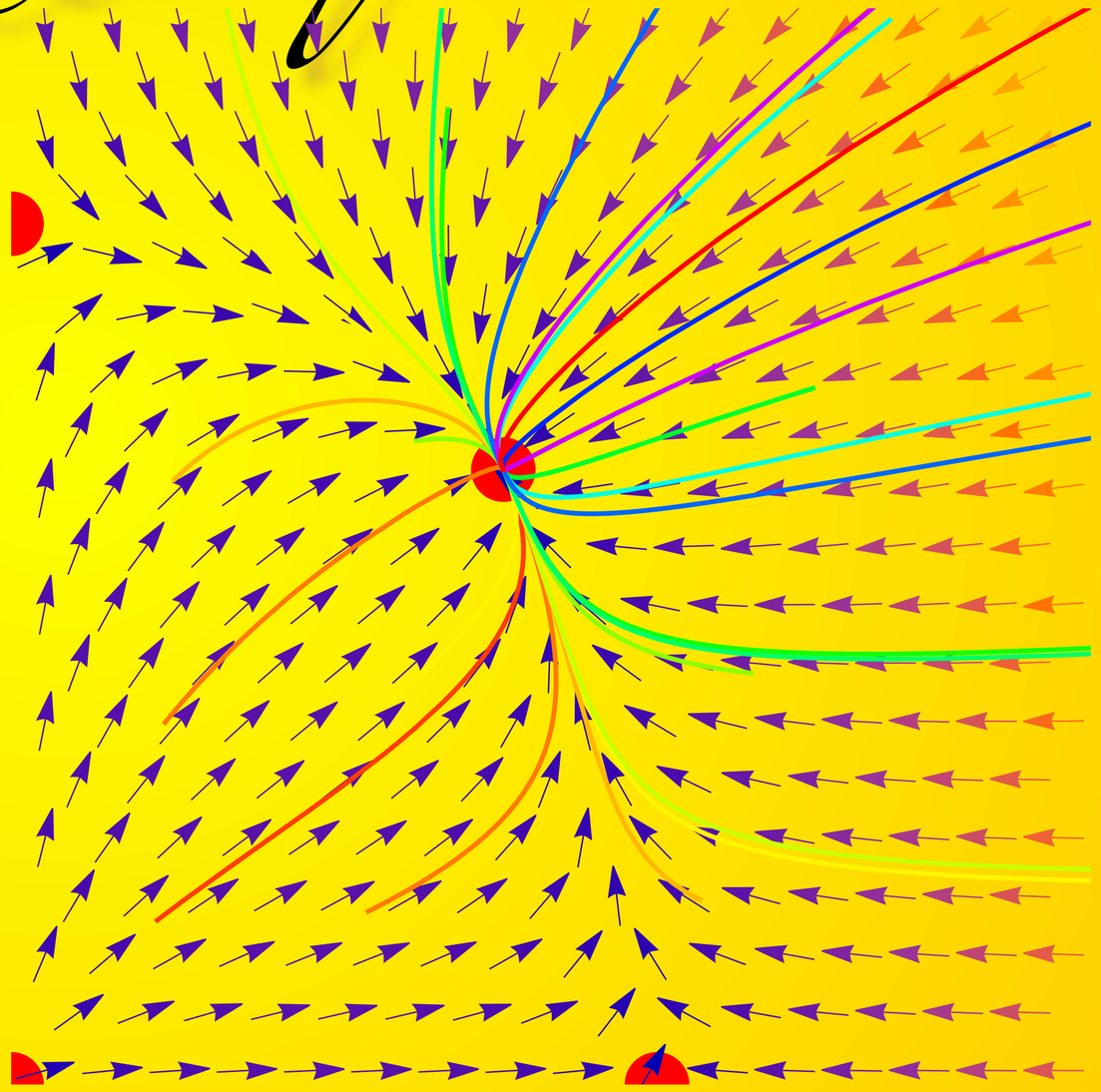
$$x''(t) = F/m$$

Force/Mass



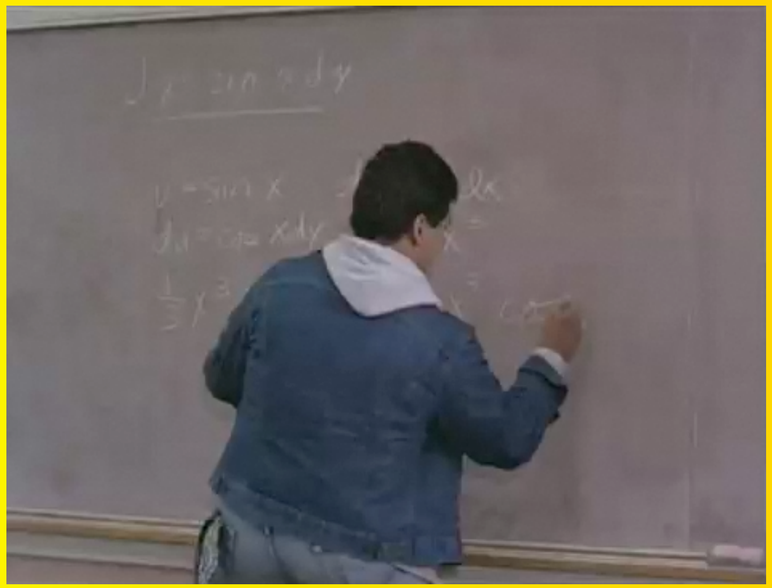
Systems of diff equations

$$x'(t) = f(x, y)$$
$$y'(t) = g(x, y)$$



11 Cool

Integrals in Movies



Improper Integrals

$$r = \frac{1}{x}$$

Surface Area $\geq$

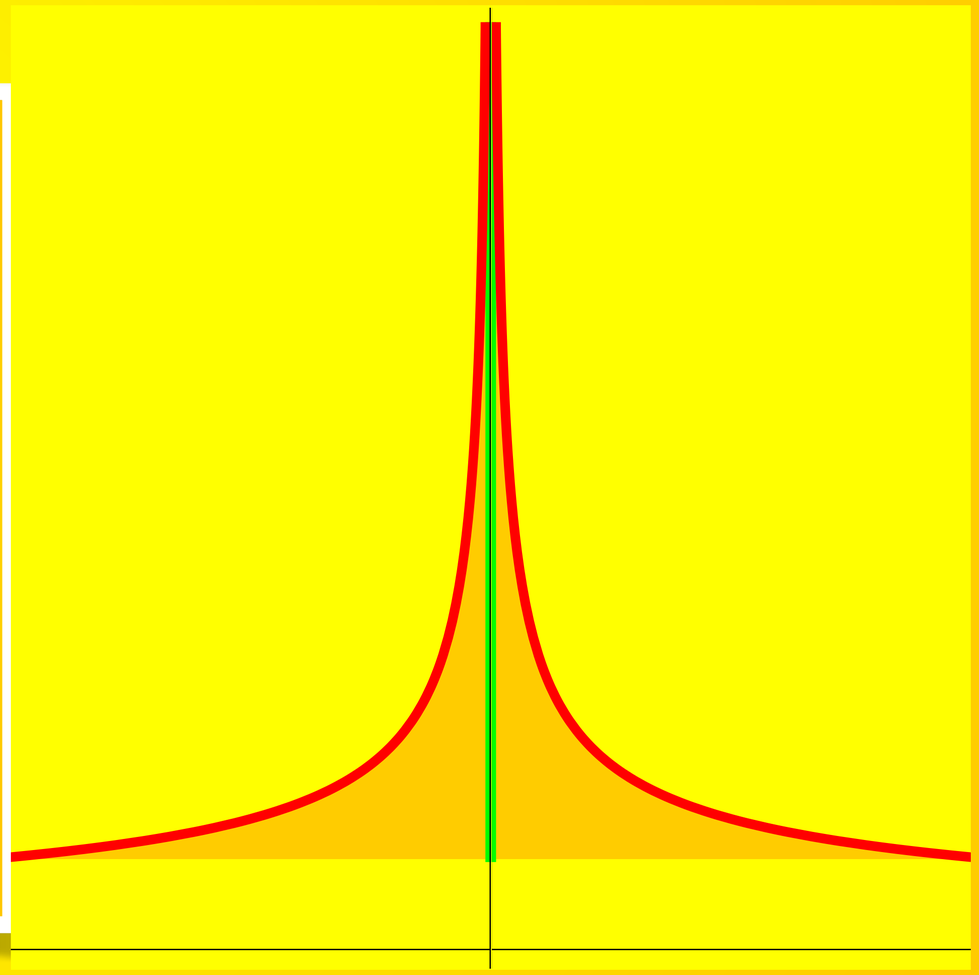
$$\int_1^b \frac{2\pi}{x}\,dx$$

Volume:

$$\int_1^b \frac{\pi}{x^2}\,dx$$

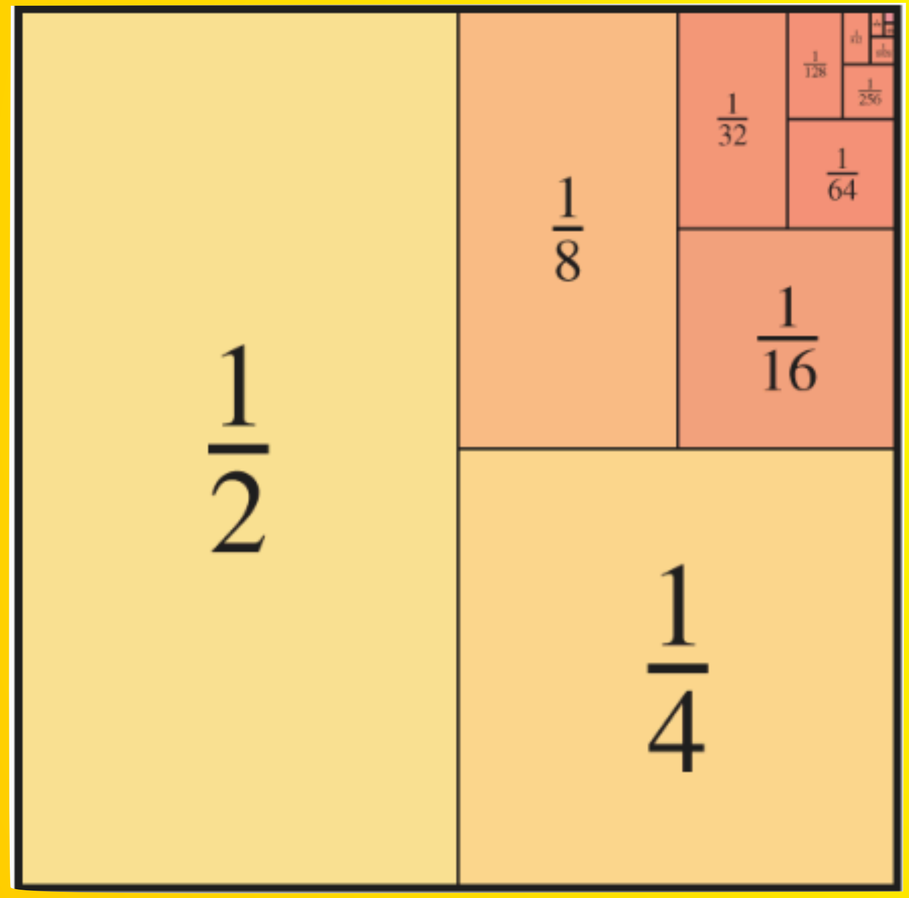
A Shock!

$$\int_{-1}^{1} \frac{1}{x^2} dx = -\frac{1}{x}\Big|_{-1}^{1} = -2$$

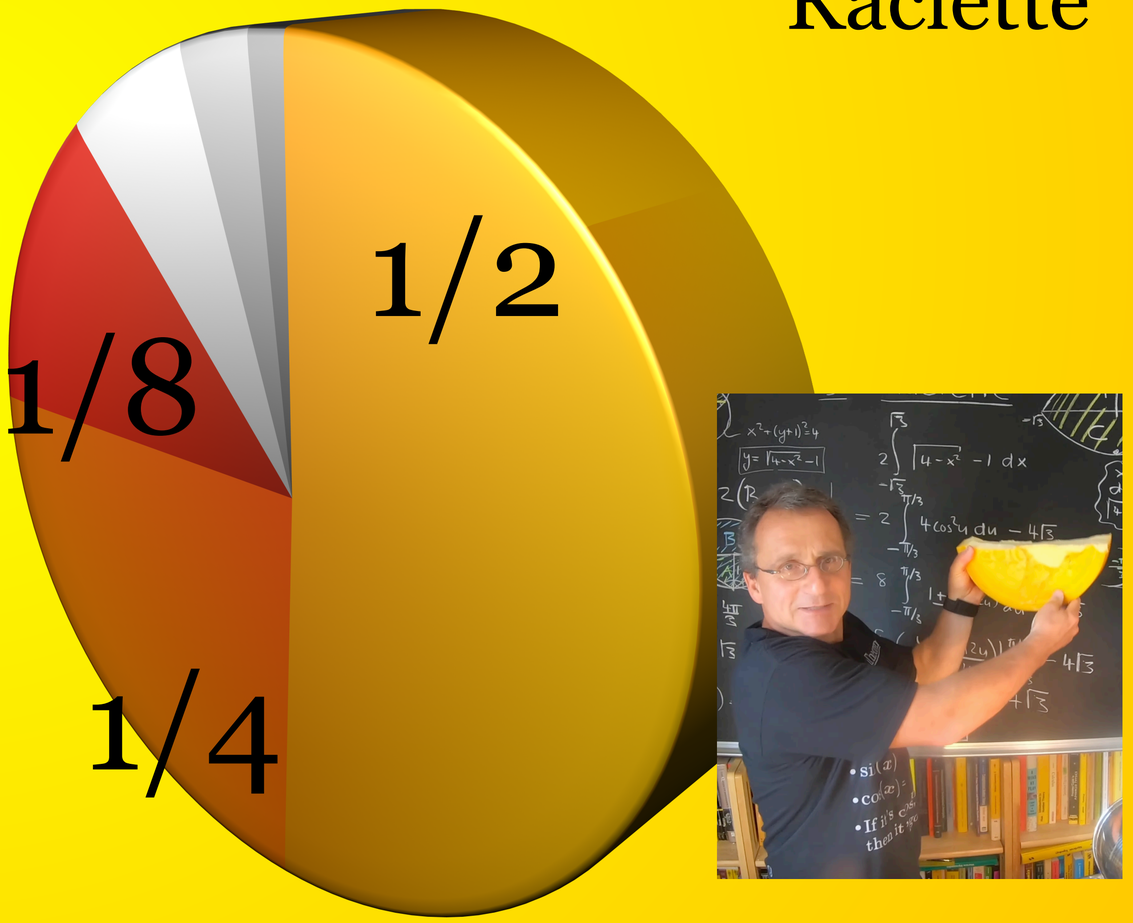


Geometric

$$s = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 2$$



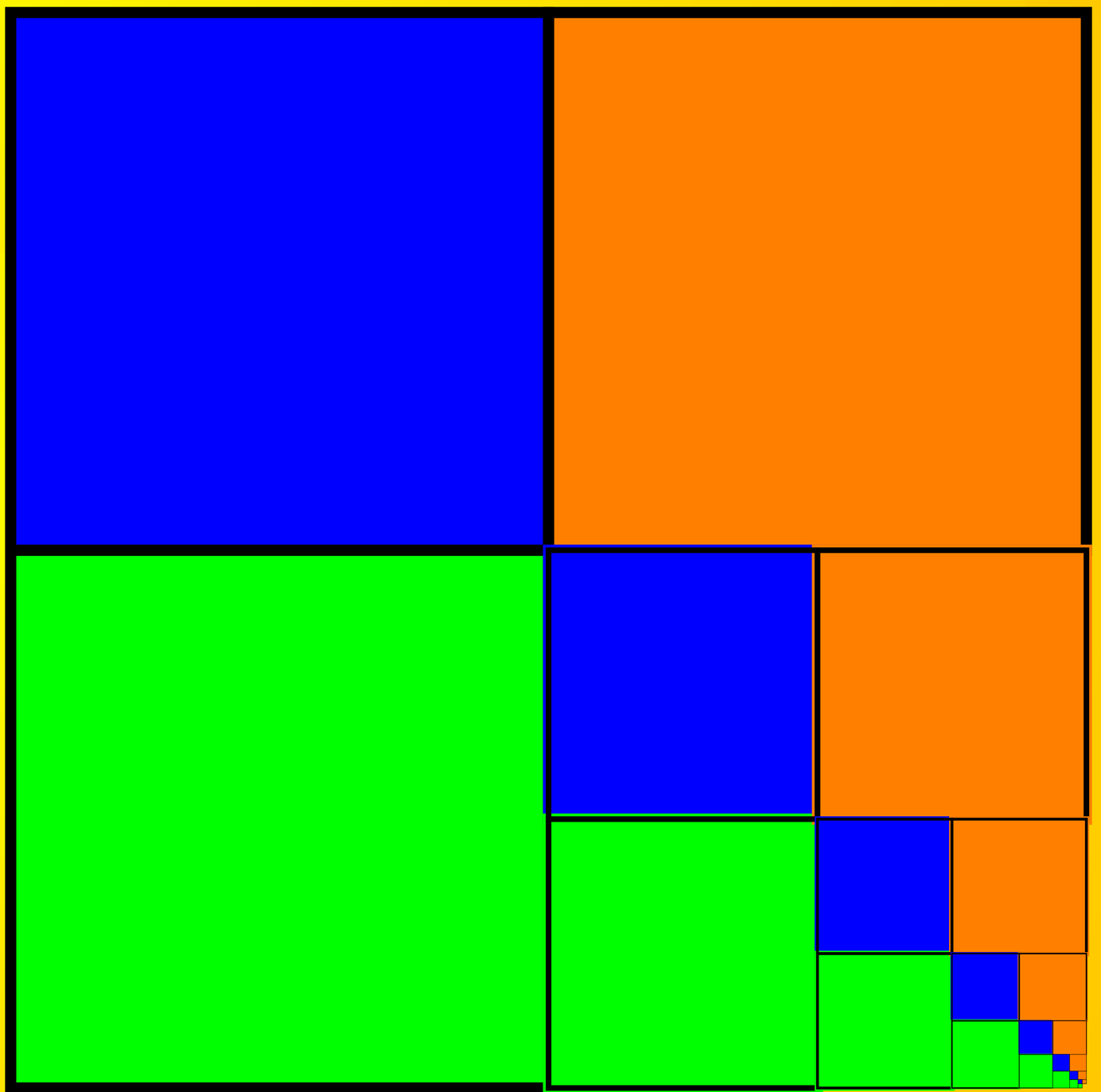
Raclette



1/4+1/16+1/64 + ...=1/3

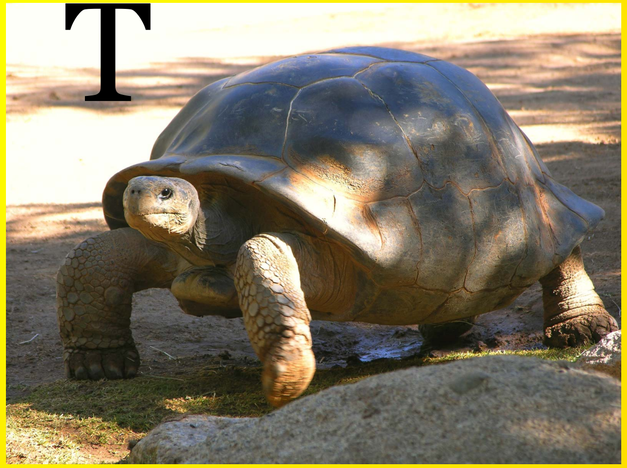
Geometric

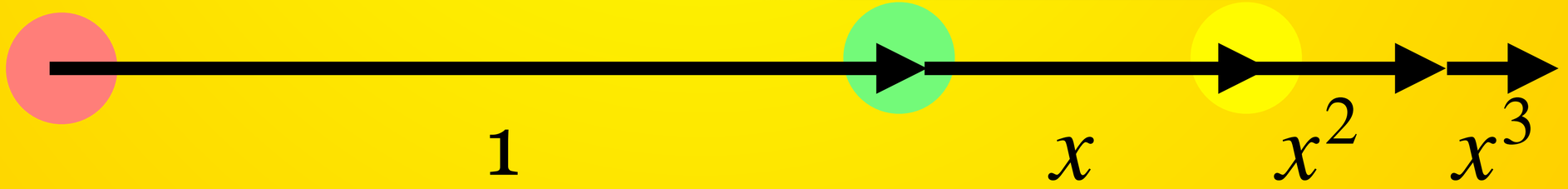
$$S = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$$



Zeno Paradox

speed(Tortois) = x * speed(Achilles)





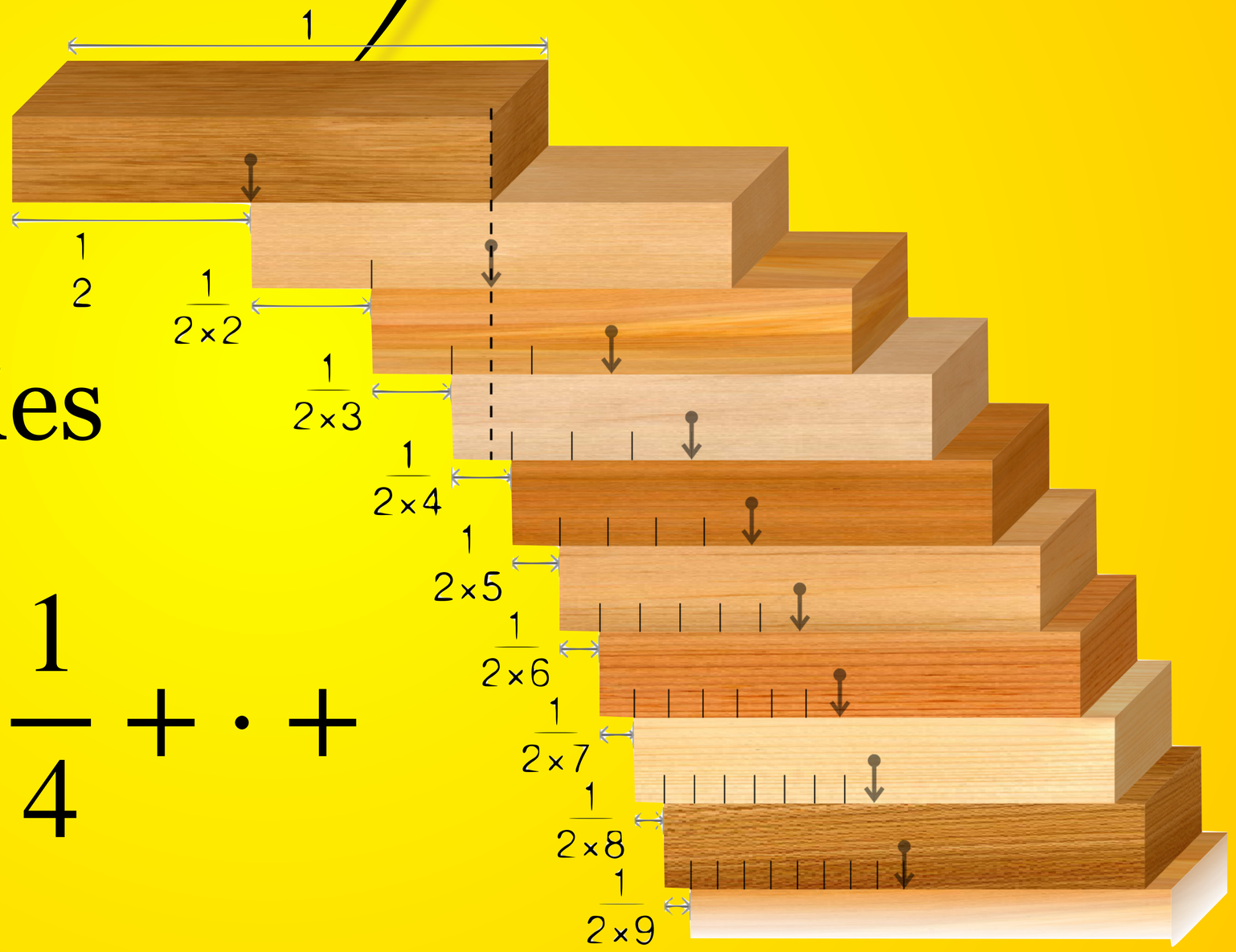
Piaget question

Larger or Equal?

$x = 1.000000000000000000\ldots$

$y = 0.999999999999999999\ldots$

Tower of Lire



Harmonic series

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdot +$$

Leibniz

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k+1}$$

$$= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

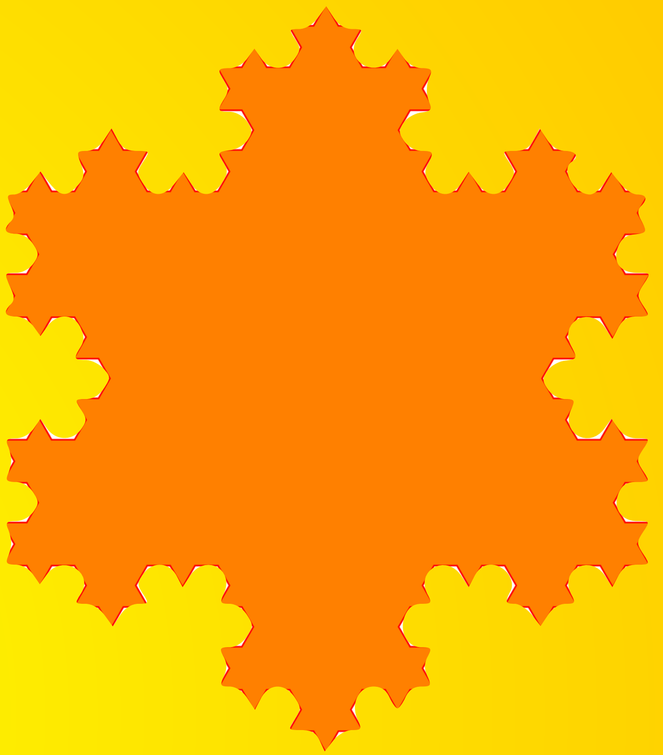
$$= \frac{\pi}{4}$$

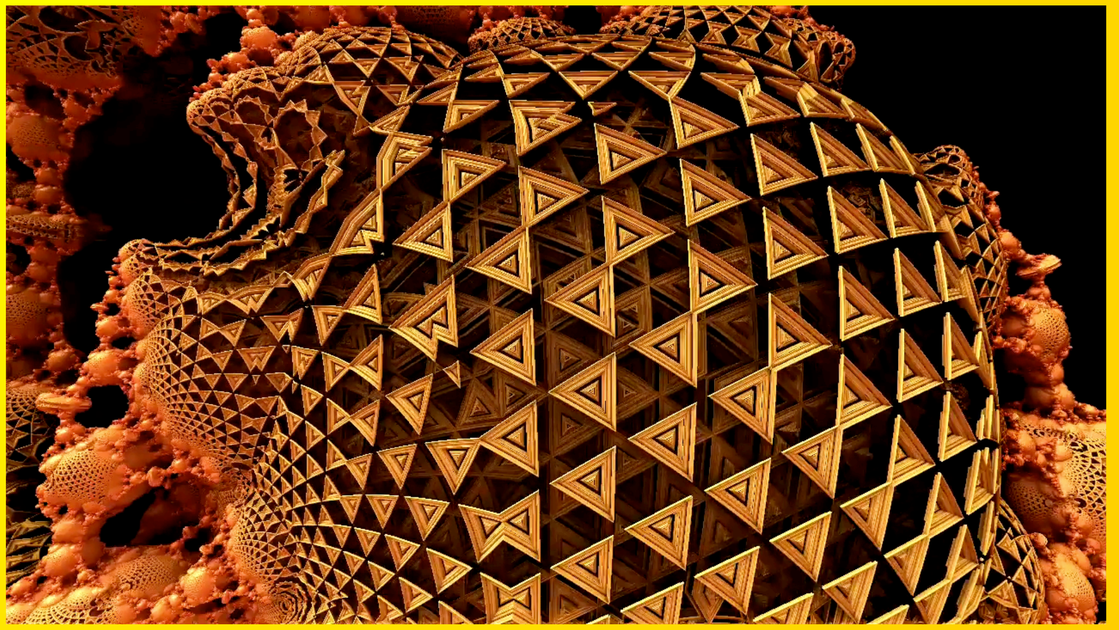
Grandi's Series

$$S = 1 - 1 + 1 - 1 + 1 - 1...$$

Luigi Grandi 1671 – 1742

Fractals

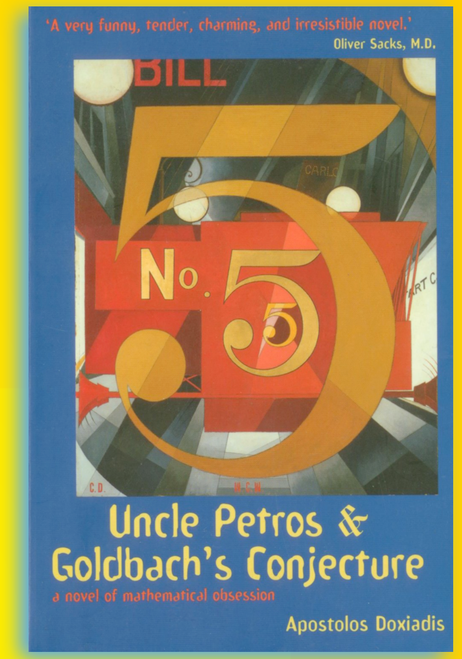




Goldbach Series

$$\sum_{p \text{ prime}} x^p = x^2 + x^3 + x^5 + x^7 + x^{11} + \ldots.$$

$$S(x)^2 = \sum_k b_k x^k \qquad \text{has non-zero } b_{2k}$$



$\mathcal{P}$ - $\mathcal{NP}$

P decision Problems

NP-decision Problems

polynomial $x^a \ll a^x$ exponential

Taylor for integration

$$\sin(x^3) = x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} + \dots$$

integrate each term!

$$\int \sin(x^3)\,dx = \frac{x^4}{4} + \frac{x^{10}}{10 * 3!} + \frac{x^{16}}{16 * 5!} + \dots$$

III Some Jokes

What did the zero say to the eight

Nice belt!

Why did the two fours skip lunch?

Because they already 8.

How do you keep warm in a cold room.

Go to a corner. It is always 90 degrees!

Why is the obtuse triangle alwauys upset?

Because it is never right!

Differential calculus professors suck.

They always go off on tangents.

Why are differential equation courses so dry?

Because the problems are all about losing liquids at varying rates.

What do you call a sudden urge to solve differential equations?

Calculust

What did one math book say to the other

Don't bother me. I've got my own problems!

What would you call this poem written in the honour of a Mathematician?

$$dy/dx - 3x = 2$$

An ODE (Ordinary Differential Equation)

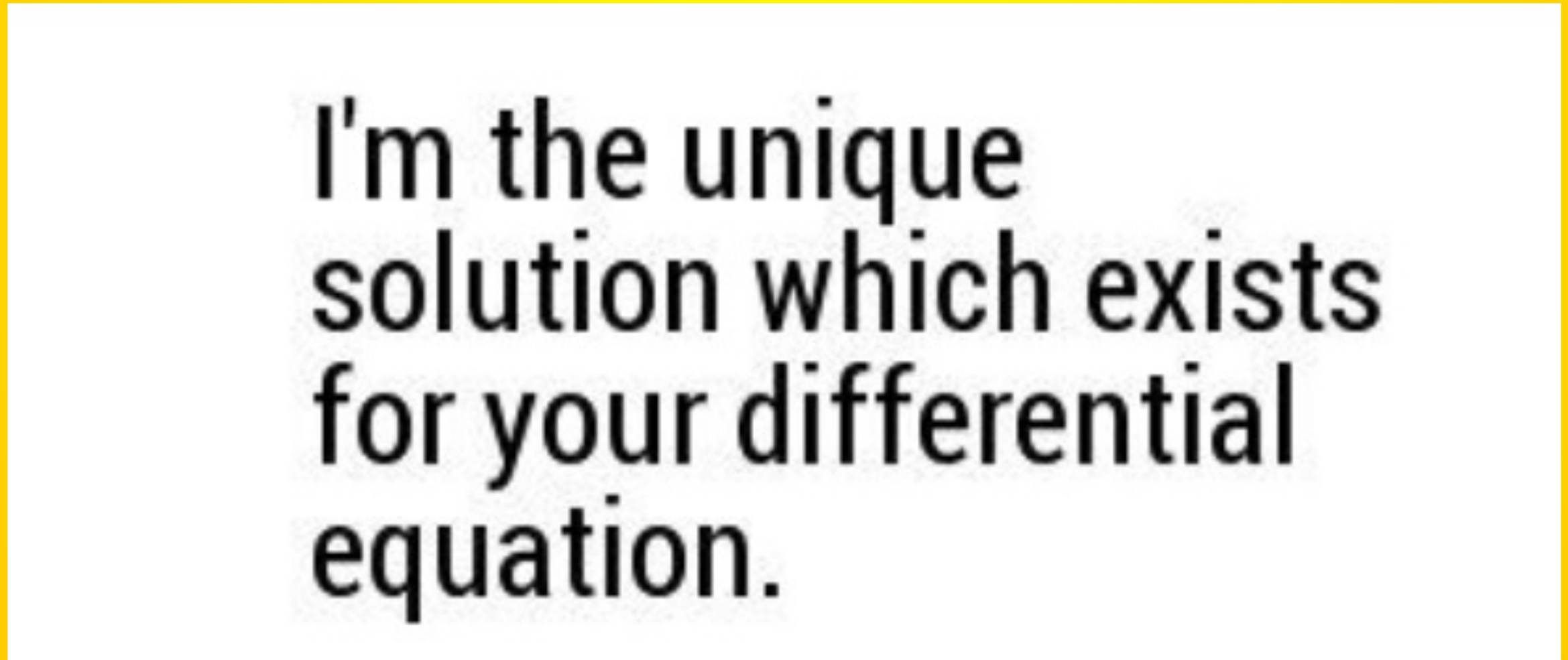
I failed my Calculus exam because I was seated between two identical twins.

It was hard to differentiate between them.

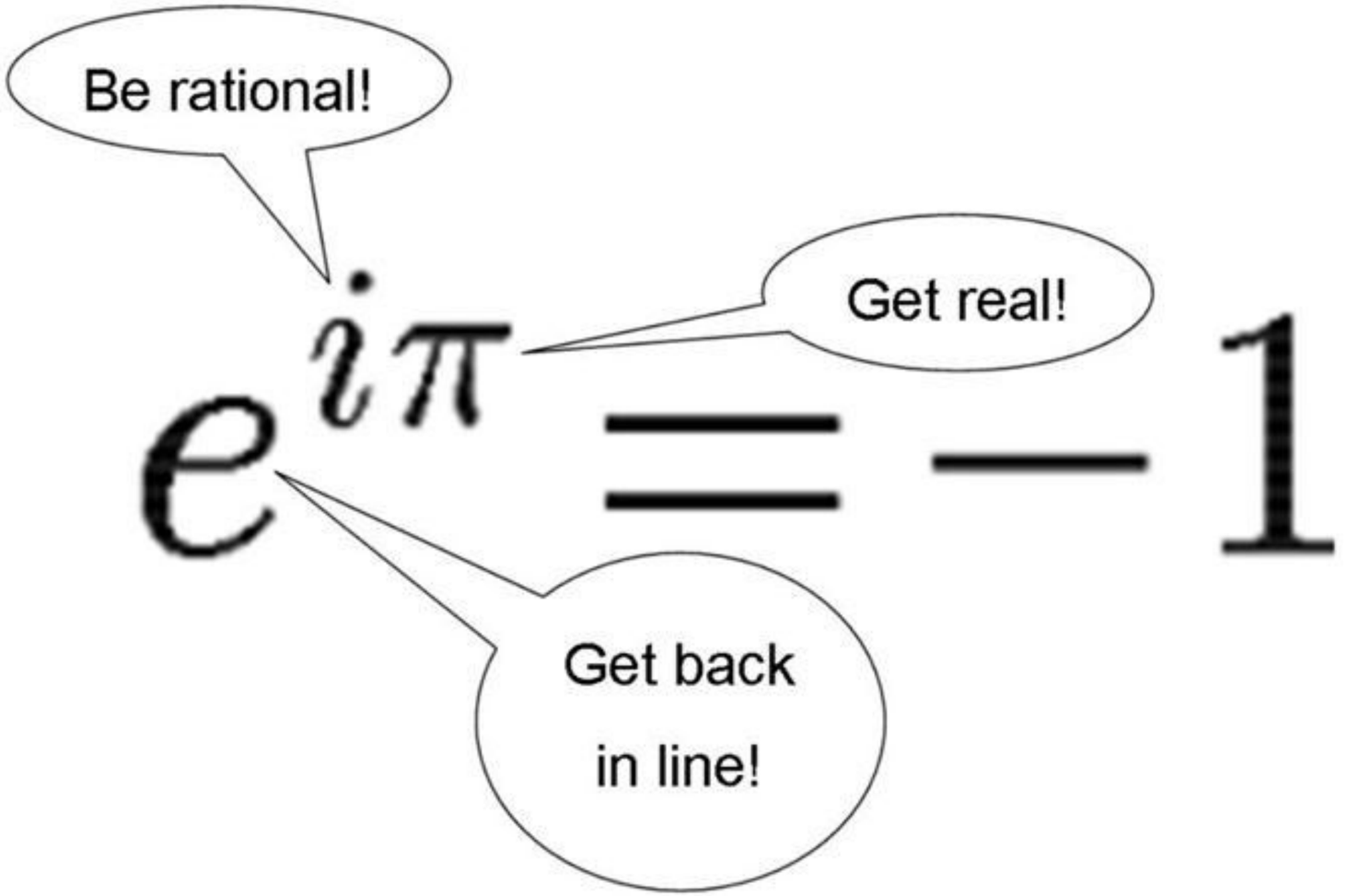
I saw my math teacher with
a piece of graph paper yesterday

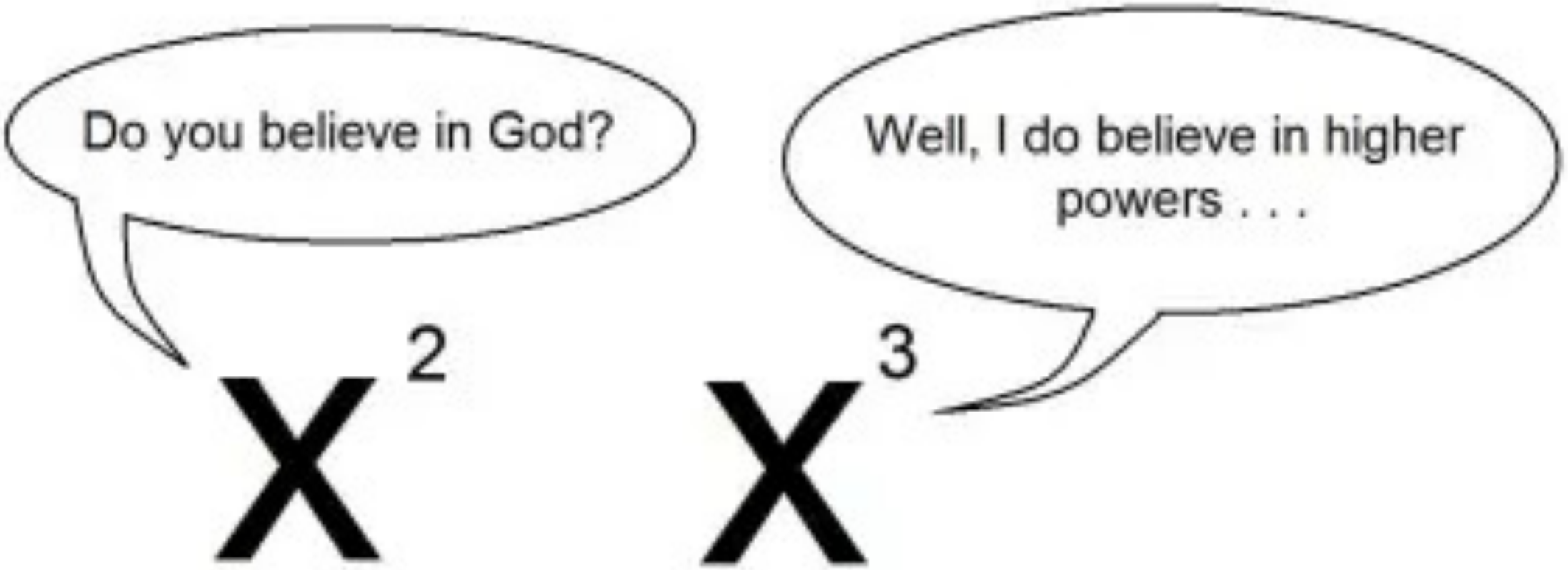
He must be plotting
something!

Not so cool pick-up line:



e, i, and π are very negative when they get together.



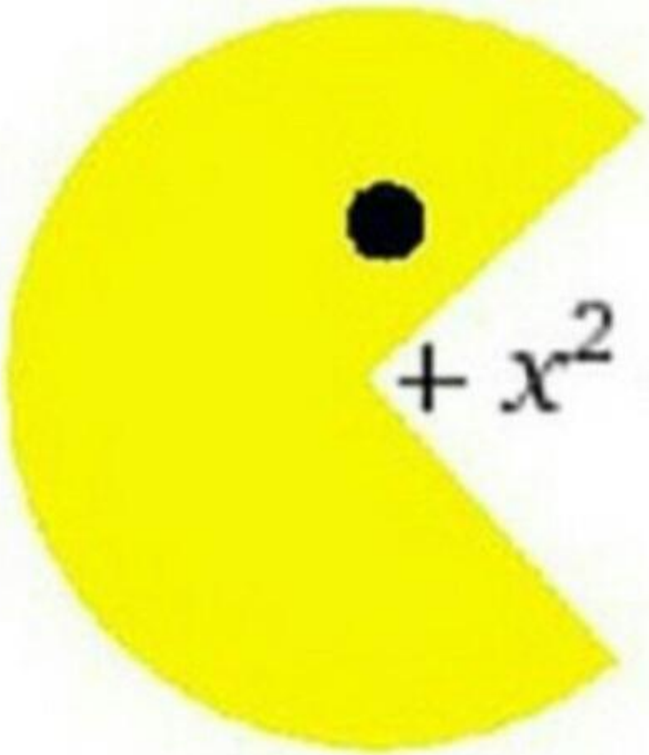

Do you believe in God?
Well, I do believe in higher powers . . .
X 2
X 3

WHAT PART OF

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r},t)\right)\Psi(\vec{r},t)$$

DON'T YOU UNDERSTAND?

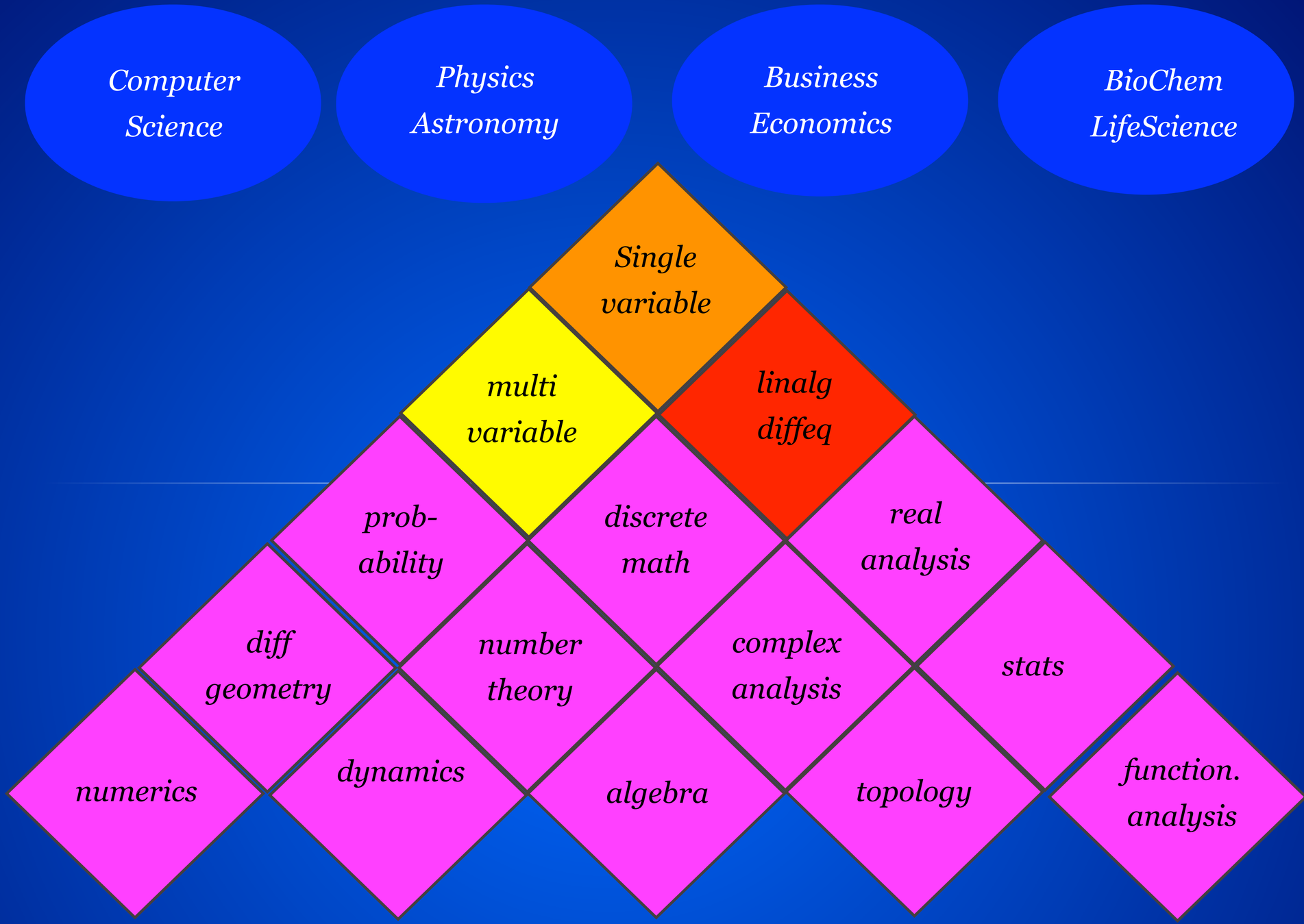
Polynom-nom-nom-nomial



$$+ x^2 + x^3 + x^4 + x^5 + \cdots$$

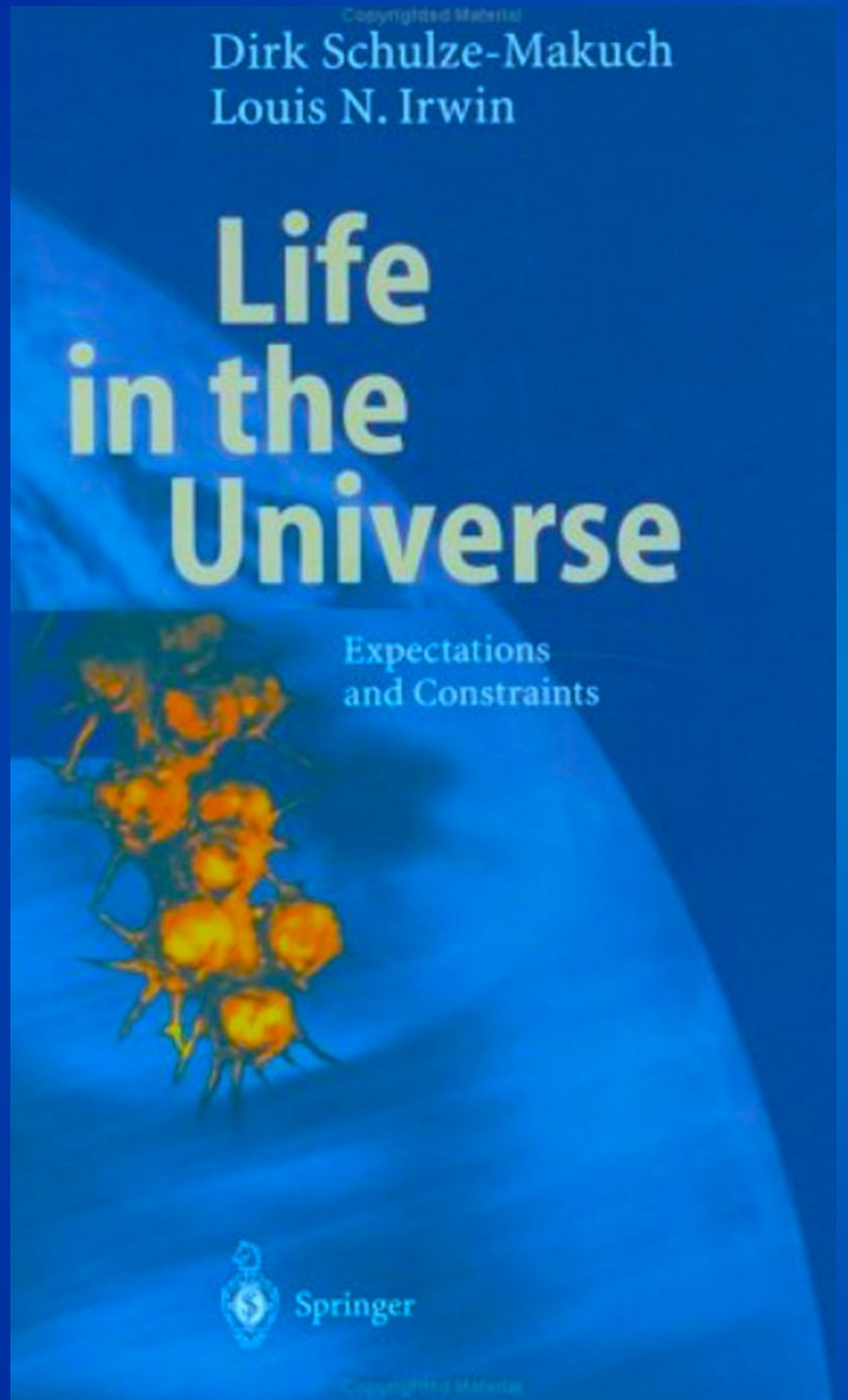
IV Practice Problems

V Beyond Calculus


Computer Science
Physics Astronomy
Business Economics
BioChem LifeScience
Single variable
multi variable
linalg diffeq
prob-ability
discrete math
real analysis
diff geometry
number theory
complex analysis
stats
numerics
dynamics
algebra
topology
function. analysis

Are there other ways to do calculus?

Contact



(1989) suggested that zeolites may also be used by silicon-based life as enzyme mimics. Still, the problem of a suitable solvent at these temperatures, as discussed for silicone, remains.

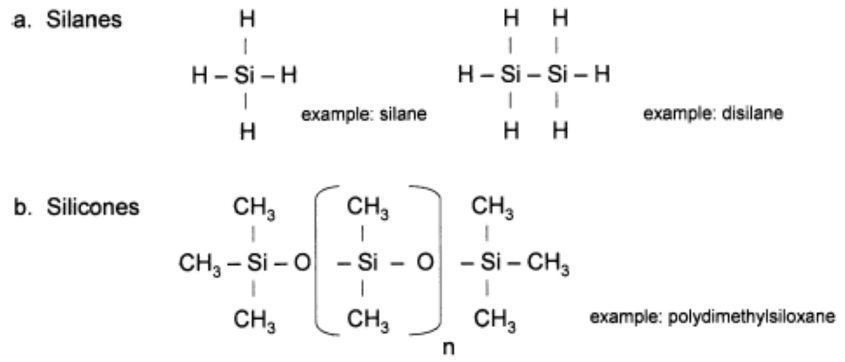
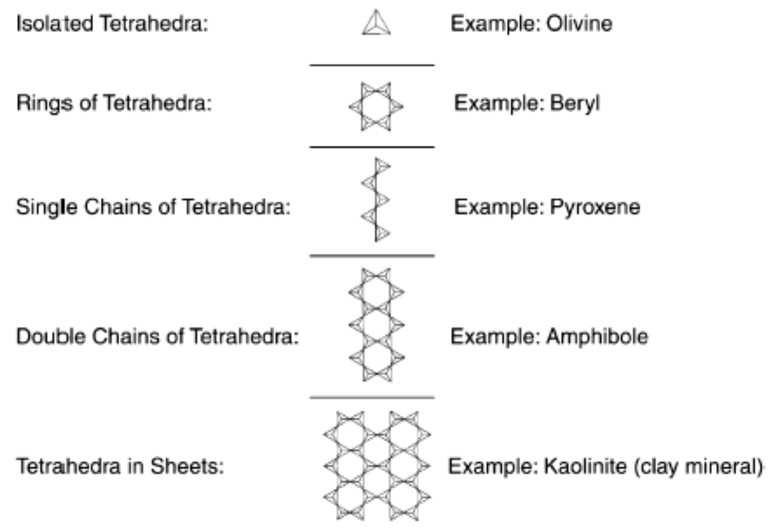
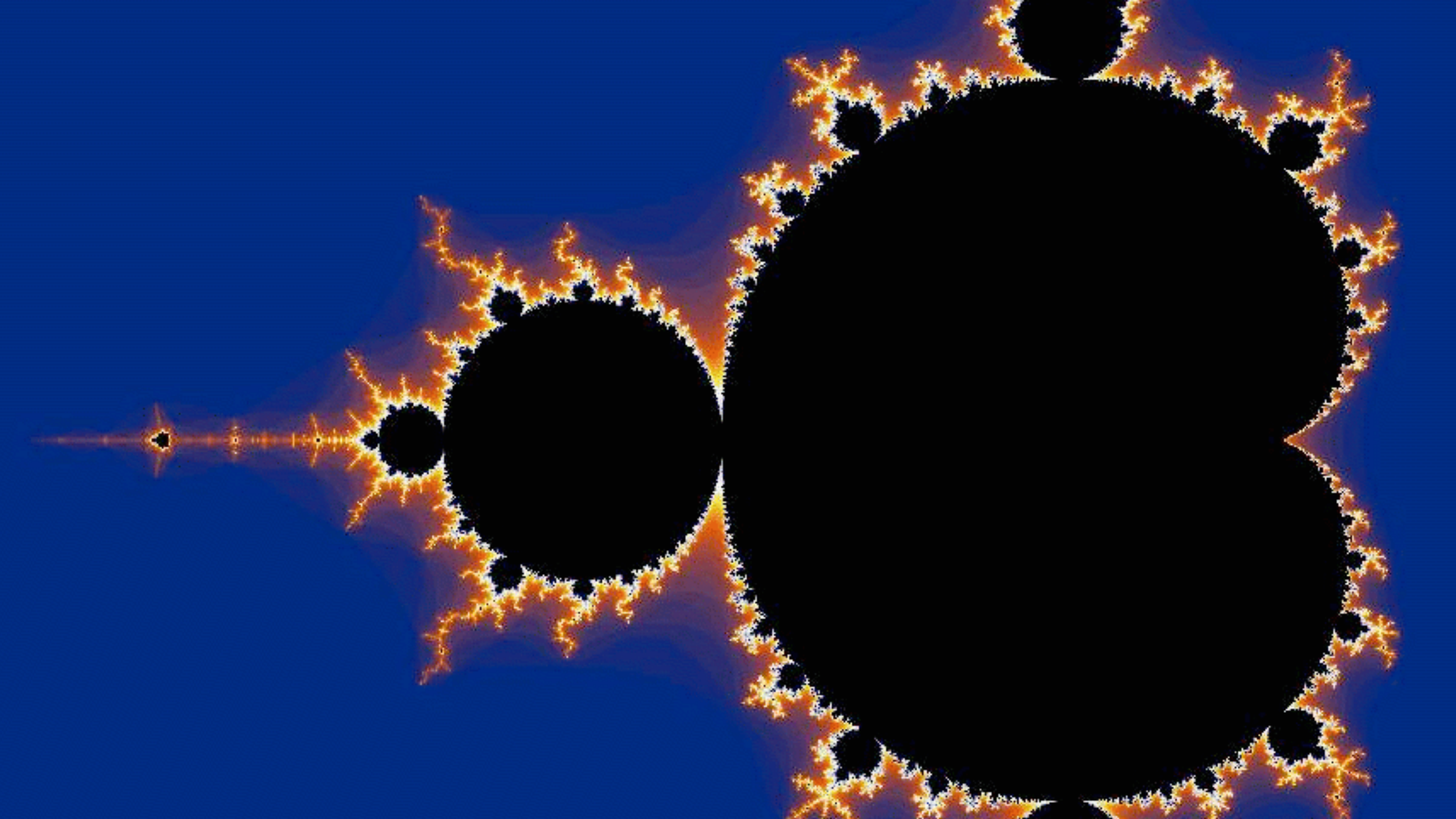


Fig. 5.4 Structure of (a) silanes, (b) silicones, and (c) silicates.

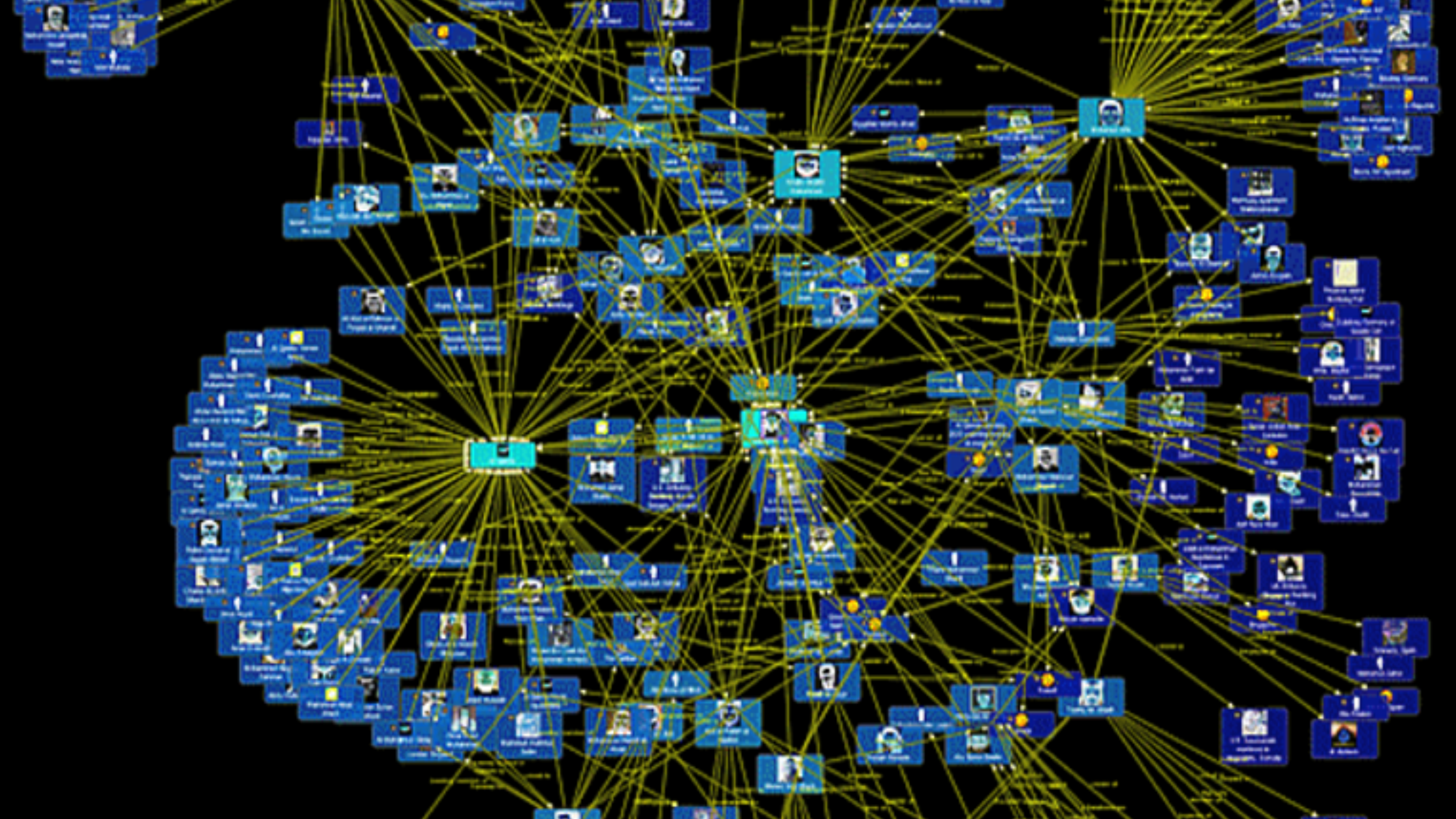


In the Complex




BASED ON AN ORIGINAL IDEA BY
Brian Price and Mike Betar

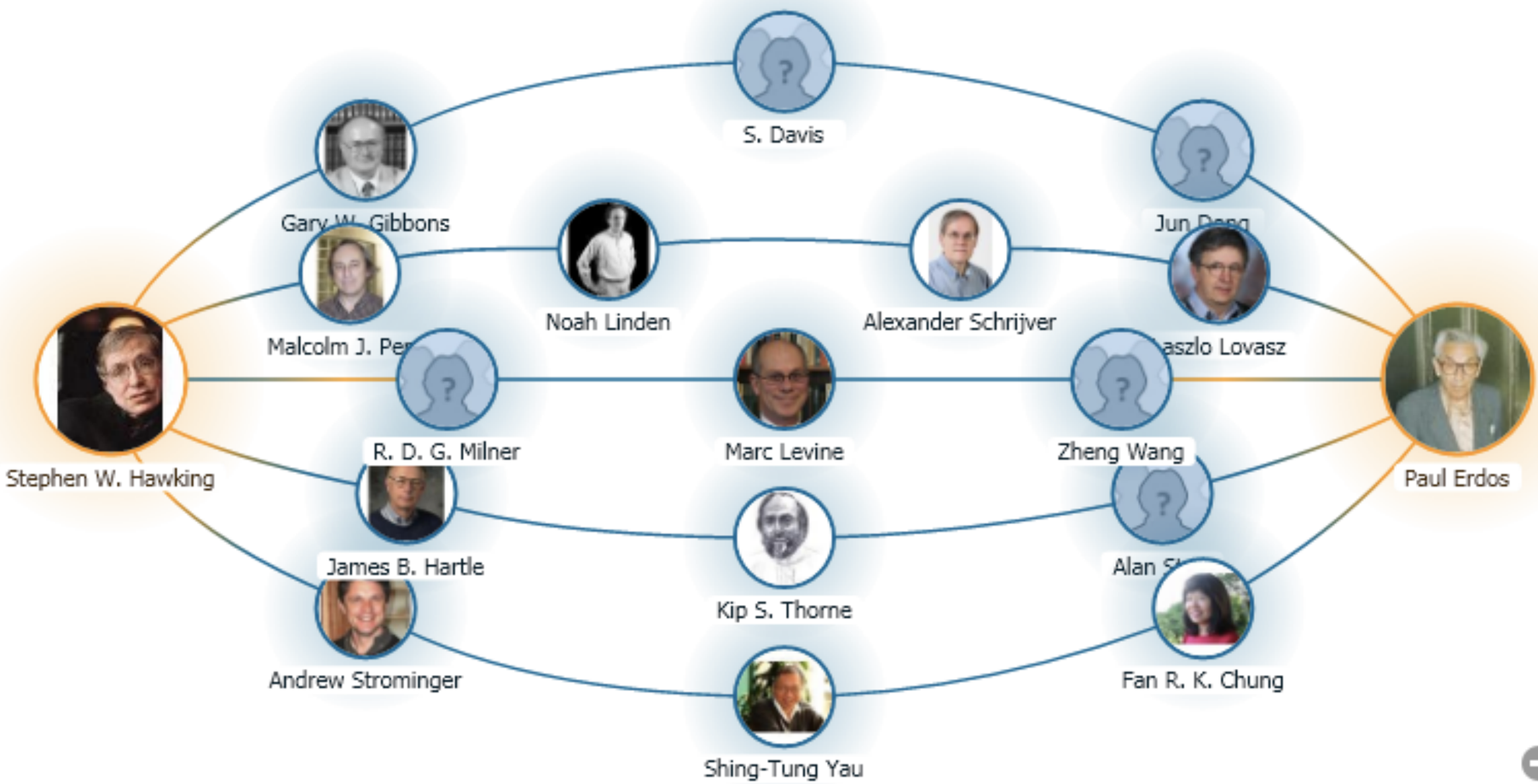
In the Discrete



Beta
emic
Search

Stephen W. Hawking > Visual Explorer

Co-author Graph | Co-author Path | Citation Graph



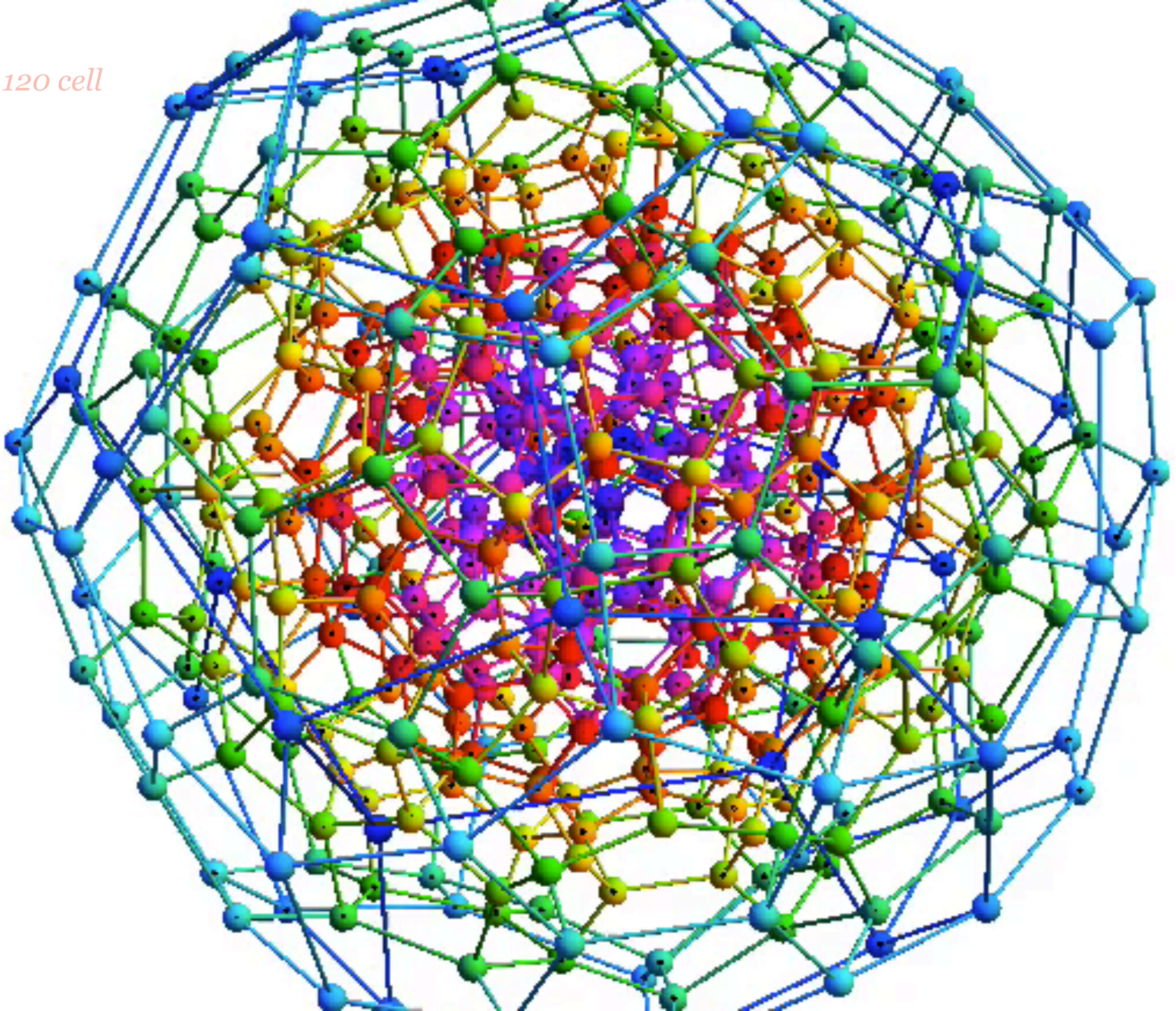
Paul Erdos

Result

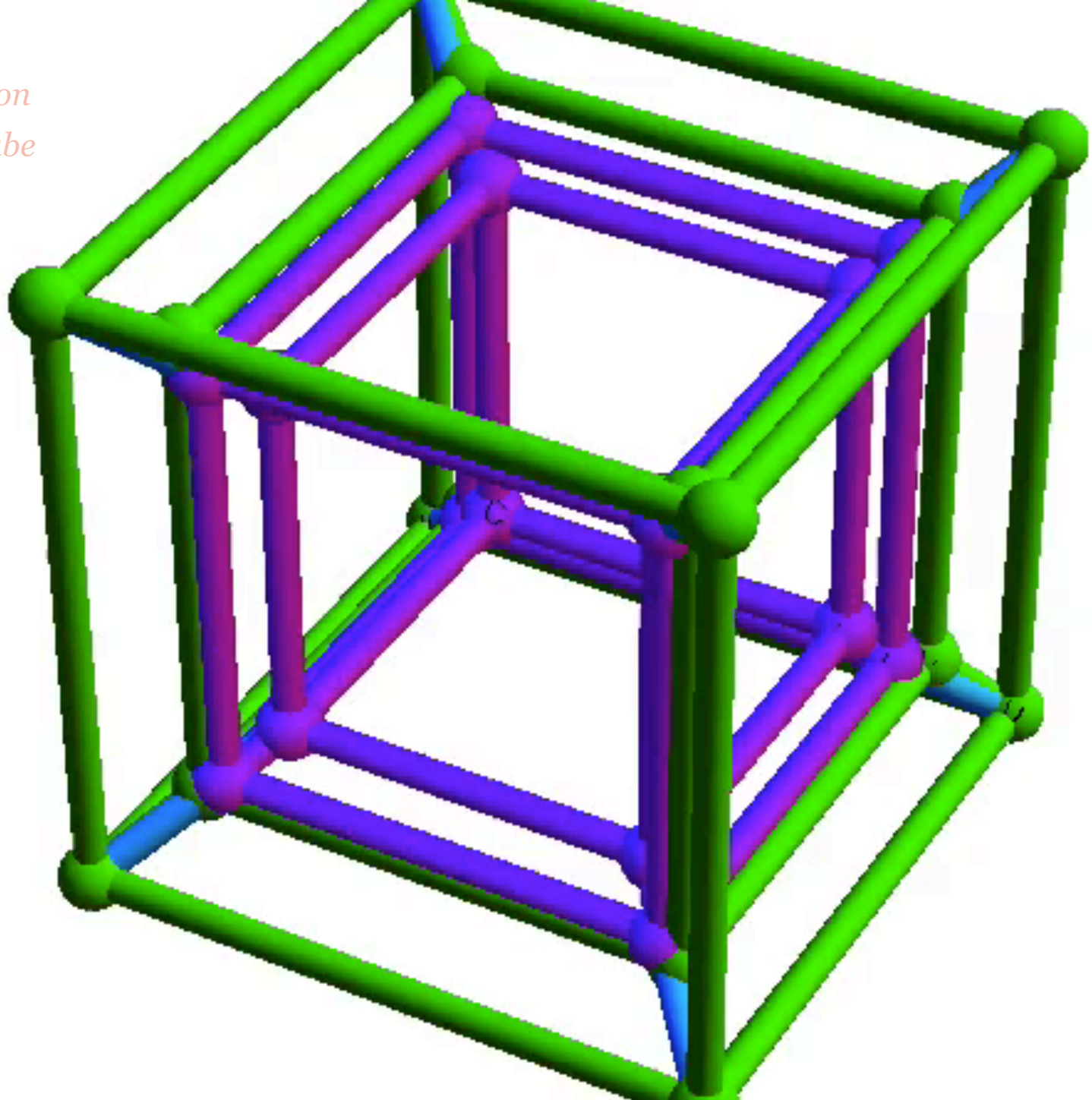
Paul Er

In higher dimensions

120 cell



Polyteron
hypercube



The End