# Lecture 3: Area and Volume

AREA

3.1. Here is an idea that goes back to the time of Archimedes. In order to compute the area of a region, we slice the region and take the limit of the Riemann sums. An important case is if the region is bound between two curves. If a function is non-negative then $\int_a^b f(x)\, dx$ is the area below the graph of f and above the interval $[a, b]$. Remember that this is a limit $\sum_{k=1}^n f(x_k)\Delta x$ with $x_k = a + k\Delta x$ and $\Delta x = (b-a)/n$.

If $f(x) \geq 0$, then $\int_a^b f(x)\, dx$ is the **area** below the graph of f.

3.2. Example: Find the area of the triangular shaped region that is bound by the cos function, the x-axis and the y-axes.
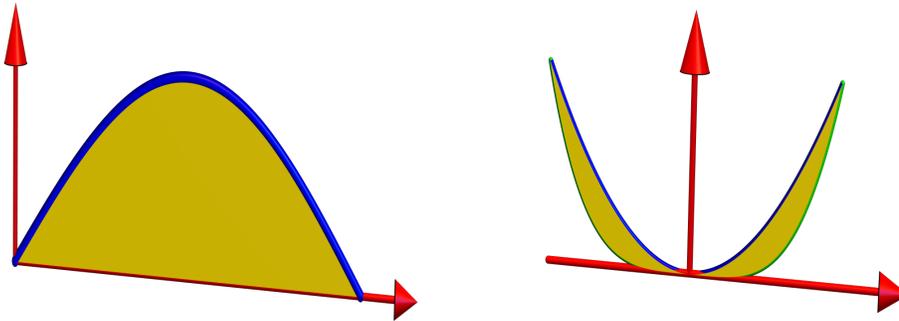
3.3.

The area of a region enclosed by two graphs $f \leq g$ and bound by $a \leq x \leq b$ is

$$\int_a^b g(x) - f(x)\, dx$$

.

3.4. The reason is that this is a limit of a sum of slice areas $(g(x_k) - f(x_k))\Delta x$ with $x_k = a + k\Delta x$ and $\Delta x = (b-a)/n$.
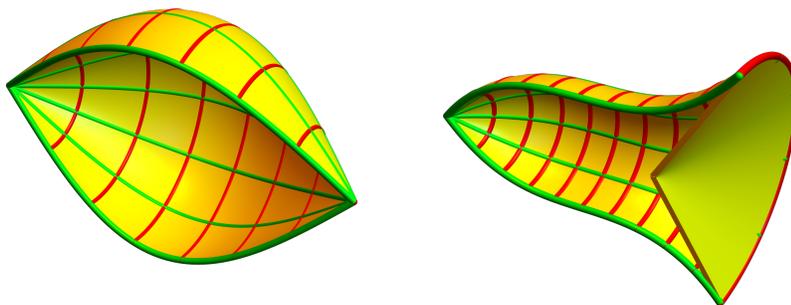
3.5. Find the area of the region enclosed by the graphs $f(x) = x^2$ and $f(x) = x^4$.

<div align="center">VOLUME</div>

3.6. The **volume of a solid** can be approximated by a sum of slices of volumes. If $A(x) = \pi f(x)^2$ is the **area of the slice** and the body is enclosed between a and b then

$$V = \int_a^b \pi f(x)^2 \ dx$$

is the **volume** of the body. The integral adds up slices of $A(x_k)\Delta x$, the volume of the slices.

3.7. Example: Rotate the function $f(x) = \sin(x)$ around the x-axes. This gives a **lemon**. **Solution:** The area of $\pi \sin^2(x)$. The integral $\int_0^\pi \pi \sin^2(x) \ dx$ is $\frac{\pi^2}{2}$.

3.8. Sometimes, it is better to slice into washers meaning **cylinder shells**. In the case of rotated sine-function, this is not so good as we have to deal with inverse trig functions and leads to a hard problem

$$V = \int_0^1 [\pi - 2\arcsin(r)]2\pi r \ dr = \frac{\pi^2}{2} \ .$$

In one of the work sheet problems, we will see that the washer method is better. The picture to the right illustrates this problem.